Thermalization of scalar and gauge theory with off-shell entropy

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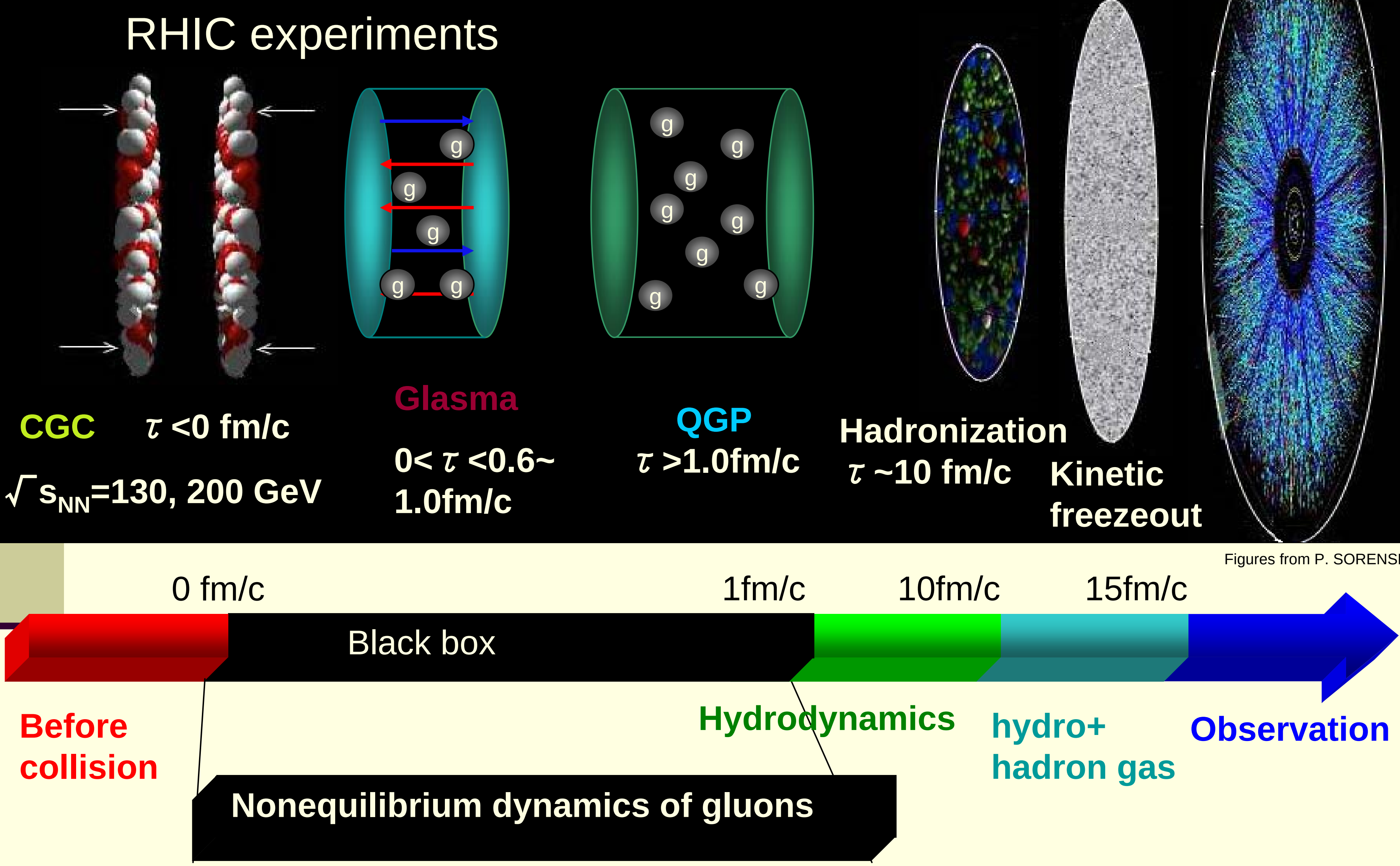
Background

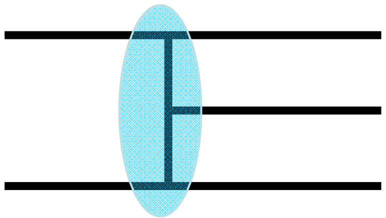

RHIC experiments
g
CGC τ <0 fm/c
$\sqrt{s_{NN}}$=130, 200 GeV
Glasma
0< τ <0.6~ 1.0fm/c
QGP
τ >1.0fm/c
Hadronization
τ ~10 fm/c
Kinetic freezeout
Figures from P. SORENSEN
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before collision
Hydrodynamics
hydro+ hadron gas
Observation
Nonequilibrium dynamics of gluons

Topics in nonequilibrium gluodynamics

- **Early thermalization for Partons**

0.6-1fm/c (Exp.) < 2-3fm/c (Perturbative Analysis

Hydrodynamics gg→gg, gg→ggg



‘Soft’: field Yang Mills equation

‘Hard’ :parton Vlasov-Boltzmann equation

However

Dense system (Boltzmann eq. should not be applied)

No consideration of particle number changing process g→gg, g→ggg (Off-shell effect)

Purpose of this talk

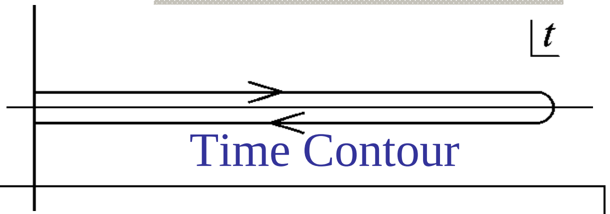
Introduction for the kinetic entropy based on Kadanoff-Baym equation

Proof for the H-theorem for scalar O(N) Φ^4 theory.

Study for the non-Abelian gauge theory

2. Kadanoff-Baym eqn

in Closed-Time Path formalism



2 Particle-Irreducible Effective Action Γ

$$\Gamma[G] = \frac{i}{2}\mathrm{Tr}\ln(G)^{-1} - \frac{1}{2}(\partial^2 + m^2)G + \frac{1}{2}\Phi[G]$$

$$G^{ab}(x,y) = \langle T_C \tilde{\phi}(x)\tilde{\phi}(y)\rangle \quad a,b = 1,2$$

Schwinger-Dyson equation

$$\frac{\delta\Gamma}{\delta G} = 0 \quad G_0^{-1}G - \Sigma G = i$$

Mean field is omitted.

Kadanoff-Baym equation

in terms of statistical (distribution) and spectral functions

$$F(x,y) = \frac{1}{2}\left\langle \left\{ \tilde{\phi}(x), \tilde{\phi}(y) \right\} \right\rangle$$

$$\rho(x,y) = \left\langle \left[\tilde{\phi}(x), \tilde{\phi}(y) \right] \right\rangle$$

For a free field

$$F(p^0, p) = 2\pi\delta(p^2 - m^2)\left(1 + \frac{1}{e^{\beta|p^0|} - 1}\right)$$

Boson

$\gamma \to 0$

$$\rho(p^0, p) = \frac{\gamma}{(p^0 - \omega)^2 + \gamma^2/4} \to 2i\pi\epsilon(p^0)\delta(p^2 - m^2)$$

Breit-Wigner type

The Kadanoff-Baym equation:

Time evolution of statistical (distribution) and spectral function

$$(-G_0^{-1} + \Sigma_\delta)F(x_0, y_0, \mathbf{p}) = \int_0^{y_0} dz_0 \Sigma_F \rho - \int_0^{x_0} dz_0 \Sigma_\rho F$$

$$(-G_0^{-1} + \Sigma_\delta)\rho(x_0, y_0, \mathbf{p}) = -\int_{y_0}^{x_0} dz_0 \Sigma_\rho \rho$$

$G_0^{-1} = -\partial^2 - m^2$ Σ =Self-energies

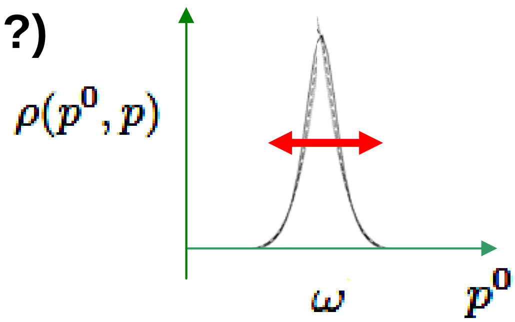
Memory integral

Merit

(Why do we use KB eq, not Boltzmann eq?)

- Spectral function

  Time evolution of spectral function
  distribution function
- Off-shell effect



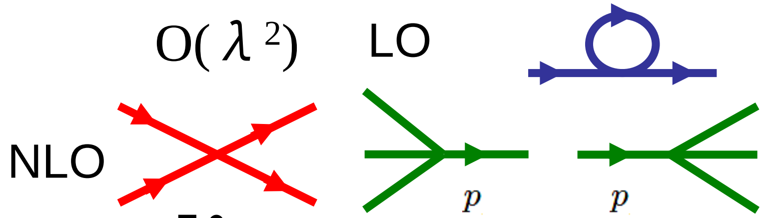
We can trace partons which are unstable by its particle number changing process in addition to collision effects. we can extract

gg→g (2 to 1) and ggg→g (3 to 1) and the inverse prohibited kinematically in Boltzmann simulation. This process might contribute the early thermalization.

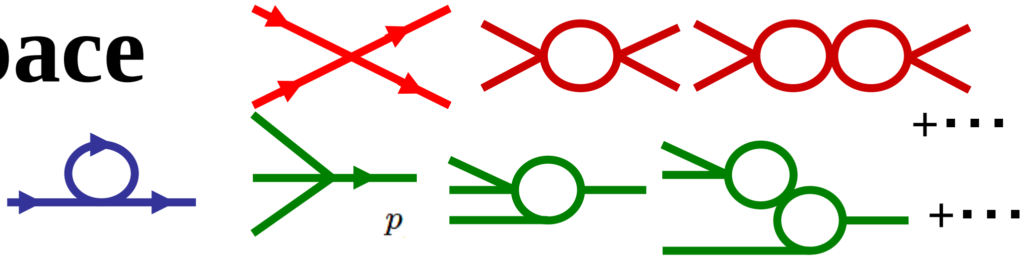
Scalar theory as a toy model

Application for BEC, Cosmology (or reheating) and DCC dynamics?

- ϕ^4 **theory with no condensate** $<\phi>=0$
- **Homogeneous in space**
- **1+1 dimensions**
- **Next Leading Order of coupling**



- **O(N) theory with no condensate** $<\phi_a>=0$
- **Homogeneous in space**
- **1+1 dimensions**
- **Next Leading Order in 1/N expansion**



Entropy in Rel. Kadanoff-Baym equation

- **Nonrel. case: Ivanov, Knoll and Voskresenski (2000), Kita (2006)**
- **The first order gradient expansion of the Schwinger-Dyson equation.**

[ ] Entropy flow spectral function

$$s^\mu = \int \frac{d^{d+1}p}{(2\pi)^{d+1}} \left[\frac{\rho}{i} \left(p^\mu - \frac{1}{2} \frac{\partial \mathrm{Re}\Sigma_R}{\partial p_\mu} \right) + \frac{\Sigma_\rho}{i} \frac{1}{2} \frac{\partial \mathrm{Re} G_R}{\partial p_\mu} \right] \sigma$$

$$\sigma = -f \ln f + (1+f) \ln(1+f)$$

$$\partial_\mu s^\mu(X) = \int \frac{d^{d+1}p}{(2\pi)^{d+1}} \frac{1}{2} \ln \frac{G^{12}}{G^{21}} C \geq 0$$

For NLO $\lambda^2(\Phi^4)$

For NLO of 1/N (O(N))

H-theorem needs not to be based on the quasiparticle picture.

In the quasiparticle limit

We reproduce the entropy for the boson.

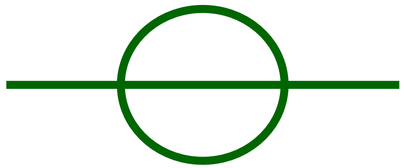
$$s^0 \rightarrow s^0 = \int \frac{d^d p}{(2\pi)^d} \left[-n_\mathbf{p} \ln n_\mathbf{p} + (1+n_\mathbf{p}) \ln(1+n_\mathbf{p}) \right]$$

Sketch of H-theorem (O(N))

Φ^4

Coupling expansion

Self-energy



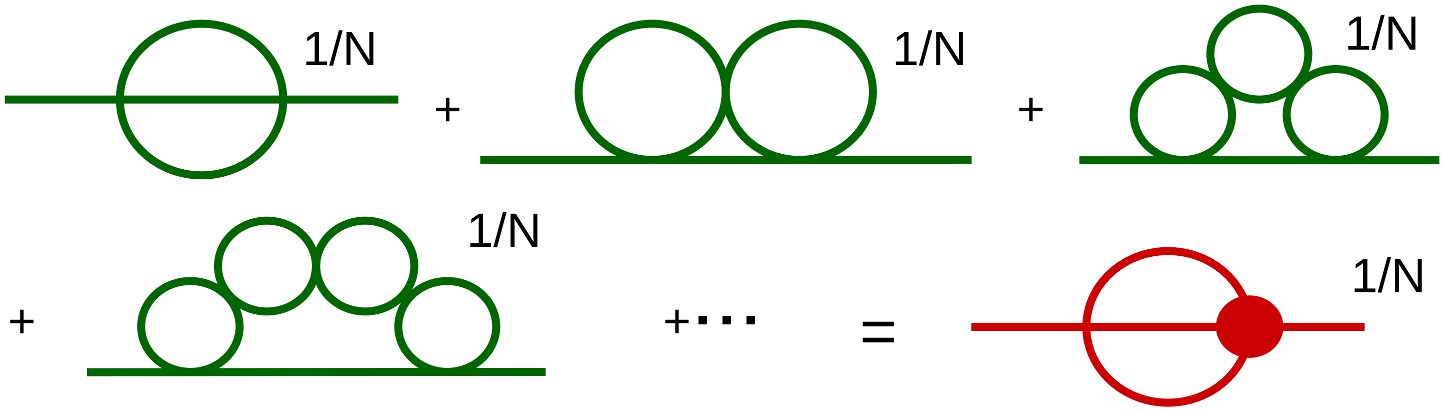
$\sim \lambda^2$

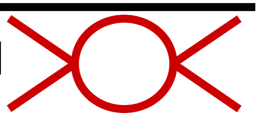
$$\partial_\mu s^\mu(X) = \lambda^2 \times (x\text{-}y)\log(x/y) \geqq 0$$

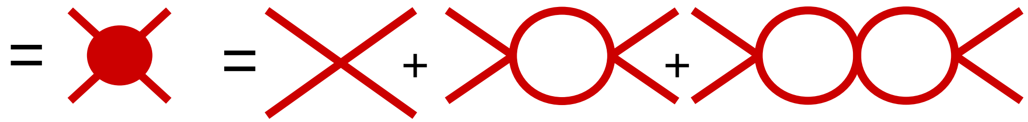
O(N)

1/N expansion

$$-\frac{\lambda}{4!N}(\hat{\phi}_a\hat{\phi}_a)^2$$



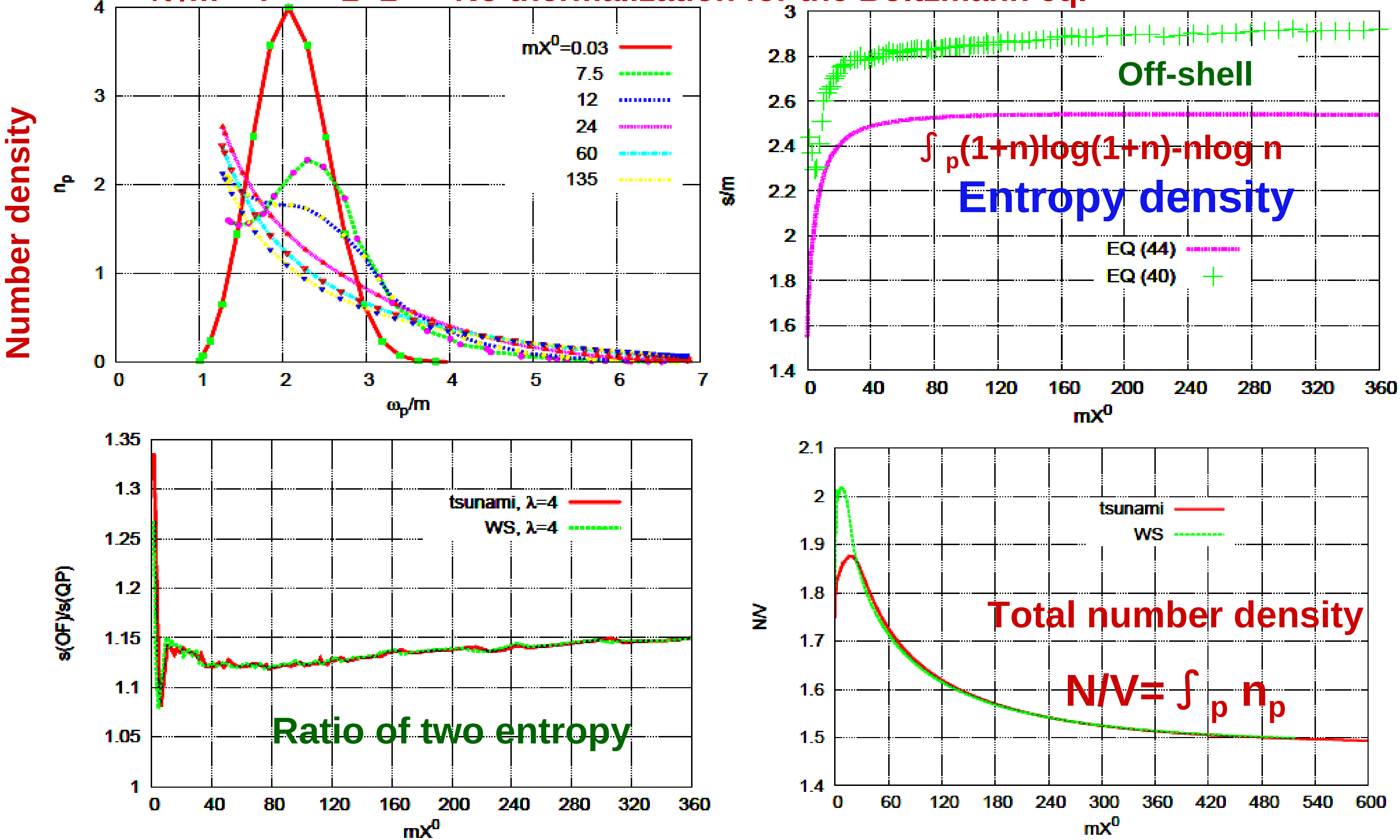
$\lambda_{\mathrm{eff}} = \dfrac{\lambda}{1 - \text{1/N}}$ 



$$\partial_\mu s^\mu(X) = \lambda \times \lambda_{\mathrm{eff}} \times (x\text{-}y)\log(x/y) \geqq 0$$

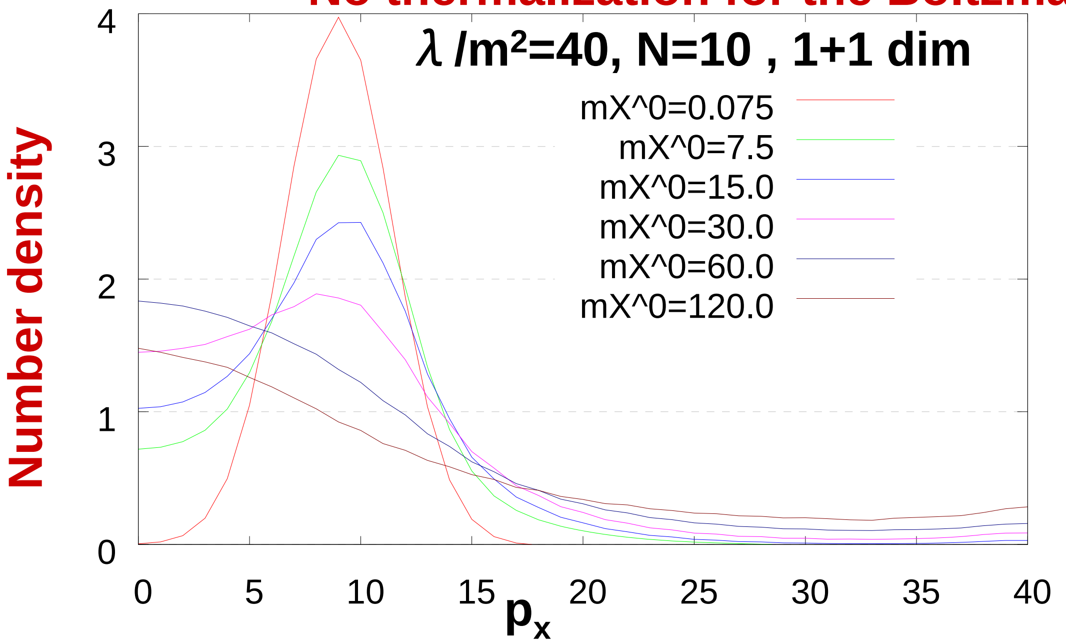
Evolution of kinetic entropy (Φ^4)

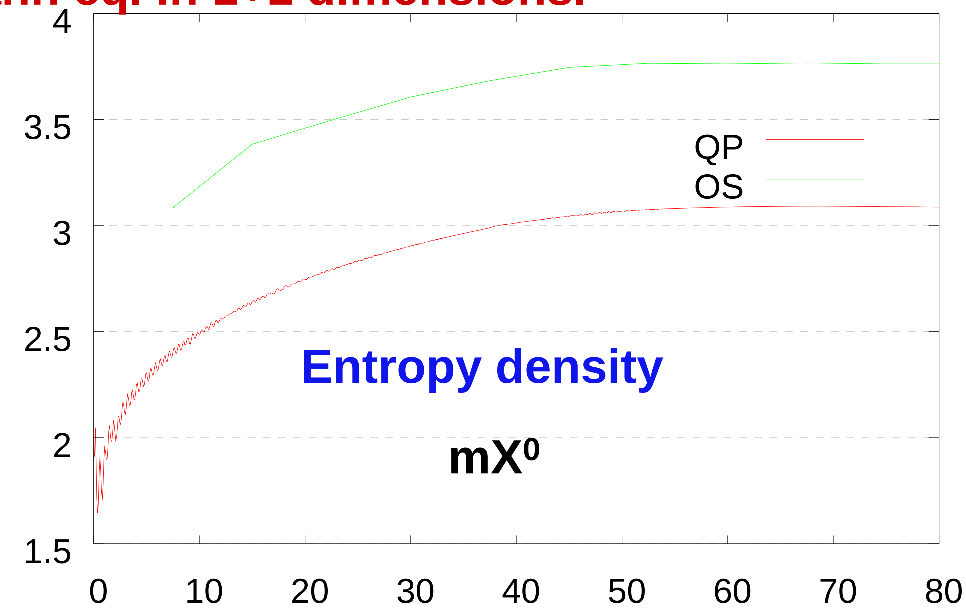
$\lambda/m^2=4$ 1+1 No thermalization for the Boltzmann eq.

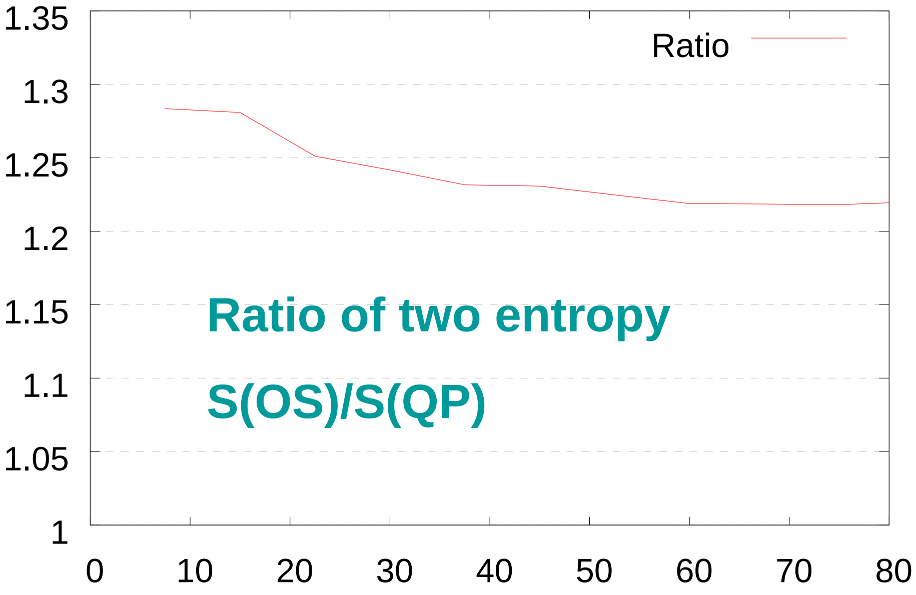


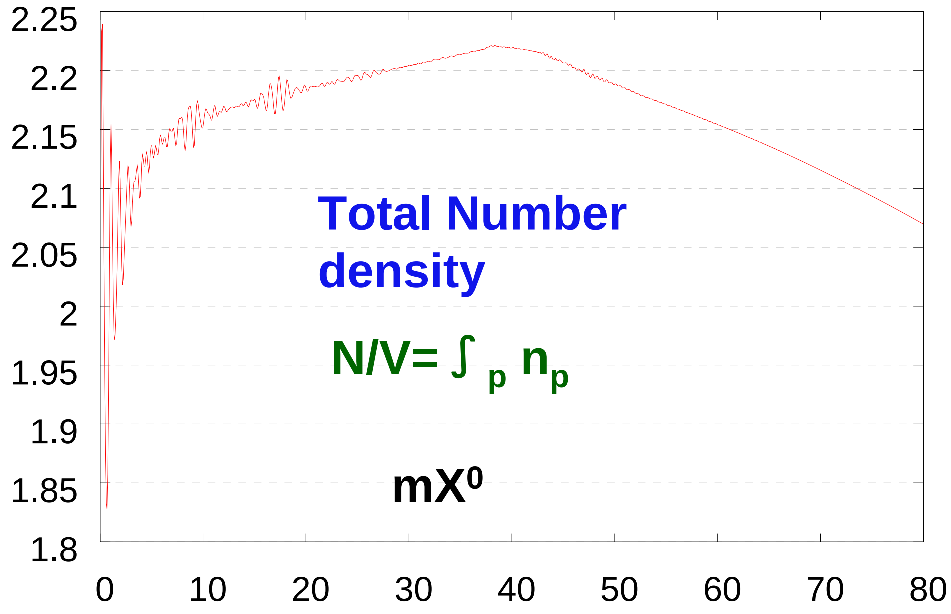
Evolution of kinetic entropy (O(N))

No thermalization for the Boltzmann eq. in 1+1 dimensions.









Non-Abelian Gauge Theory

- Controlled gauge dependence of effective action

(Smit and Arrizabaraga (2002), Carrington et al (2005))

Exact Γ_{2PI} **at** $\frac{\delta\Gamma}{\delta D}=0 \Leftrightarrow D^{-1}(x,y)=D_0^{-1}(x,y)-\Pi(x,y)$

Gauge invariant Nielsen (1975)

Green's function **Self-energy**

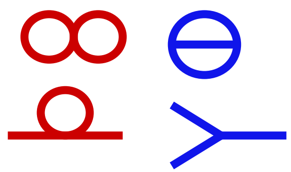
Γ_{2PI} **⇒ Gauge invariant Energy, Pressure and Entropy derived from δ Γ / δ T**

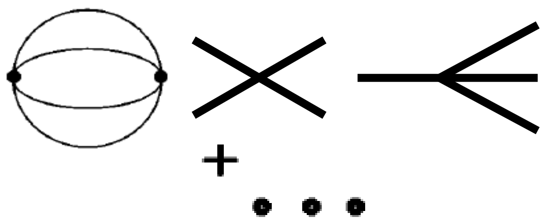
This might not the entropy in the previous page.

Truncated effective action

$$\Gamma_{2PI}=\Gamma_L+\Gamma_{ex} \qquad \Gamma_L \sim O(g^{2L-2}) \; / \; \Gamma_{ex}=O(g^{2L})$$

Expansion of coupling of self energy





Stationary point

$$\frac{\delta\Gamma_L}{\delta D}=0 \quad \Leftrightarrow$$ **Schwinger-Dyson equation**

Under gauge transformation $\delta\Gamma_L \sim g^2\Gamma_L$ **Higher order gauge dependence**

Γ_L **⇒ Energy, Pressure and Entropy derived from δ Γ / δ T has controlled gauge dependence. Gauge invariance is reliable in the truncated order.**

4. Summary and Remaining Problems

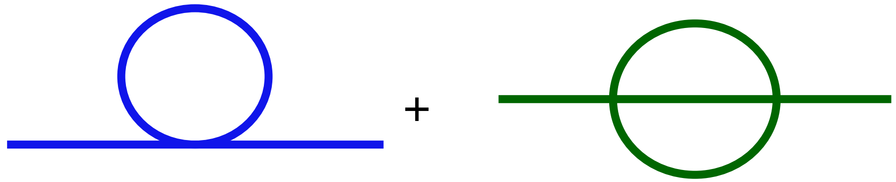
- We have introduced the kinetic entropy based on the Kadanoff-Baym equation.
- The kinetic entropy satisfies H-theorem for NLO of λ (Φ^4) and 1/N (O(N)).
- S(OS)/S(QP) is nearly constant, but S(QP) tends to be affected by the total number density.
- Gauge dependence is controlled in deriving thermodynamic variables (energy, pressure and entropy and so on).

- Longer time simulation in the O(N) case
- Thermal solution for the SD eq. for the LO of g^2 for the gauge theory (2+1 dimensions)
- H-theorem for the gauge theory, gauge invariance of the entropy

Proof of H-theorem (O(N))

Φ^4

Coupling expansion

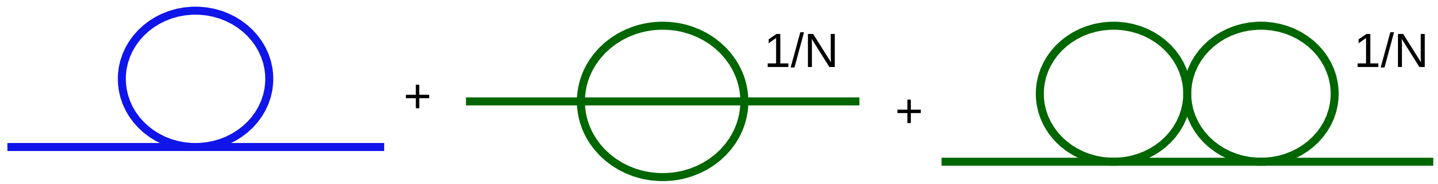


$\sim \lambda^2$

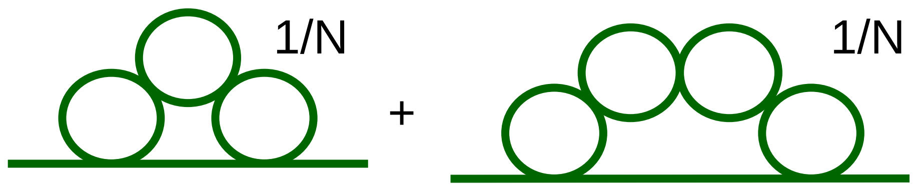
$$\partial_\mu s^\mu(X) = \lambda^2 \times (x-y)\log(x/y) \geqq 0$$

O(N)

1/N expansion

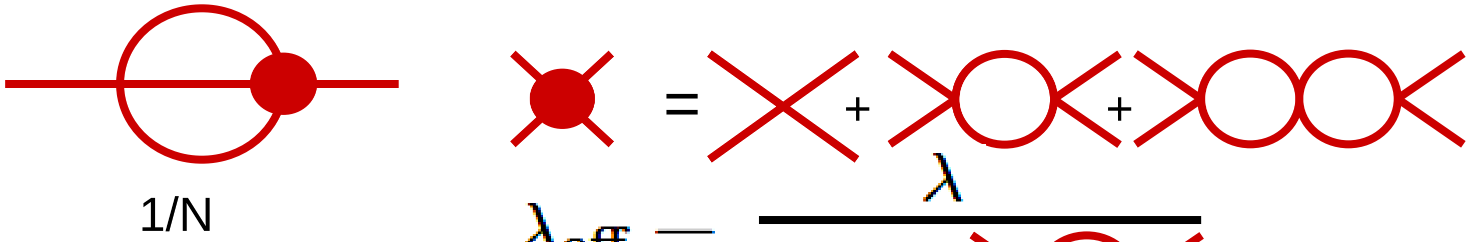


$-\frac{\lambda}{4!N}(\hat{\phi}_a\hat{\phi}_a)^2$ +

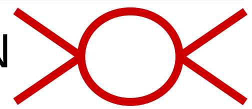


$+\cdots$

=



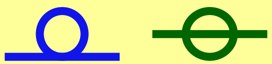
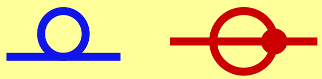
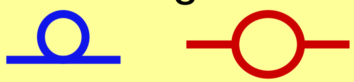
$$\lambda_{\text{eff}} = \frac{\lambda}{1 - 1/N \times \text{(diagram)}}$$



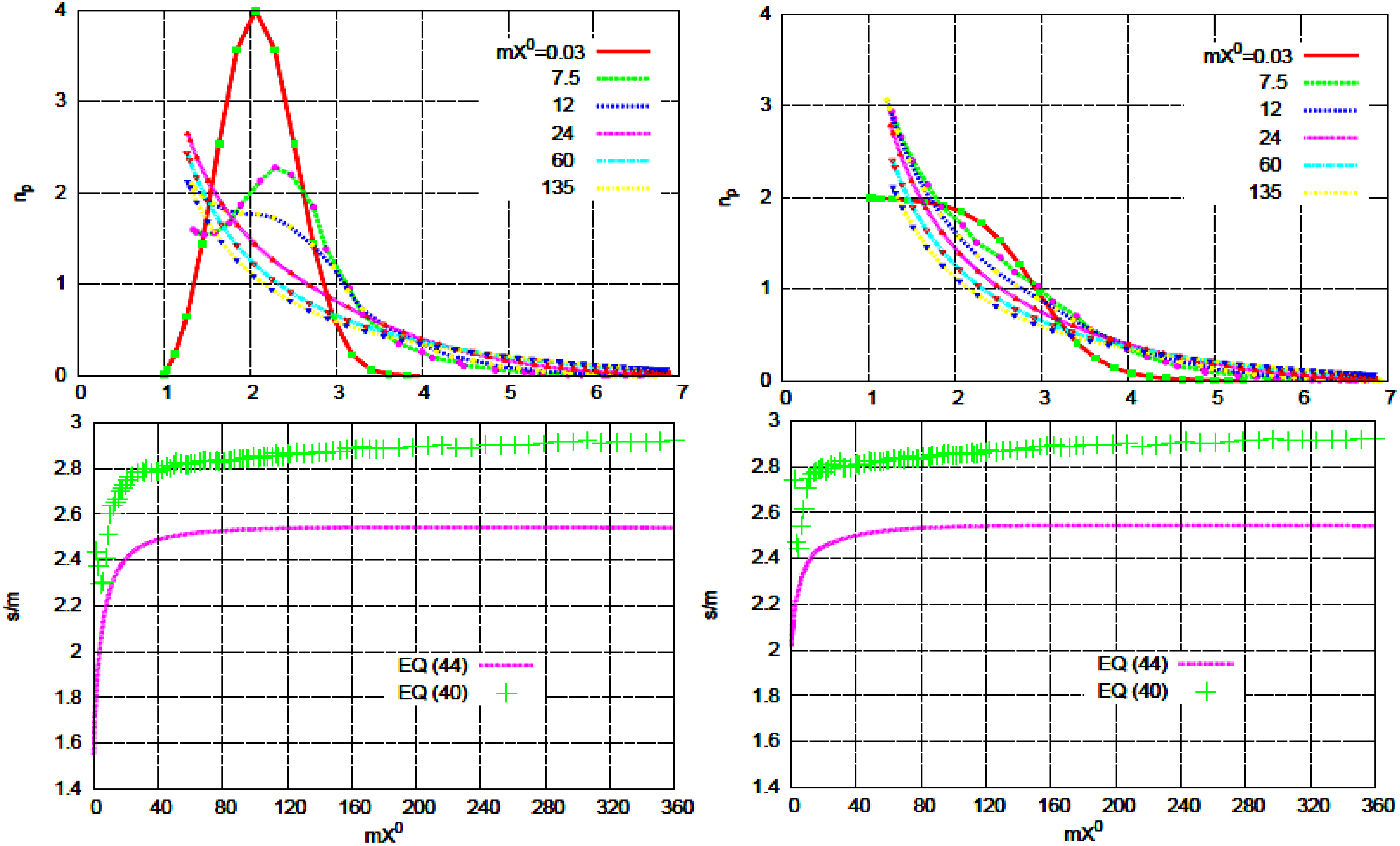
$$\partial_\mu s^\mu(X) = \lambda \times \lambda_{\text{eff}} \times (x-y)\log(x/y) \geqq 0$$

Time irreversibility

Symmetric phase $\langle \Phi \rangle = 0$

	$\lambda \Phi^4$	O(N)	SU(N)
Exact 2PI (no truncation)	×	×	×
Truncation	NLO of λ  △	NLO of 1/N  △	LO of g^2  ?
LO of Gradient expansion H-theorem	○	○	?

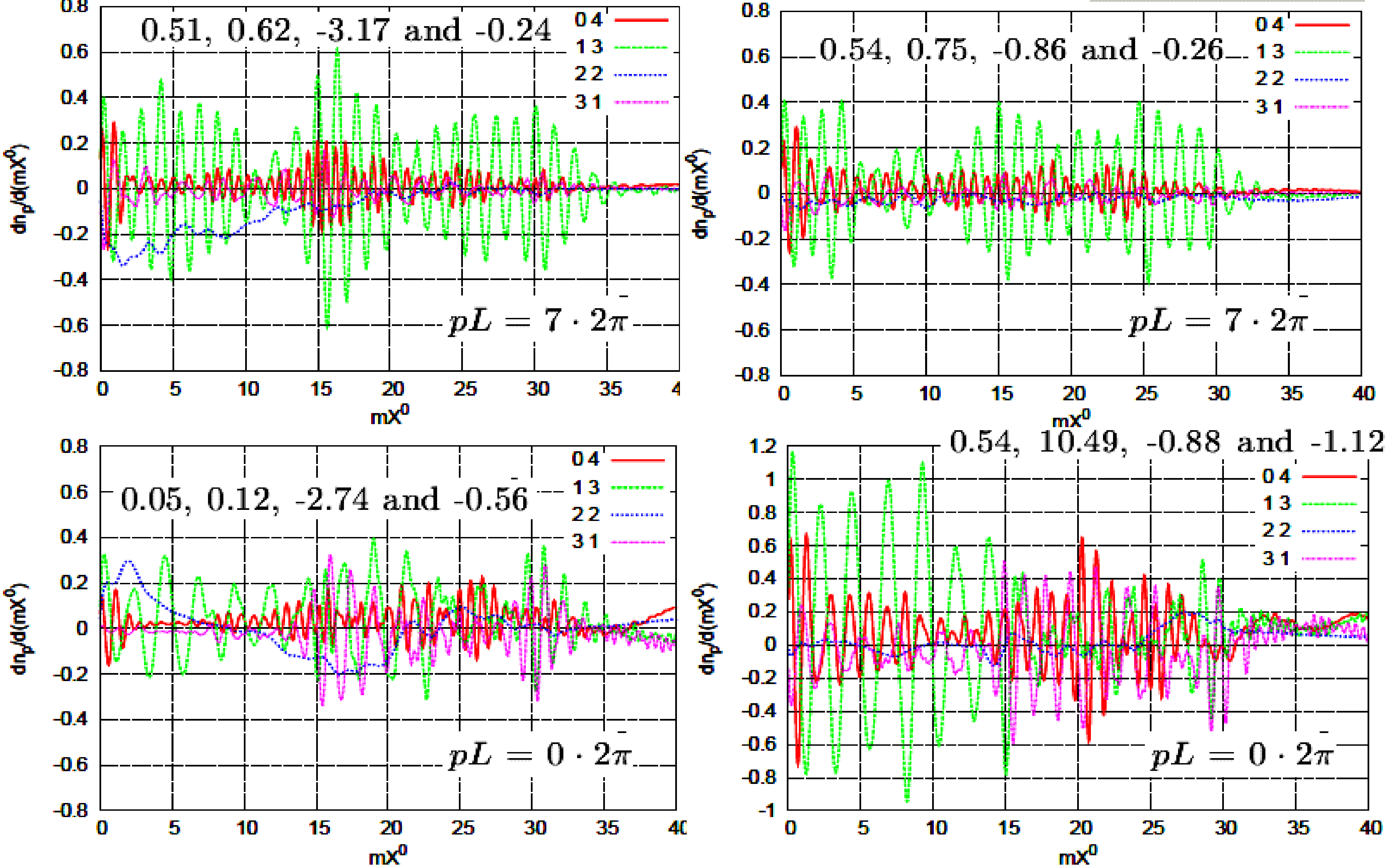
Time evolution of number density



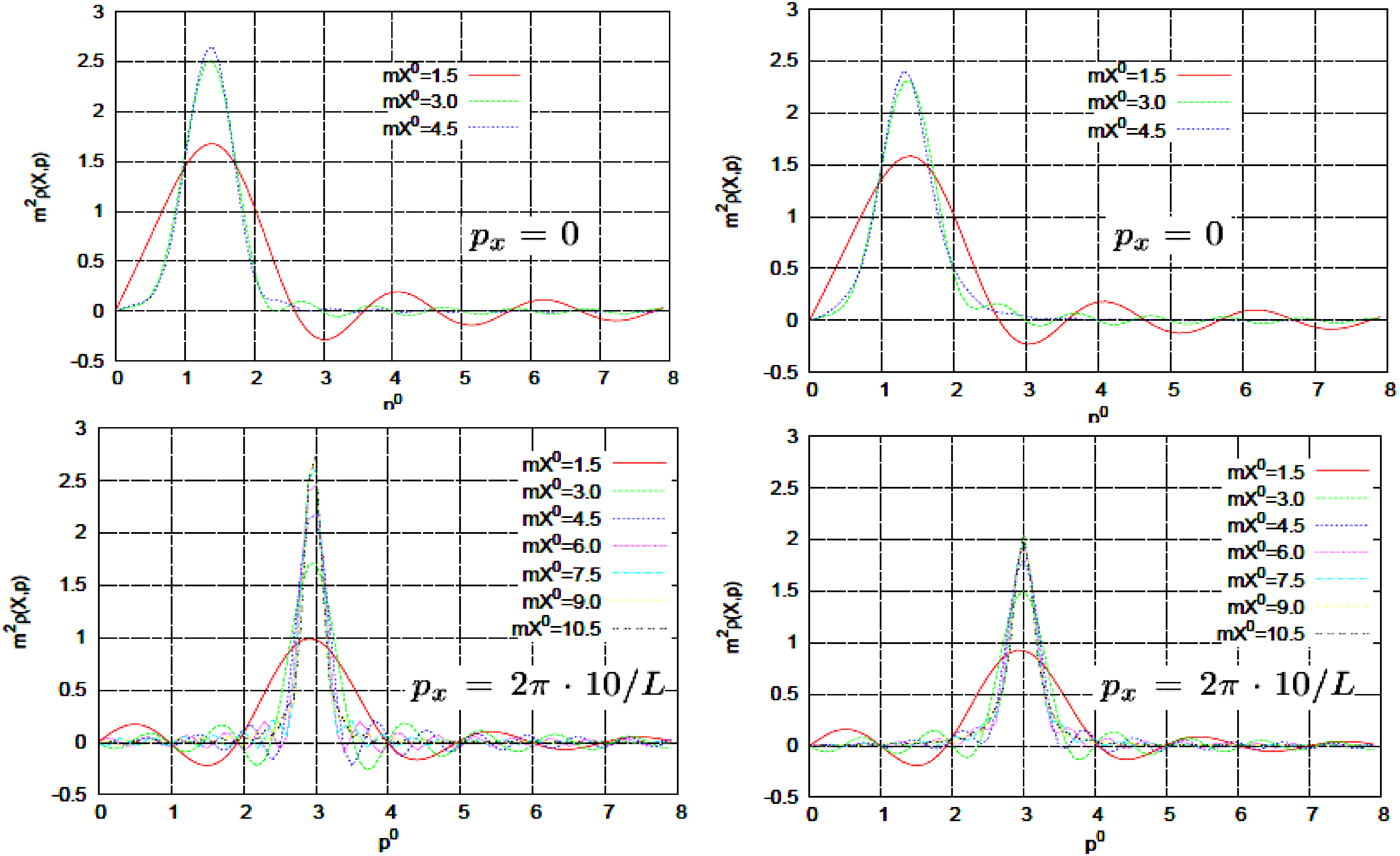
For quasiparticle approximation

$$
\begin{aligned}
\partial_t n_{\mathbf{p}}(t) &= \frac{\lambda^2}{3}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\frac{d^d\mathbf{k}}{(2\pi)^d}\int_{t_0}^{t}dt'\frac{1}{2\tilde{\omega}(\mathbf{p})2\tilde{\omega}(\mathbf{q})2\tilde{\omega}(\mathbf{k})2\tilde{\omega}(\mathbf{p}-\mathbf{k}-\mathbf{q})} \\
& \Big\{ [(1+n_{\mathbf{p}})(1+n_{\mathbf{q}})(1+n_{\mathbf{k}})(1+n_{\mathbf{p}-\mathbf{k}-\mathbf{q}}) - n_{\mathbf{p}}n_{\mathbf{q}}n_{\mathbf{k}}n_{\mathbf{p}-\mathbf{k}-\mathbf{q}}(t')] \\
\times & \cos[(\tilde{\omega}(\mathbf{p})+\tilde{\omega}(\mathbf{q})+\tilde{\omega}(\mathbf{k})+\tilde{\omega}(\mathbf{p}-\mathbf{k}-\mathbf{q}))(t-t')] \qquad \mathbf{0 \Leftrightarrow 4} \\
& +3[(1+n_{\mathbf{p}})(1+n_{\mathbf{q}})(1+n_{\mathbf{k}})n_{\mathbf{p}-\mathbf{k}-\mathbf{q}} - n_{\mathbf{p}}n_{\mathbf{q}}n_{\mathbf{k}}(1+n_{\mathbf{p}-\mathbf{k}-\mathbf{q}})(t')] \\
\times & \cos[(\tilde{\omega}(\mathbf{p})+\tilde{\omega}(\mathbf{q})+\tilde{\omega}(\mathbf{k})-\tilde{\omega}(\mathbf{p}-\mathbf{k}-\mathbf{q}))(t-t')] \qquad \mathbf{1 \Leftrightarrow 3} \\
& +3[(1+n_{\mathbf{p}})(1+n_{\mathbf{q}})n_{\mathbf{k}}n_{\mathbf{p}-\mathbf{k}-\mathbf{q}} - n_{\mathbf{p}}n_{\mathbf{q}}(1+n_{\mathbf{k}})(1+n_{\mathbf{p}-\mathbf{k}-\mathbf{q}})(t')] \\
\times & \cos[(\tilde{\omega}(\mathbf{p})+\tilde{\omega}(\mathbf{q})-\tilde{\omega}(\mathbf{k})-\tilde{\omega}(\mathbf{p}-\mathbf{k}-\mathbf{q}))(t-t')] \qquad \mathbf{2 \Leftrightarrow 2} \\
& +[(1+n_{\mathbf{p}})n_{\mathbf{q}}n_{\mathbf{k}}n_{\mathbf{p}-\mathbf{k}-\mathbf{q}} - n_{\mathbf{p}}(1+n_{\mathbf{q}})(1+n_{\mathbf{k}})(1+n_{\mathbf{p}-\mathbf{k}-\mathbf{q}})(t')] \\
\times & \cos[(\tilde{\omega}(\mathbf{p})-\tilde{\omega}(\mathbf{q})-\tilde{\omega}(\mathbf{k})-\tilde{\omega}(\mathbf{p}-\mathbf{k}-\mathbf{q}))(t-t')]\Big\}. \qquad \mathbf{3 \Leftrightarrow 1}
\end{aligned}
$$

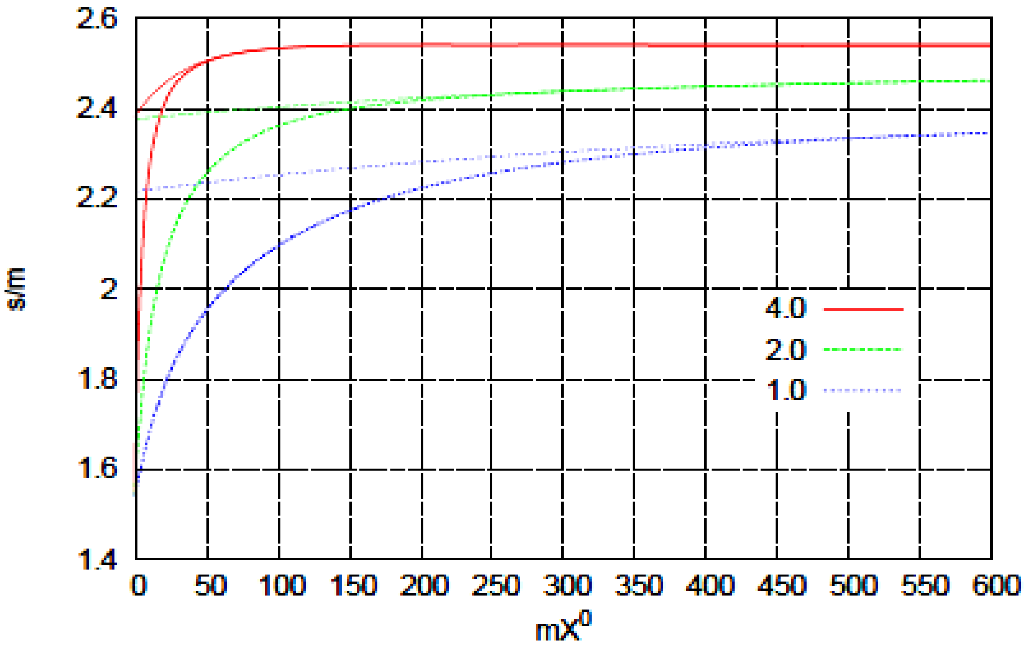
Microscopic processes

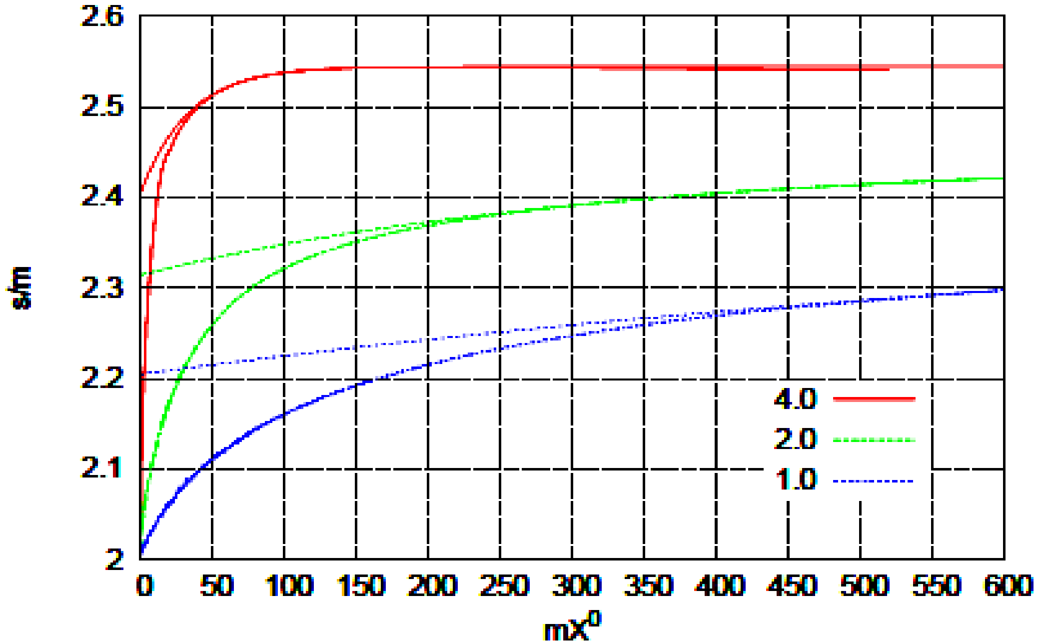


Spectral function



Coupling dependence





$$s(X^0) = s_{\max} - Ae^{-\gamma(mX^0)}$$

λ	γ_0	$s_{\max}$	A	γ	γ_0	$s_{\max}$	A	γ	
4(OS)	-	2.93	0.16	0.0071	-	2.93	0.15	0.0071	
4	0.24	2.54	0.16	0.030	0.14	2.54	0.14	0.030	$100 \le mX^0 \le 150$
2	0.084	2.48	0.10	0.0031	0.036	2.44	0.13	0.0031	$300 \le mX^0 \le 600$
1	0.027	2.39	0.17	0.0024	0.0085	2.39	0.19	0.0011	$600 \le mX^0 \le 900$

Self-energy

Schwinger-Dyson eqn

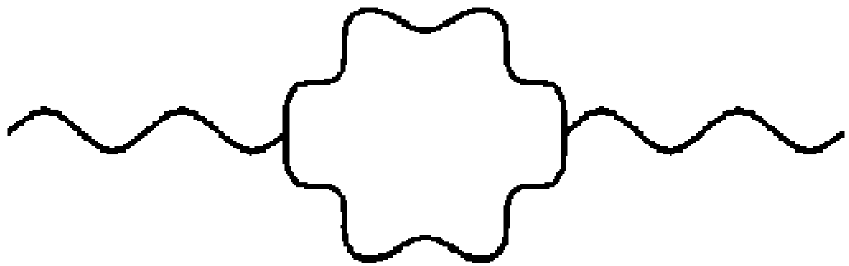
$$D^{-1}(x,y) = D_0^{-1}(x,y) - \Pi(x,y)$$

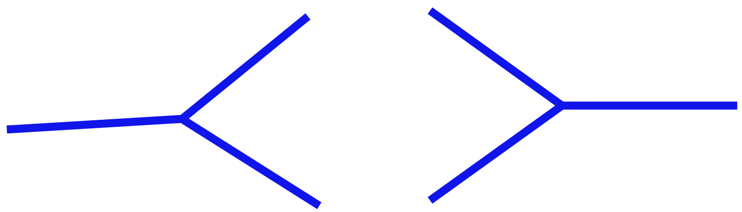
For perturbative Green's functions

Imaginary part

$$\mathrm{Im}\,\Pi_{\mu\nu}(\omega,\mathbf{p}) = -\pi m_D^2 \omega \int \frac{d\Omega}{4\pi} v_\mu v_\nu \delta(\omega - \mathbf{v}\cdot\mathbf{p})$$

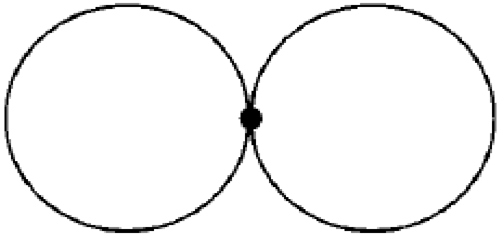
contributes to the particle number changing process g⇔gg



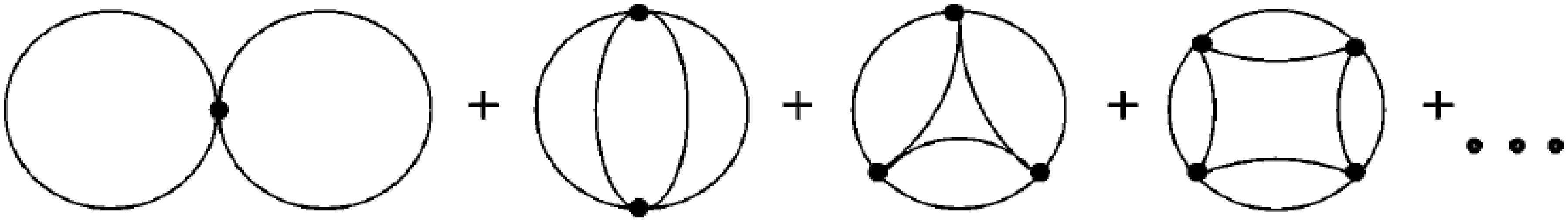


O(N): 1/N expansion

LO O(N)



NLO
O(1)



O(N): spectral function