

格子QCDシミュレーションによる QGP媒質中の重いクォーク間ポテンシャルの研究

WHOT-QCD Collaboration

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WHOT-QCD Coll. arXiv:0907.4203

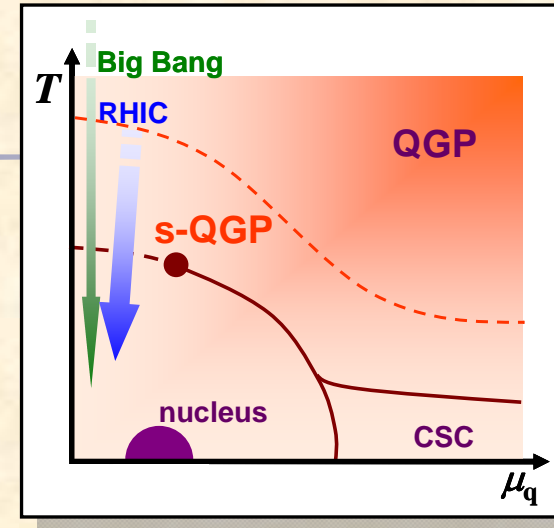
Introduction

Study of **Quark-Gluon Plasma (QGP)**

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- Early universe after Big Bang
 - Relativistic heavy-ion collision

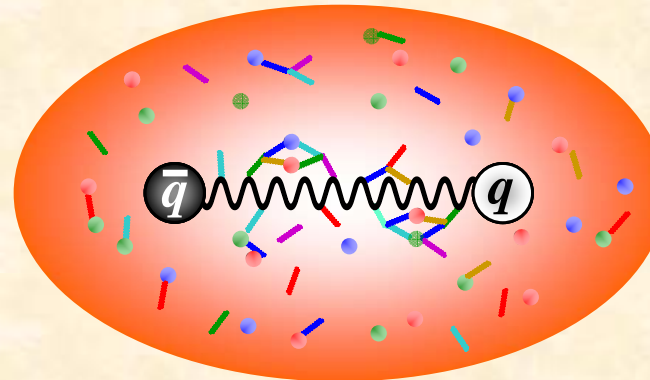
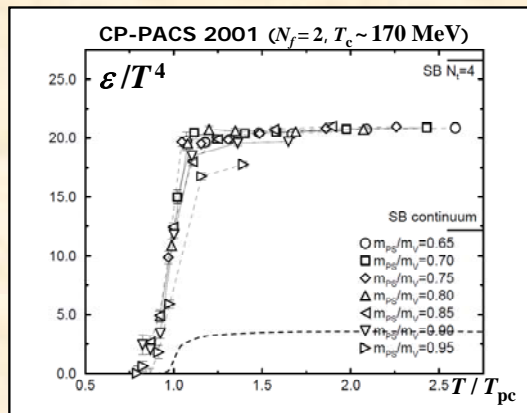
Theoretical study based on first principle (QCD)

- **Lattice QCD simulation at finite (T, μ_q)**



Bulk properties of QGP $(p, \varepsilon, T_c, \dots)$
are well investigated.

Internal properties of QGP
are still uncertain.



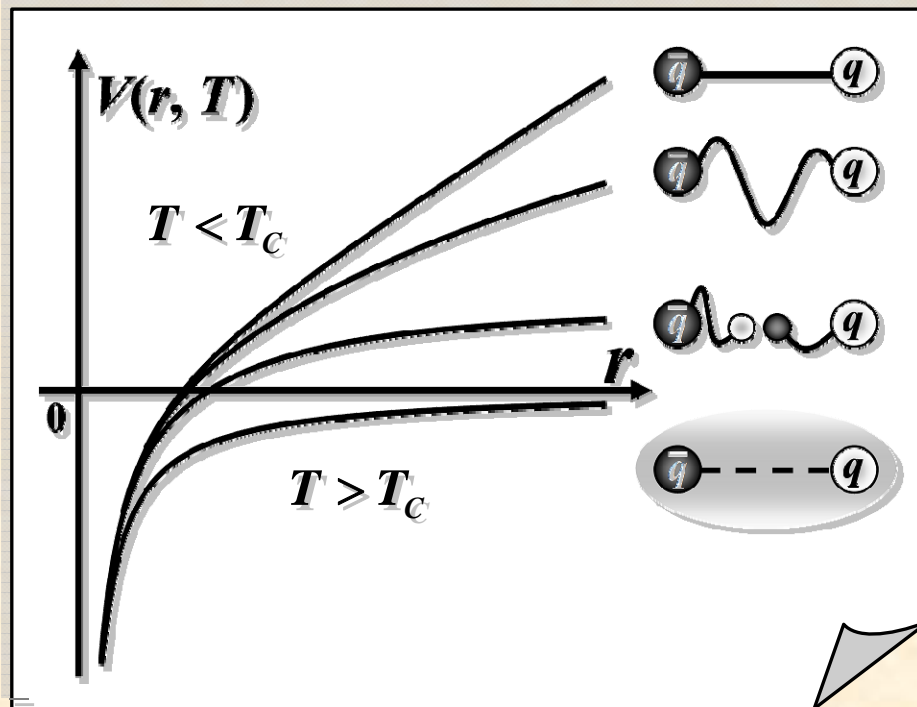
Properties of quarks and gluons in QGP

- **Heavy-quark potential:** free energy btw. static charged quarks in QGP
 1. Relation of inter-quark interaction between zero and finite T
 2. Temperature dependence of Debye screening length

Heavy-quark potential at finite T

Heavy-quark potential

→ interaction btw. static quark (Q) and antiquark (\bar{Q}) in QCD matter



➤ $T = 0, V(r) = -\frac{\alpha}{r} + \sigma r$

➤ $T > 0$, string tension (σ) decreases, string breaking at r_c

➤ At $T > T_c$, screening effect in QGP

$$V(r, T) = -\frac{\alpha_{\text{eff}}(T)}{r} e^{-m_D(T)r}$$

Properties of heavy-quark potential in quark-gluon plasma

- Heavy-quark bound state ($J/\psi, \Upsilon$) in QGP
- Screening effect in QGP
- Inter-quark interaction btw. $\bar{Q}Q$ and QQ

→ Lattice QCD simulations

Heavy-quark free energy

To characterize an interaction b/w static quarks,

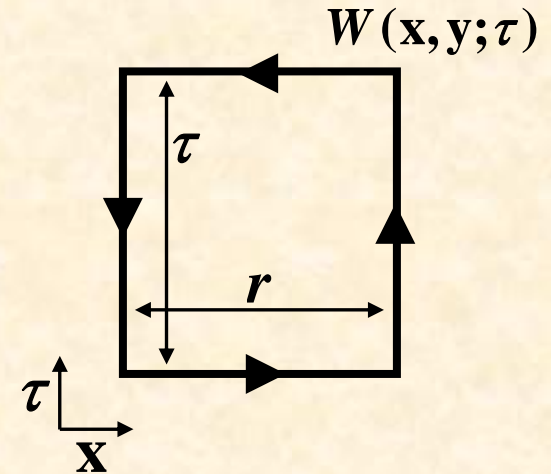
$$T = 0$$

Heavy-quark potential Brown and Weisberger (1979)

← Wilson-loop operator

$$V(r) = -\lim_{\tau \rightarrow \infty} [\tau^{-1} \ln \langle W(\mathbf{x}, \mathbf{y}; \tau) \rangle]$$

$$= -\frac{\alpha}{r} + \sigma r$$



$$T > 0$$

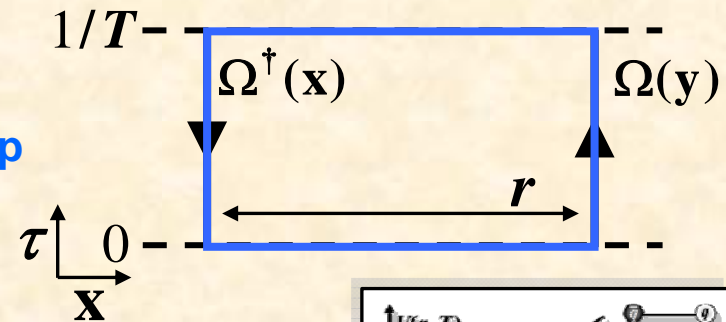
Heavy-quark free energy Nadkarni (1986)

Polyakov-line $\Omega(\mathbf{x}) = \prod_{\tau=1}^{N_t} U_4(\mathbf{x}, \tau)$: static quark at \mathbf{x}

Correlation b/w Polyakov-lines projected to a color singlet channel in the Coulomb gauge

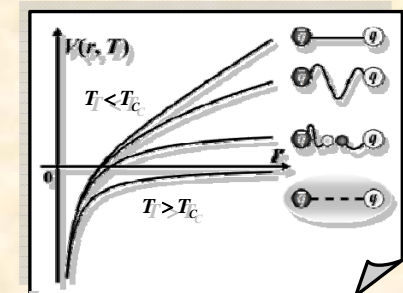
→ $F^1(r, T) = -T \ln \langle \text{Tr} \Omega^\dagger(\mathbf{x}) \Omega(\mathbf{y}) \rangle$

Operator similar to the Wilson-loop at finite T



Heavy-quark free energy may behave in QGP as:

- short r : $F^1(r, T) \sim V(r)$ (no effect from thermal medium)
- mid r : screened by plasma
- long r : two single-quark free energies w/o interactions



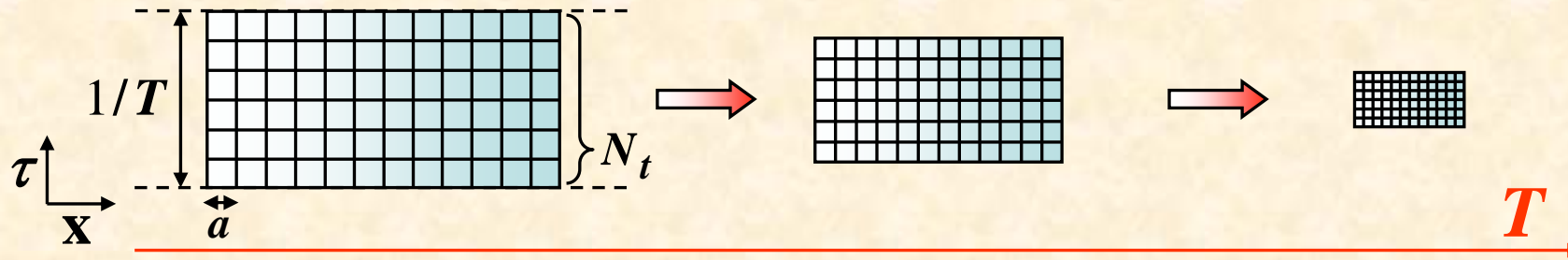
Approaches of lattice simulations at finite T

To vary temperature on the lattice,

$$T = \frac{1}{a N_t} \quad \begin{array}{l} a: \text{ lattice spacing} \\ N_t: \text{ lattice size in E-time direction} \end{array}$$

➤ Fixed N_t approach

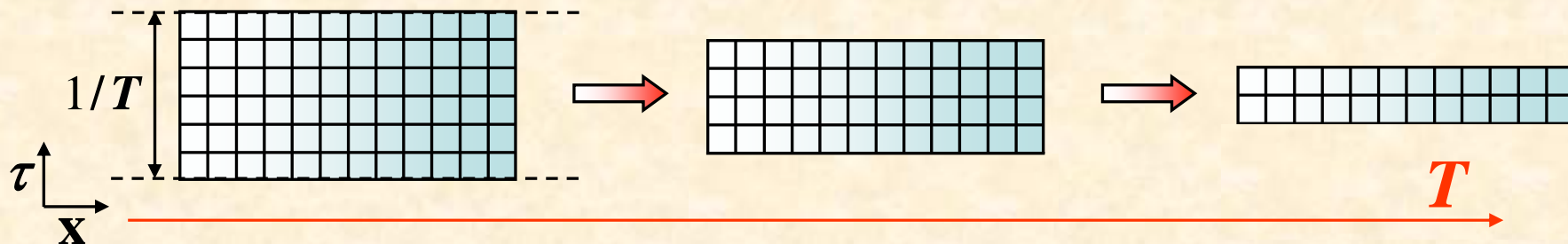
- Changing a (controlled by the gauge coupling ($\beta = 6/g^2$)), fixing N_t



Advantages: T can be varied arbitrarily, because β is an input parameter.

➤ Fixed scale approach WHOT-QCD, PRD 79 (2009) 051501.

- Changing N_t , fixing a



Advantages: investigation of T dependence w/o changing spatial volume possible.

renormalization factor dose not change when T changes.

Results of numerical simulations

1. Heavy-quark potential at $T = 0$ and $T > 0$
2. Heavy-quark potential for various color-channels
3. Heavy-quark potential at finite density (μ_q)

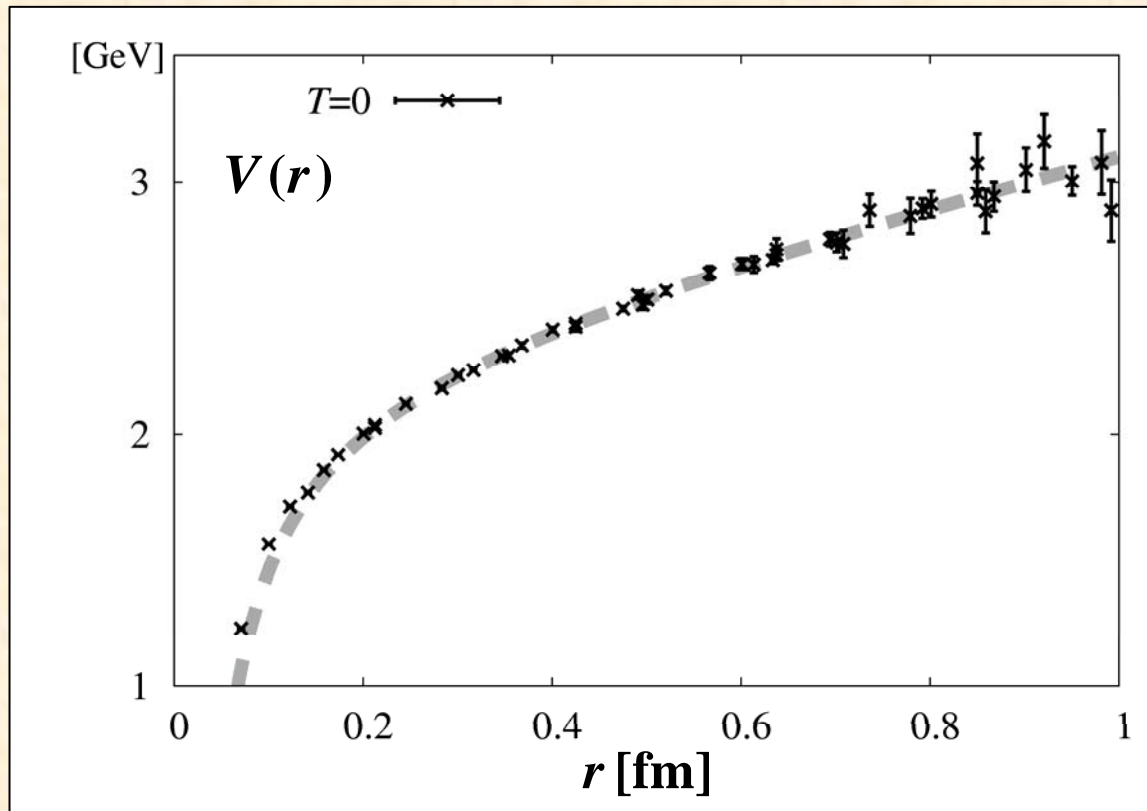
Simulation details

$N_f = 2+1$ full QCD simulations

- Lattice size: $N_s^3 \times N_t = 32^3 \times 12, 10, 8, 6, 4$
 \implies Temperature: $T \sim 200\text{-}700$ MeV (5 points)
- Lattice spacing: $a = 0.07$ fm
- Light quark mass*: $m_s/m_\rho = 0.6337(38)$
- Strange quark mass*: $m_N/m_{K^*} = 0.7377(28)$
- Scale setting: Sommer scale, $r_0 = 0.5$ fm

*CP-PACS & JLQCD Coll., PRD78 (2008) 011502.

Heavy-quark potential at $T = 0$



Data calculated by
CP-PACS & JLQCD Coll.
on a $28^3 \times 58$ lattice

Phenomenological potential

$$V(r) = -\frac{\alpha_0}{r} + \sigma r + V_0$$

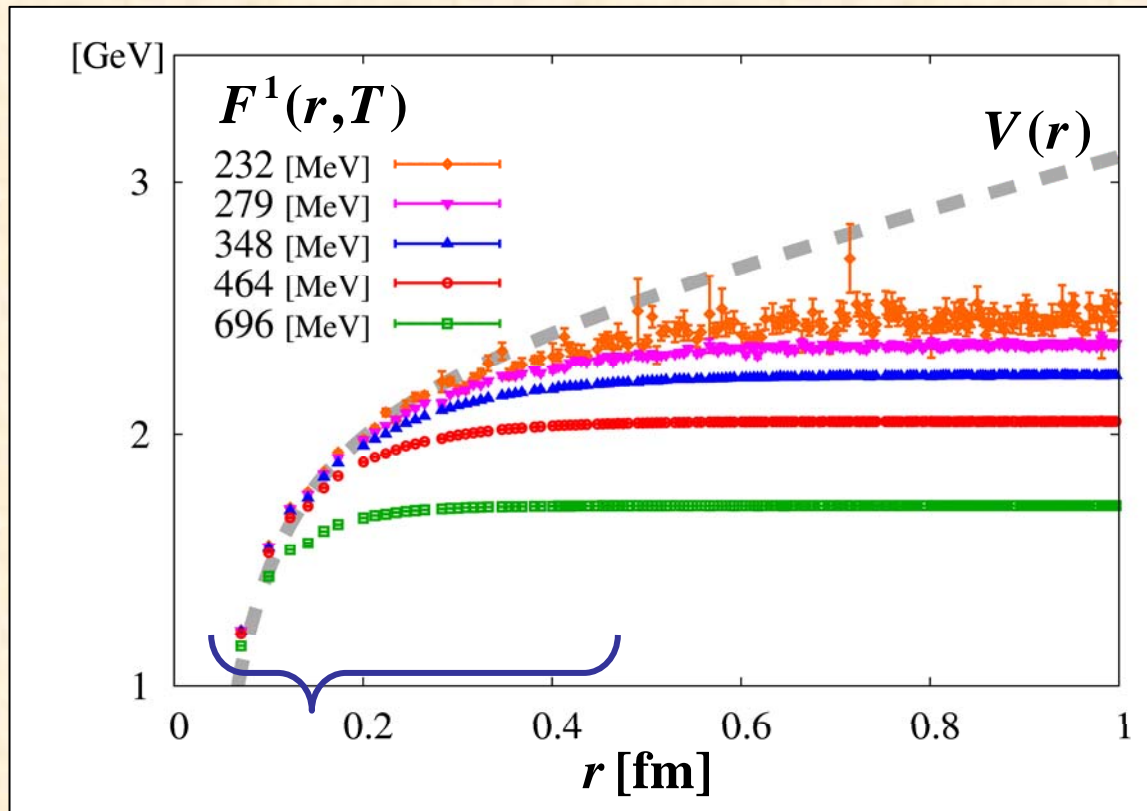
= Coulomb term at short r
+ Linear term at long r
+ const. term



Our fit results

$$\alpha_0 = 0.441$$
$$\sqrt{\sigma} = 0.434 \text{ GeV}$$
$$V_0 = 2.23 \text{ GeV}$$

Heavy-quark free energy at $T > 0$



At short distance

$F^1(r, T)$ at any T

converges to

$V(r) = F^1(r, T=0)$.



Short distance physics is insensitive to T .

Note:

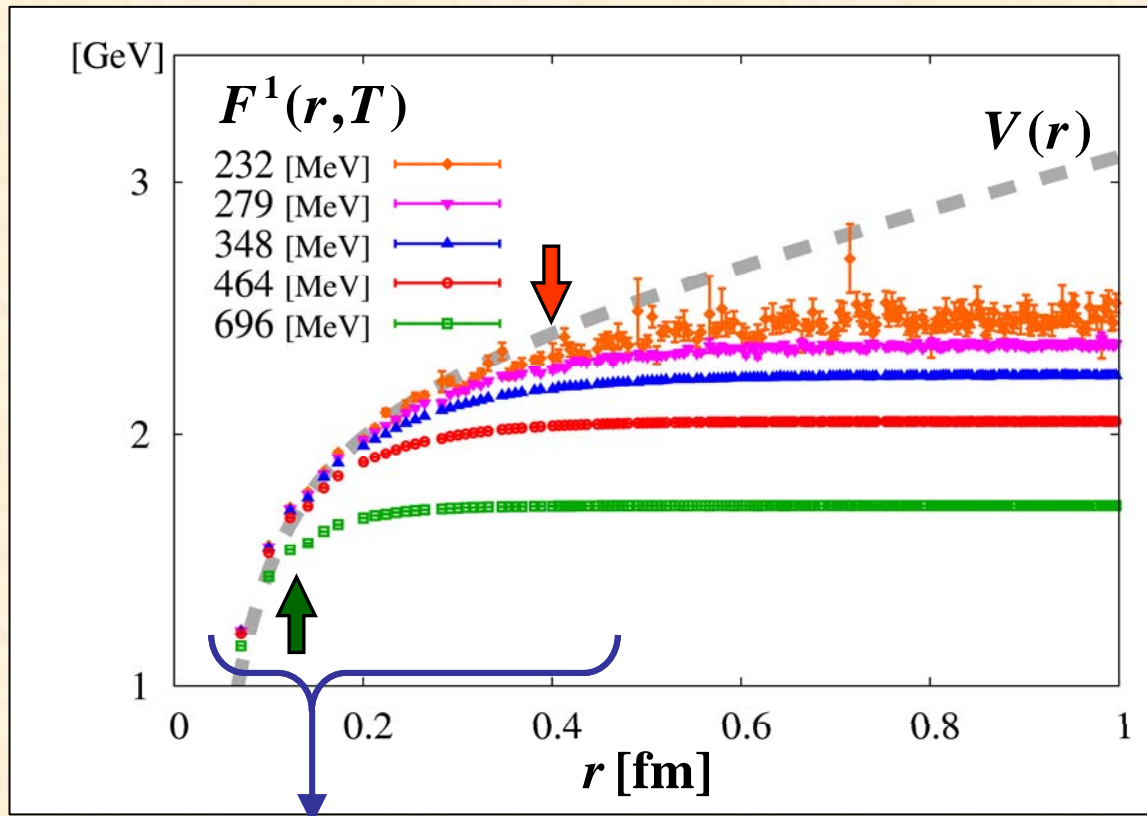
In the **fixed N_f approach**, this property is used to adjust the constant term of $F^1(r, T)$.

In our **fixed scale approach**, because the renormalization is common to all T , no further adjustment of the constant term is necessary.



We can confirm the expected insensitivity!

Heavy-quark free energy at $T > 0$



At short distance

$F^1(r, T)$ at any T
converges to

$$V(r) = F^1(r, T=0).$$



Short distance physics is
insensitive to T .

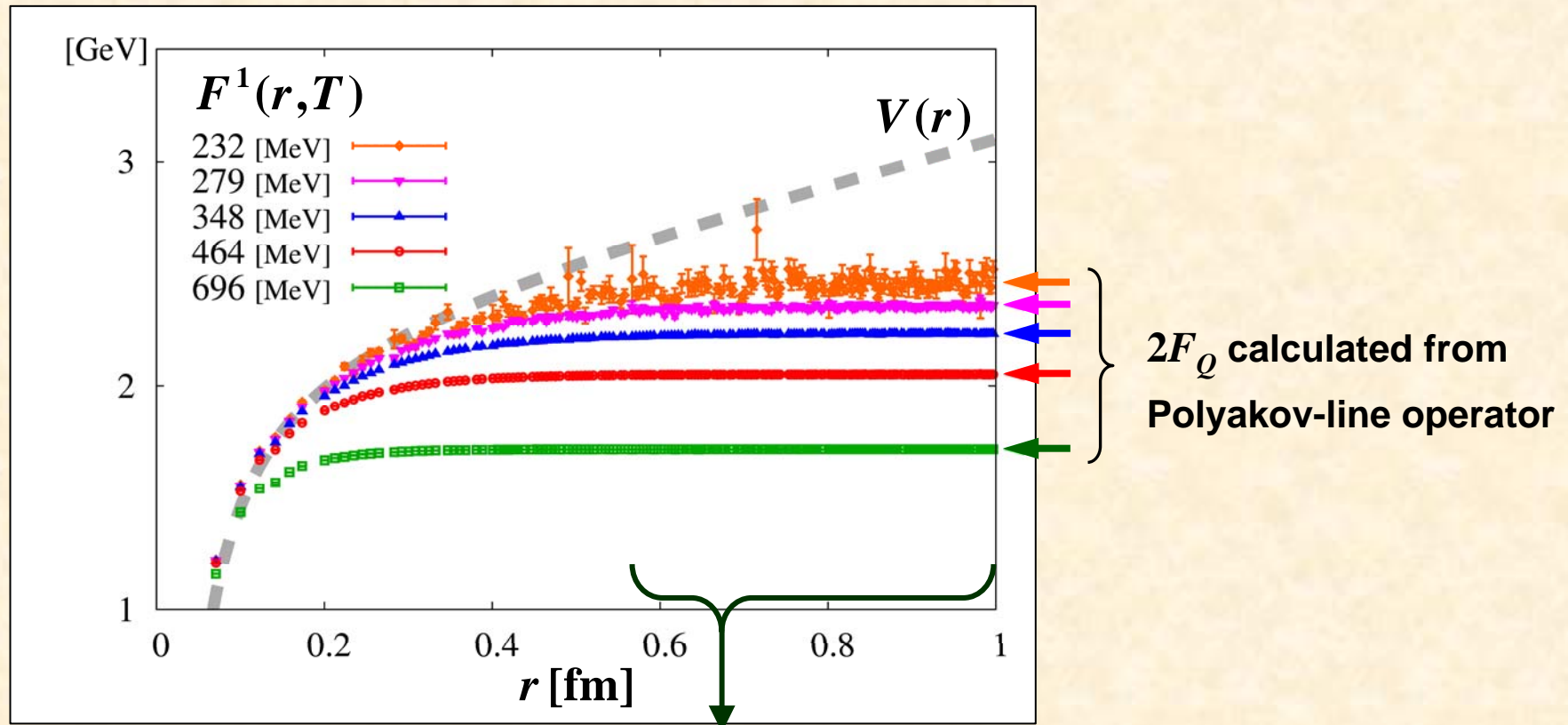
$T \sim 200$ MeV: $F^1 \sim V$ up to 0.3--0.4 fm

$T \sim 700$ MeV: $F^1 \not\sim V$ even at 0.1 fm

} Range of thermal effect is T -dependent

⇒ Debye screening effect

Heavy-quark free energy at $T > 0$



Long distance

$F^1(r, T)$ becomes flat and has no increase lineally.

⇒ Confinement is destroyed due to a thermal medium effect.

At large r , correlation b/w Polyakov-lines will disappear,

$$F^1(r, T) = -T \ln \langle \text{Tr} \Omega^\dagger(\mathbf{x}) \Omega(\mathbf{y}) \rangle \xrightarrow{r \rightarrow \infty} -T \ln \langle \text{Tr} \Omega \rangle^2 = 2F_Q$$

⇒ F^1 converges to 2x(single-quark free energy) at long distance

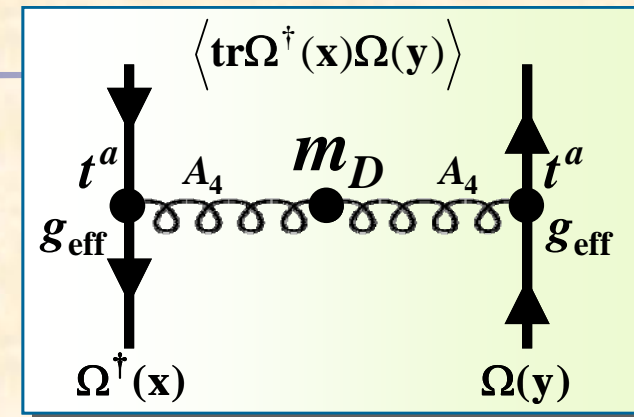
Debye screening effect in QGP

Single gluon exchange ansatz

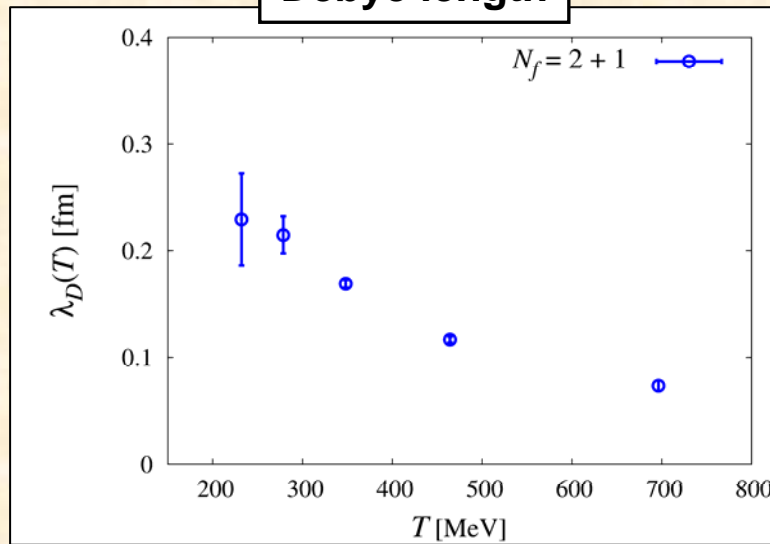
$$F^1(r, T) = -T \ln \langle \text{Tr} \Omega^\dagger(\mathbf{x}) \Omega(\mathbf{y}) \rangle \rightarrow -\frac{4}{3} \frac{\alpha_{\text{eff}}(T)}{r} e^{-m_D(T)r} + 2F_Q$$

$g_{\text{eff}} \equiv \sqrt{4\pi\alpha_{\text{eff}}}$: effective running coupling

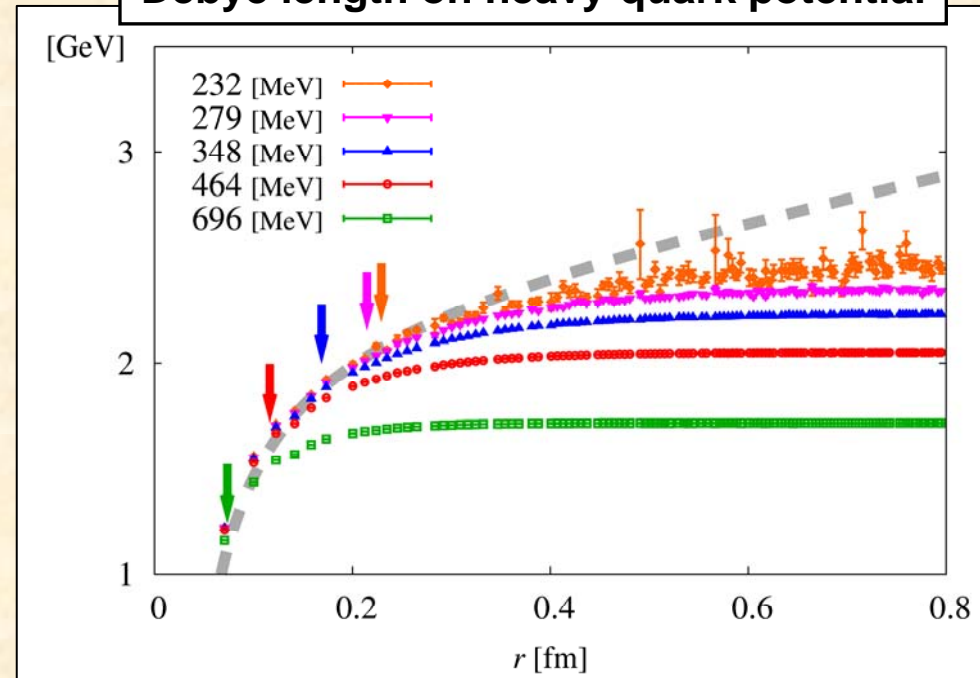
$m_D (= 1/\lambda_D)$: Debye screening mass (length)



Debye length



Debye length on heavy-quark potential



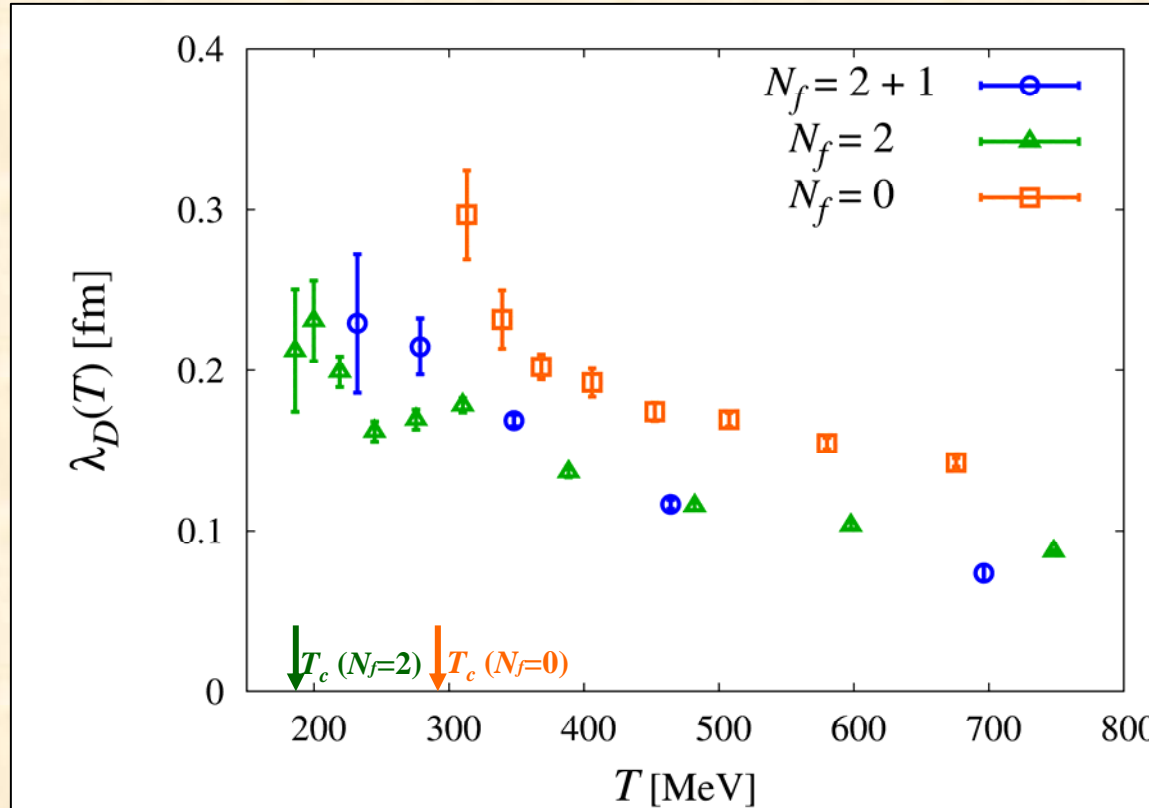
➤ λ_D small when T increases

➤ λ_D : distance where $F^1(r, T)$ departs from $V(r)$

$$\Rightarrow r < \lambda_D : F^1(r, T) \sim -\frac{\alpha_0}{r}, \quad r > \lambda_D : F^1(r, T) \sim -\frac{\alpha(T)}{r} \exp(-r/\lambda_D)$$

Debye screening effect in QGP

Flavor dependence of Debye length



$N_f = 2$: fixed Nt ap.

$16^3 \times 4, m_s/m_p = 0.65$

WHOT, PRD75 (2007) 074501

$N_f = 0$: fixed scale ap.

$20^3 \times (26-8)$

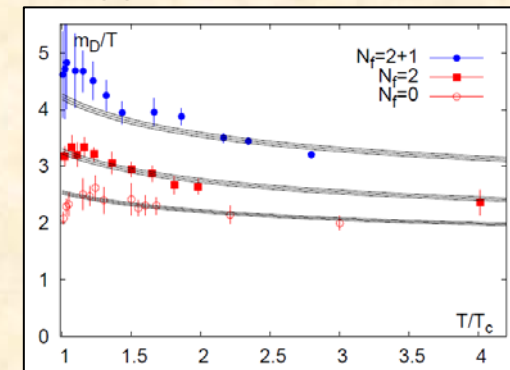
WHOT, in preparation

Magnitude of Debye length

$$\lambda_D^{N_f=2+1} \sim \lambda_D^{N_f=2} < \lambda_D^{N_f=0}$$

Dynamical quark effects are important for Debye effect.

c.f.) Staggered-type quark action



RBC-Bielefeld collaboration

Fixed Nt ap., $m_s \sim 220$ MeV

Summary

Properties of quark-gluon plasma

⇒ **Heavy-quark potential ($F^1(r, T)$)** defined by free energy btw. static charged quarks

1. Heavy-quark potential at $T = 0$ and $T > 0$

➤ At short r : F^1 at any T converges to heavy-quark potential at $T = 0$

⇒ Short distance physics is insensitive to T .

➤ At mid r : screening effect appears

➤ At long r : F^1 becomes flat and has no linear behavior.

⇒ Correlation b/w Polyakov-lines disappears and F^1 converges to $2F_Q$

2. Debye screening effect in QGP

➤ Debye length by single gluon exchange ansatz: $F^1(r, T) = -\frac{\alpha(T)}{r} \exp(-r / \lambda_D) + 2F_Q$

⇒ λ_D small when T increases
 $r < \lambda_D$: Coulomb attraction, $r > \lambda_D$: screening by plasma

$$F^1(r, T) \sim -\frac{\alpha_0}{r}, \quad F^1(r, T) \sim -\frac{\alpha(T)}{r} \exp(-r / \lambda_D)$$

Dynamical quark effects are important for Debye effect.

