有限温度における閉じ込め・非閉じ込め相転移における磁気的モノポールの役割
Yang-Mills 場の新しい定式化による解析

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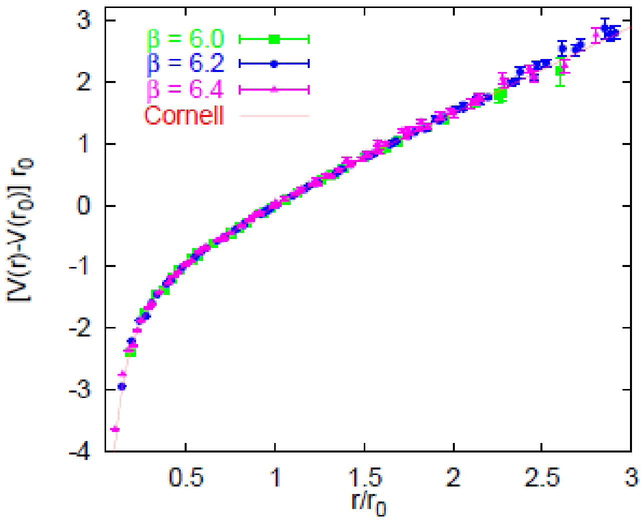
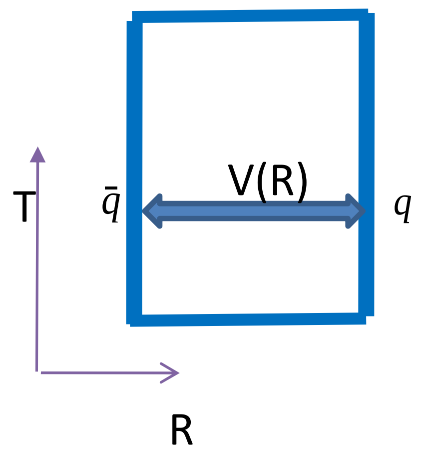
Introduction

- Important feature of QCD
  - Strong coupling gauge theory
  - Asymptotic freedom, dynamical chiral symmetry breaking
  - Quark confinement/ color confinement

$$V(r) = -C\frac{g^2_{\rm YM}(r)}{r} + \sigma r \qquad F(r) = -\frac{d}{dr}V(r) = -C\frac{g^2_{\rm YM}(r)}{r^2} - \sigma + \cdots \ (C, \sigma > 0)$$





➔The dual superconductivity picture can be promising mechanism for quark confinement

Dual super conductivity picture for quark confinement

Nambu(1974), 'tHooft(1975) ,Mandelstam(1976)

- Super conductivity (type II)
  - Cuper pair

➔Missner effect

Magnetic flux tube = monopole—monopole connections

➔linear potential between monopole-monopole

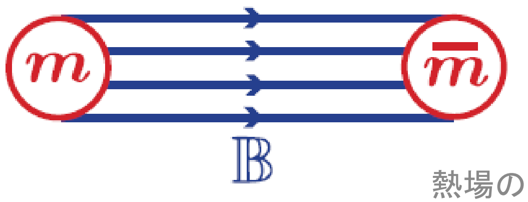
- dual super conductivity picture for Yang-mills theory
  – Magnetic monopole condensation

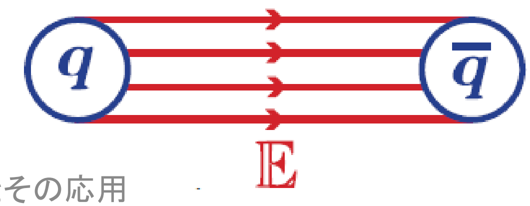
➔Missner effect

– formation of a hadron string (electric flux tube)

➔linear potential between quark-quark potential

Electro-magnetic duality





Dual superconductor picture from lattice studies

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\mathrm{no\ GF}} \sim e^{-\sigma_{NA}|S|}$,

- Numerical simulations support this picture:
  - **Abelian dominance** ⇔ $\sigma_{Abel} \simeq \sigma_{NA}\ (92 \pm 4)\%$
    - [Suzuki & Yotsuyanagi, PRD42,4257,1990]
  - **(Abelian) Monopole dominance** ⇔ $\sigma_{monopole} \simeq \sigma_{Abel}\ (95)\%$
    - [Stack, Neiman and Wensley, hep-lat/9404014],[Shiba & Suzuki, hep-lat/9404015]

SU(2) case

Abelian-projected Wilson loop $\left\langle \exp\left\{ig\oint_C dx^\mu A_\mu^3(x)\right\}\right\rangle_{\mathrm{YM}}^{\mathrm{MAG}} \sim e^{-\sigma_{Abel}|S|}$!?

Dual superconductor picture from lattice studies (cont')

- Center vortex dominance *[Greensite ,xxxxx]*

➔ These are obtained by using special gauge such as maximal Abelian gauge (MAG), Laplasian gauge, maximal center gauge.

⇔ gauge dependent,
color symmetry is broken

- **We have given a new description of the lattice Yang-Mills theory a la Cho-Faddeev-Niemi-Shabanov (CFNS) decomposition**
  - SU(2) case :*PLB645 67-74(2007), PLB653 101-108(2007)*
  - SU(3) case: *PLB669:107-118(2008),PoS(LATTICE 2008)268*

Introduction (cont')

We have given a new procedure called reduction for obtaining a gauge-independent magnetic monopole from a given Yang-Mills field.

- quark-quark potential from Wilson loop operator
- *gauge-independent "Abelian" Dominance*
- The decomposed V field reproduced the potential of original YM field.

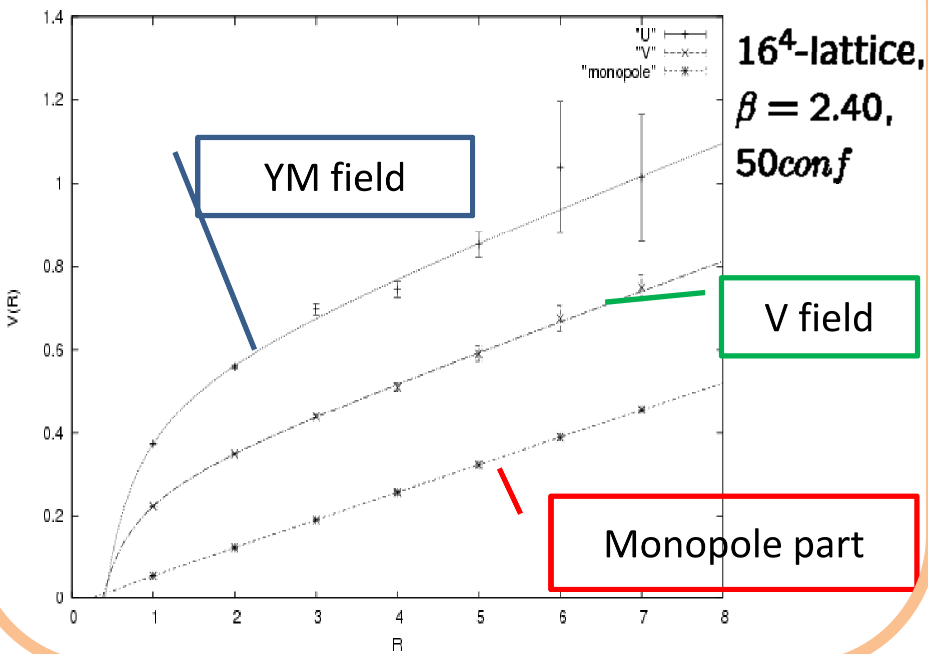
$$\sigma_{full} \sim \sigma_V \quad (93 \pm 16\%)$$

- *gauge-independent monopole dominance*
- The string tension is reproduced ole part.

$$\sigma_V \sim \sigma_{monopole} \quad (94 \pm 9\%)$$

$$\sigma_{full} \sim \sigma_{monopole} \quad (88 \pm 13\%)$$

$$V(R) = c + \frac{\alpha}{R} + \sigma R$$



Introduction (cont')

- The magnetic monopole plays a central role in quark confinement.

➔Investigation of magnetic monopoles as a quark confiner

➔study of the relations monopoles and phase transition of confinement and deconfinement

CONTENTS:

- *(gauge invariant) magnetic monopole loops and contribution to string tension*
- *magnetic monopoles in topological Yagn-Mills configurations*
- *Monopole loops in lattice data*
- *Conclutions and discussion*

Decomposition of link variable

- Can we obtain a gauge independent decomposition of the link variable U=XV, which reproduces the V"Abelian" dominance for Wilson loop?
  - V corresponds to the conventional "Abelian" part.
  - V and X transform under the SU(N) gauge transformation

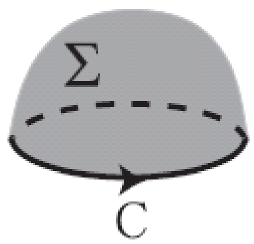
- Non-Abrelian Stokes' theorm *e.g. K.-I. Kondo PRD77 085929(2008)*

$$W_C[A] = \mathrm{tr} P \exp ig \oint_C dx^\mu A_\mu(x)/\mathrm{tr}(1)$$

$$= \int [d\mu(\xi)]_C \exp ig \oint_C V_\mu(x)$$

$$= \int [d\mu(\xi)]_\Sigma \exp ig \int_{S:C=\partial S} dS^{\mu\nu} F_{\mu\nu}[V]$$

$$V_\mu(x) = \langle \Lambda, \xi | A_\mu(x) | \Lambda, \xi \rangle$$



A new description of lattice Yang-Mills theory

PLB645 67-74(2007), PLB653 101-108(2007)
Kondo-Shibata, arXive:0801.4203[hep-th];

- Defining equation for the decomposition $U=XV$ for given link variables U and color filed $\boldsymbol{n}$

$$V_{x,\mu}\mathbf{n}_{x+\mu} = \mathbf{n}_x V_{x,\mu}$$
$$tr[\mathbf{n}_x\{X_{x,\mu} - X^\dagger_{x,\mu}\}] = 0$$

- Reduction condition defines **equipollent theorem**

$$\mathrm{n}_x = arg\,\mathrm{min}_{\mathrm{n}} F_R$$
$$F_R[\mathrm{n}, U] = \sum_x tr[\mathrm{n}_x U_{x,\mu} \mathrm{n}_{x+\hat{\mu}} U^\dagger_{x,\mu}]$$

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle\in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$
$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$
$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle\in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Wilson loop operator & magnetic monopole on a lattice

- Non-Abrelian Stokes' theorm *e.g. K.-I. Kondo PRD77 085929(2008)*

$$W_C[\mathbf{A}] = \mathrm{tr}\left[P \exp ig \oint_C dx^\mu A_\mu(x) \right] / \mathrm{tr}(\mathbf{1}) = \int [d\mu(\xi)]_\Sigma \exp\left\{ \int_{S:C=\partial S} dS^{\mu\nu} \mathcal{F}_{\mu\nu}[V] \right\}$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left\{ ig\sqrt{\frac{N-1}{2N}}(k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j, N_\Sigma) \right\}$$

$$\Xi_\Sigma := *d\Theta_\Sigma \Delta^{-1} = \delta * \Theta_\Sigma \Delta^{-1}, N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$$

D−dimensional Laplacian $\Delta = d\delta + \delta d$

Θ_Σ : the vorticity tensor with support on the surface Σ_C sppaned by Willson loop C

$$\Theta^{\mu\nu}_\Sigma(x) = \int_\Sigma dS^{\mu\nu}(X(\sigma))\delta^D(x - X(\sigma)))$$

lattice version

$$\langle W_C[V] \rangle = \langle W_C[Mono] \rangle = \left\langle \exp\left\{ i \sum_{x,\mu} k_{x,\mu} \Xi_{x,\mu} \right\} \right\rangle$$

$$n_{x,\mu} = \frac{1}{2\pi} k_\mu = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \mathcal{F}_{x,\rho\sigma}$$

$$\mathcal{F}_{x,\mu\nu} \equiv argTr[(1 + \mathbf{n}_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V^\dagger_{x+\hat{\nu},\mu} V^\dagger_{x,\nu}]$$

Property of monopoles on lattice

$$n_{x,\mu} = \frac{1}{2\pi} k_\mu = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \Theta_{x,\rho\sigma}$$

$$\mathcal{F}_{x,\mu\nu} \equiv argTr[(1 + \mathbf{n}_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V^\dagger_{x+\hat{\nu},\mu} V^\dagger_{x,\nu}]$$

- Invariant under SU(2) gauge transformation.
- Monopole currents are define as link variables on the deal lattice (shifted by a half integer for each direction.)
- They take integer values

  $n_{x,\mu}$ ={-2,-1,0,1,2}
- Current conservation:

$$\epsilon \partial_\mu n_{x,\mu} = \sum_\mu (n_{x,\mu} - n_{x-\mu,\mu}) = \sum_{\mu=\pm1,..,\pm4} n_{x,\mu} = 0$$

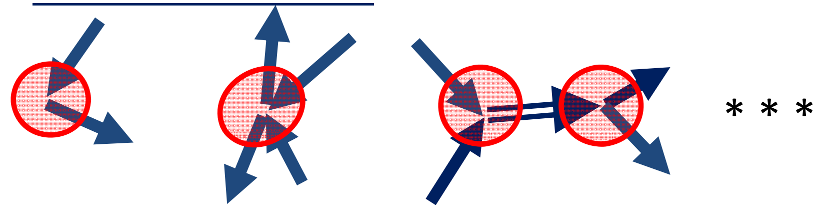
with beein $n_{x,-\mu} = n_{x-\mu,\mu}$

- Non-zero Monopole currents can be identified with geometrical objects.
  - Nonzero current ⇔ edge
  - end points (dual lattice site) ⇔ vertices
  - Sign (strength) of current ⇔ direction (waite)
- Current conservation

⇔ The same number of Incoming and outgoing links

➔ =monopole current construct loops

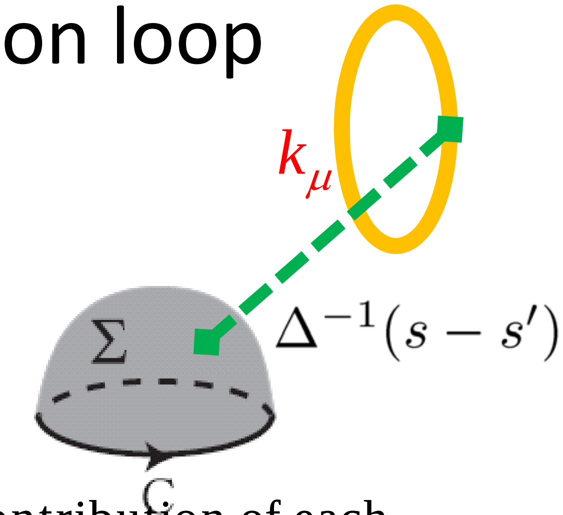
Possible verteces



Monopole contribution to the Wilson loop

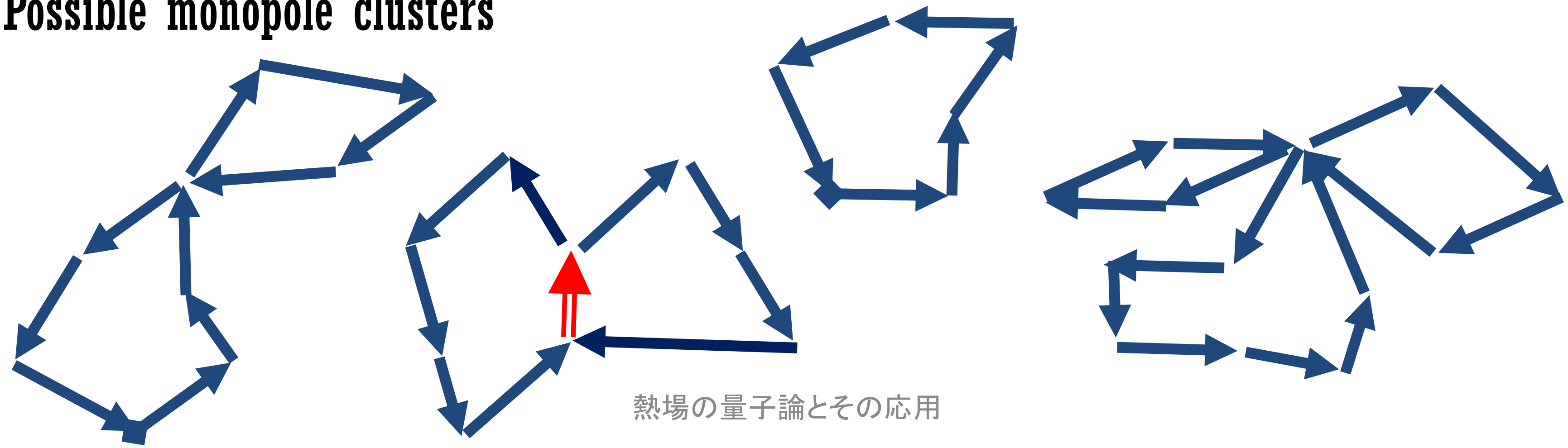
$$\langle W_C[V]\rangle \simeq \langle W_C[Mono]\rangle = \left\langle \exp\left\{ i\sum_{x,\mu} k_{x,\mu}\Xi_{x,\mu} \right\} \right\rangle$$

$$\Xi_{x,\mu} = \sum_{\sigma(y)\in\Sigma} \epsilon^{\mu\nu\alpha\beta}\partial_\nu \Delta^{-1}(x-y)\sigma^{\alpha\beta}(y)$$



- Wilson loop of the monopole part decomposed into the contribution of each monopole loop, since monopole currents are decomposed into loops.
- The small monopole give zero contribution, since integral by opposite direction of current canceled each other. ➔The large cluster of monopole loops contribute to the Wilson loop.

Possible monopole clusters



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Monopoles from topological configurations in the SU(2) Yang-Mills theory

- Investigation of monopole loop generated from some of known topological configurations in the SU(2) Yang-Mills theory such as merons and instantons
- which are characterized by the gauge-invariant topological index,
  - **topological charge (density) , magnetic charge (density)**

- Link variable is calculated by path ordered product of the parallel trans porter .

$$U_{x,\mu} = \mathcal{P}\exp\left(-ig\int_x^{x+\epsilon\mu} A_\mu(x)dx^\mu\right) = \mathcal{P}\prod_{j=0}^{N} U_{x,\mu}^{\epsilon/N}(j)$$

$$U_{x,\mu}^{\epsilon/N}(j) = \exp\left(\frac{-ig\epsilon}{2}\left\{A_\mu(x+\frac{j}{N}\epsilon\mu) + A_\mu(x+\frac{j+1}{N}\epsilon\mu)\right\}\right)$$

- The lattice is set up as finite volume box with open boundary condition.
  - Inside the box link variable is set up by solution

  Color field satisfy reduction condition

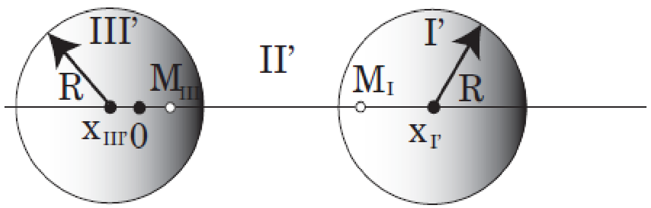
  $$\mathbf{n}_x = arg\min_{\mathbf{n}} F_R$$
  $$F_R[\mathbf{n}, U] = \sum_x tr[\mathbf{n}_x U_{x,\mu}\mathbf{n}_{x+\hat{\mu}} U_{x,\mu}^\dagger]$$

  - Outside the box $U_{x'}=1$ (V=1)

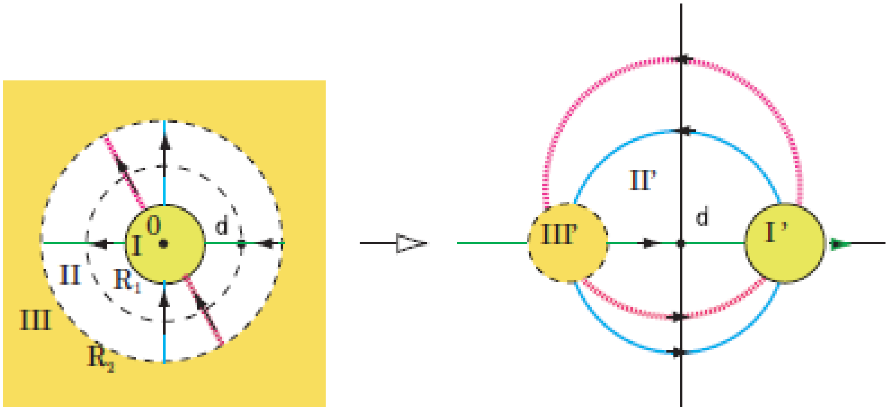
    $n_{x'}=n_x$ ($n_{x'}$ out side of box is parallel to boundary value n) to satisfy decomposition condition nV=Vn'

Two-meron Phys.Rev.D78:065033,2008.

$$\mathbf{A}^{II}(x) = \frac{\sigma^A}{2}\eta^A_{\mu\nu}\left\{\frac{(x-x'_I)_\nu}{(x-x'_I)^2} + \frac{(x-x'_{III})_\nu}{(x-x'_{III})^2}\right\}$$



$$\mathbf{A}^{I/III}(x) = \frac{\sigma^A}{2}\eta^A_{\mu\nu}\frac{(x-X')_\nu}{(x-X')^2}$$



Analytical solution

•To make action finite, we use the smeared solution with the instanton cap of size R.

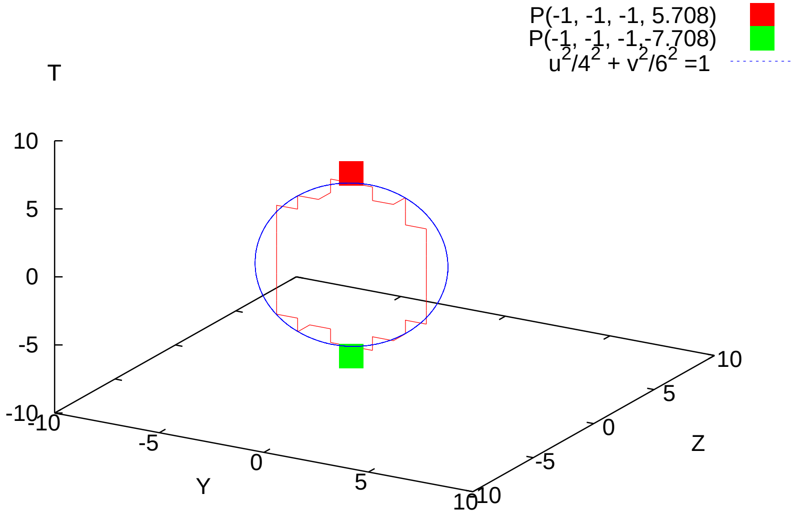
•Using conformal transformation, two-meron configuration can be transform to the concentric circle.

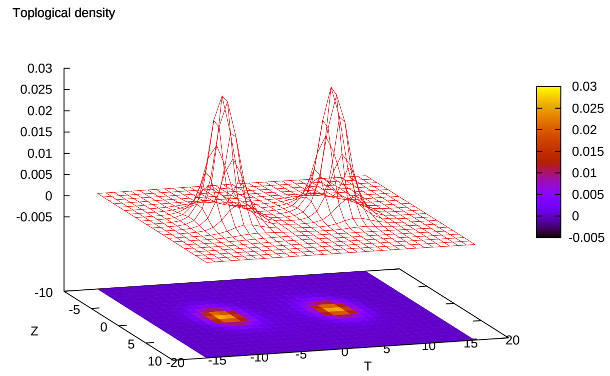
•Then reduction is condition can be solved analytically using spherical surface harmonics.

• The monopole current is given by line passing through center.

•This solution is corresponding circle passing through region II.

Monopoles from two-meron





The plot of a magnetic-monopole loop generated by a pair of (smeared) merons in 4-dimensional Euclidean space where, the gap of the energy between region II and I/III smoothed by using sooling method. The 3-dimensinal plot is obtained by projecting the 4-dimensional dual lattice space to the 3-dimensional one, i.e., (x; y; z; t) → (y; z; t). The positions of two meron sources are described by solid boxes, and the monopole loop by red solid line. In the lattice of the volume [-10, 10]3 × [-16, 16] with a lattice spacing 2 = 1, the two-merons are located at (-1; -1;-1; -1±6:078), and are smeared with the instanton cap of size R = 3:0 (d = 12, R1 = 2:833 and R2 = 50:833). The monopole loop is confined in the 3-dim. space x = -1 and in a 2-dim. plane rotated about t-axis by 0.46rad. (For guiding the eye, the monopole loop is fitted by an ellipsoid curve (blue dotted line) with the long radius 6 and the short radius 4.)

Two instanton

- Two types of solution is known,

$$A^B_\mu(x) = \eta^B_{\mu\nu}\partial_\nu log\phi(x)$$

  - ’t Hooft type $\phi(x) = 1 + \sum_{k=1}^{2} \frac{\rho_k^2}{(x-x_k)^2}$
  - JNR type $\phi(x) = \sum_{k=0}^{2} \frac{\rho_k^2}{(x-x_k)^2}$

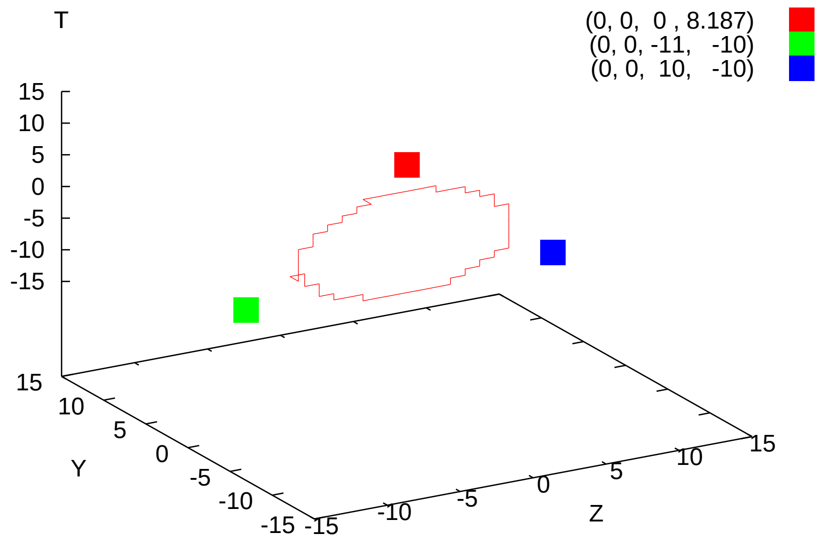
- It is hard to solve the reduction condition analytically

➔We apply our numerical method.

- There are several works using mulit-instanton configurations.
- However, these configurations can not always satisfy the Yang-Mills equation.
- Here, we use the solutions of YM equation,
- and investigate weather magnetic monopoles appear.

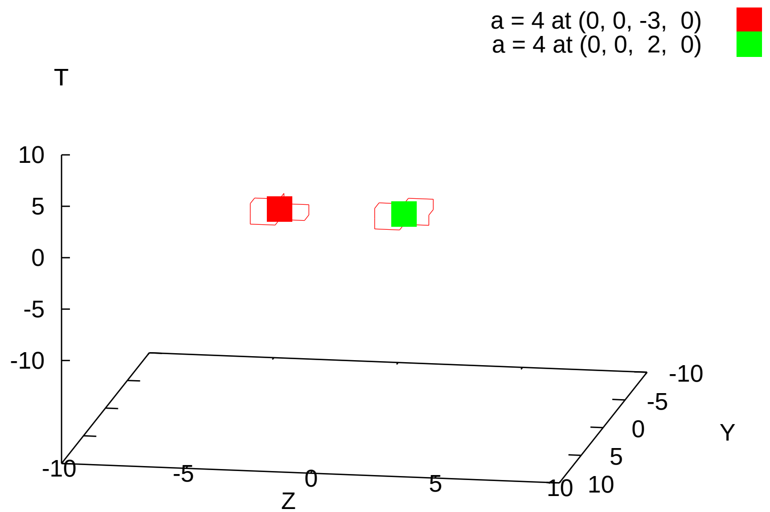
➔ We shows example where magnetic monopole loops appear

Two-instanaton JNR type



- The 3-dimensional projection, (x,y,z,t) → (y,z,t), of a magnetic-monopole loop generated by two-instanton of JNR type in 4-dimensional lattice $[-15,15]^2 ¥times [-30,30]$ with its spacing $¥epsilon=1$. the magnetic monopole is wriiten by a red solid curve and two-instanton solution is parametrized by the "size" and the "position" denoted by solid boxes; a = 4 at (0, 0, 0, 10.851), a = 4 at (0, 0,-13. -10.9), a = 4 at (0, 0, 12, -10.9).

Two-instanton(‘t Hooft type)



The monopole loop from two-instanton ('t Hooft type), which is projected into 3-dimensional dual lattice space, i.e. (x,y,z,t)=> (y,z,t). The monopole loop is plotted by red curves, and positions of the instanton source are by boxes. The two-instanton is parametrized as a = 4 at (0, 0,-3. 0) a = 4 at (0, 0, 2 0), and placed in [-12,12]^2 ¥times [-20,20]^2 lattice with its spacing ¥epslon = 1.

Calorons

- Finite temperature: Matsubara formalism

finite size and periodic boundary for temporal direction

$$\int_{-\infty}^{\infty} dt \int d^3x \quad \rightarrow \quad \int_0^{1/T} dt \int d^3x$$

- *YM equation for instanton should be modified to have Periodic boundary condition*

- ➔ *Caloron* $A_\mu^B(x) = \eta_{\mu\nu}^B \partial_\nu log\phi(x)$

$$\phi = 1 + \sum_{k=1}^{N} \frac{\rho_k^2}{2r} \frac{\sinh(r_k)}{\cosh(r_k) - \cos(\tau_k)} \text{ ('t Hooft type)}$$

$$\phi = \sum_{k=0}^{N} \frac{\rho_k^2}{2r} \frac{\sinh(r_k)}{\cosh(r_k) - \cos(\tau_k)} \text{ (JNR type)}$$

$$r = 2\pi N|x - x_k|/\beta$$

$$\tau = 2\pi N(t - t_k)/\beta$$

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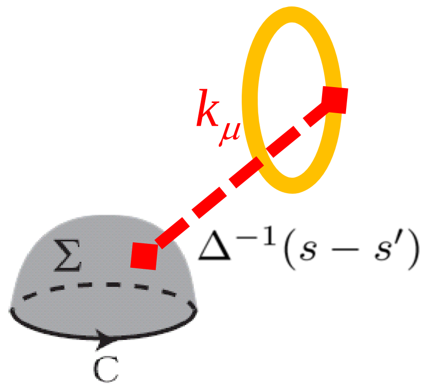
Application to lattice data

- Detecting which type of topological configurations exist in the lattice data involving magnetic-monopole loops generated by Monte Carlo simulation.

➔ Detecting the magnetic monopole loops and discriminating each closed loop from clusters of magnetic-monopole current

➔ Evaluating contrition of monopole loops to Wilson loops

  – Linking numbers of Wilson loop and monopole loop



➔ Evaluating topological index (density) in lattice data.

➔ Comparing with known topological configurations of Yang-Mills theory and lattice data in terms of magnetic monopole charge density and topological index density.

Discriminating each monopole loop from lattice data

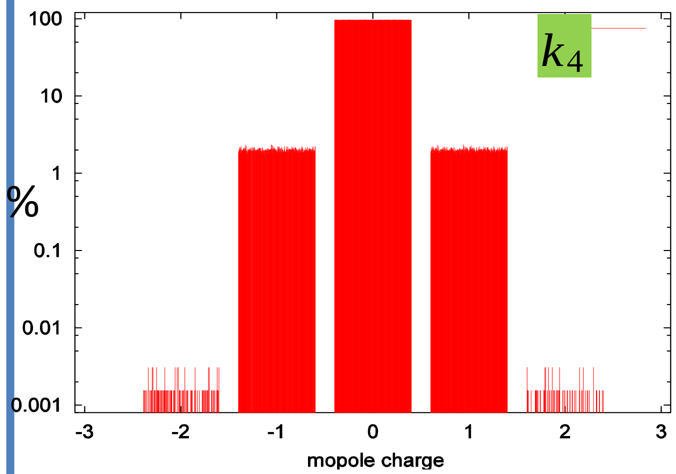
- Discriminating each closed loop from clusters of magnetic-monopole clusters.
  - huge number of non-zero current :
  - Need of computational algorithm
- Applying "computational homology" (chomp software) based on an algebraic topology algorithm .
  - monopole currents can be represented by edge of graph in 4D-space (4D-torus)
  - By using chomp one can compute Betti number and generators, very fast.

  ➔ Monopole loops are distinguished by generators of 1-dimensional Betti number.

Betti number means,

β_0 : the number of connected graph

β_1 : the number of holes (loops)



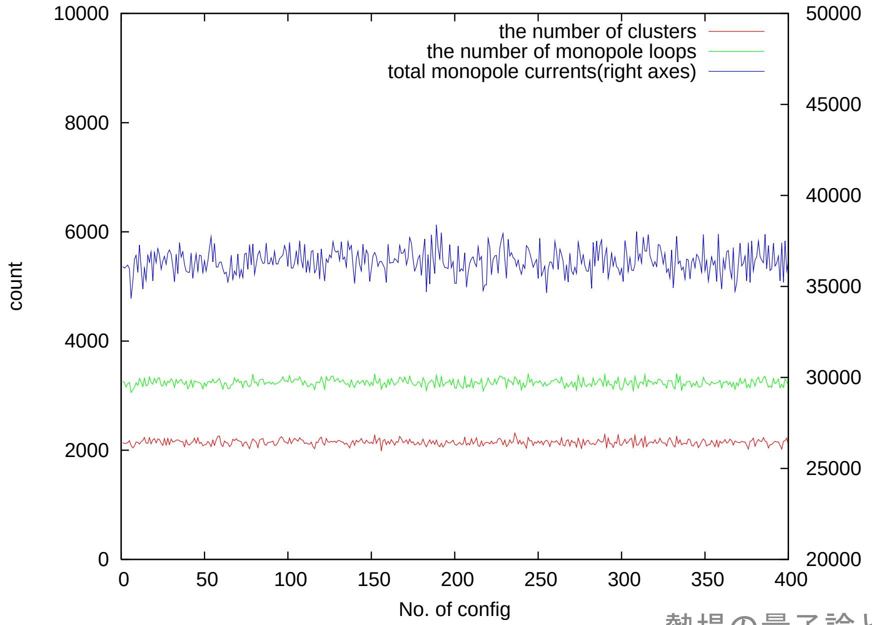
Distribution of monopole charges for 24^4 lattice (β=2.4, Wilson standard action)

About 3% current link variables have non-zero vales.

measurement of monopoles loops

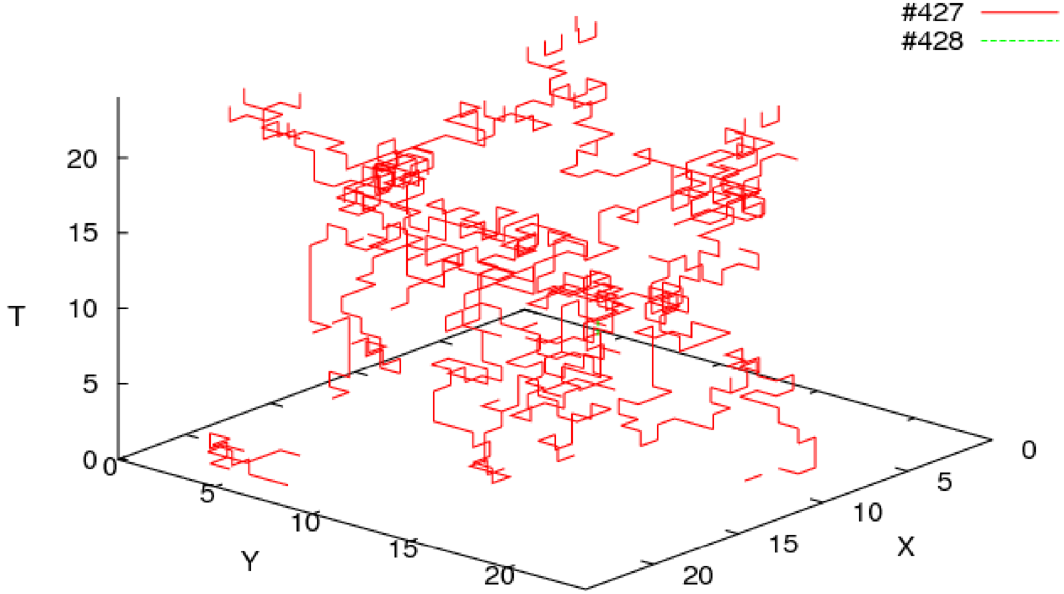
- Invetigate monopoles which contribute to Wilson loops
  - Monopole charge density
  - What kind of monopole loops are realized, check winding monopoles.
  - Distribution of monopole loops/clusters and their size

preliminary

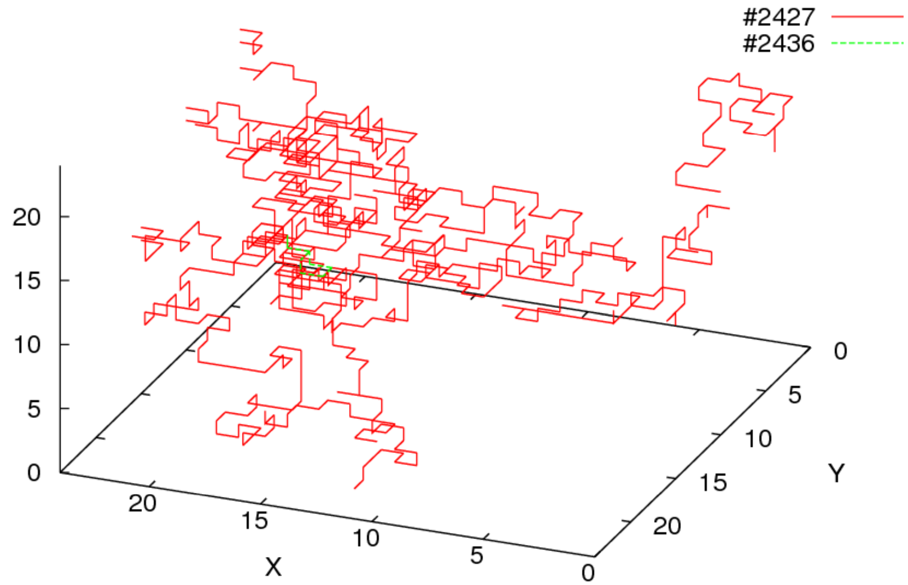


•Number of monopoles currents (blue) for each configuration
•The number of clusters (red)
•The number of loops(green)

Examples of long monopole loops



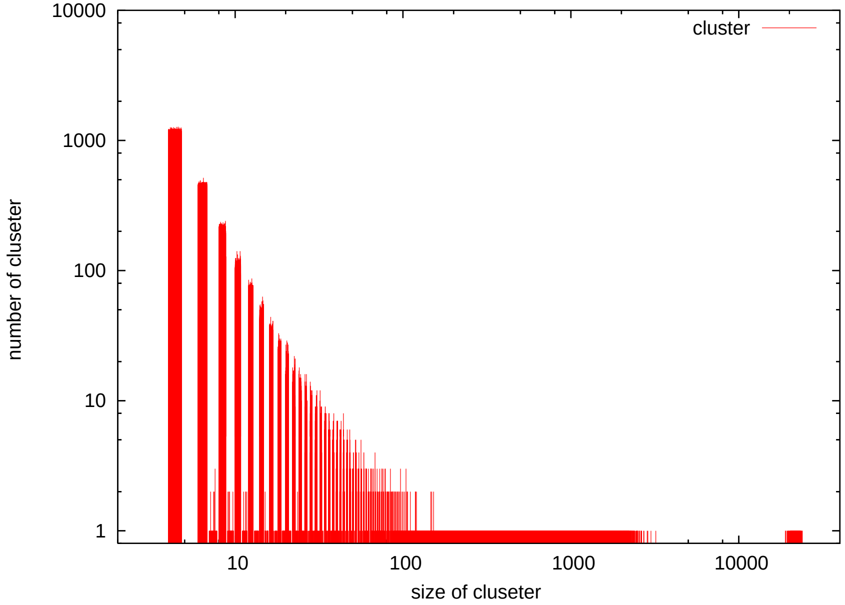
Monopole loops are plotted in 3-dimensional space (24^3 lattice with periodic boundary condition) by projection from 4D space (x,y,z,t) to 3D space (x,y,t) .



Monopole loops & clusters

preliminary

Cluster size v.s. Counts



Loop length v.s. counts