Roberge-Weiss transition and Dashen mechanism

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Imaginary chemical potential

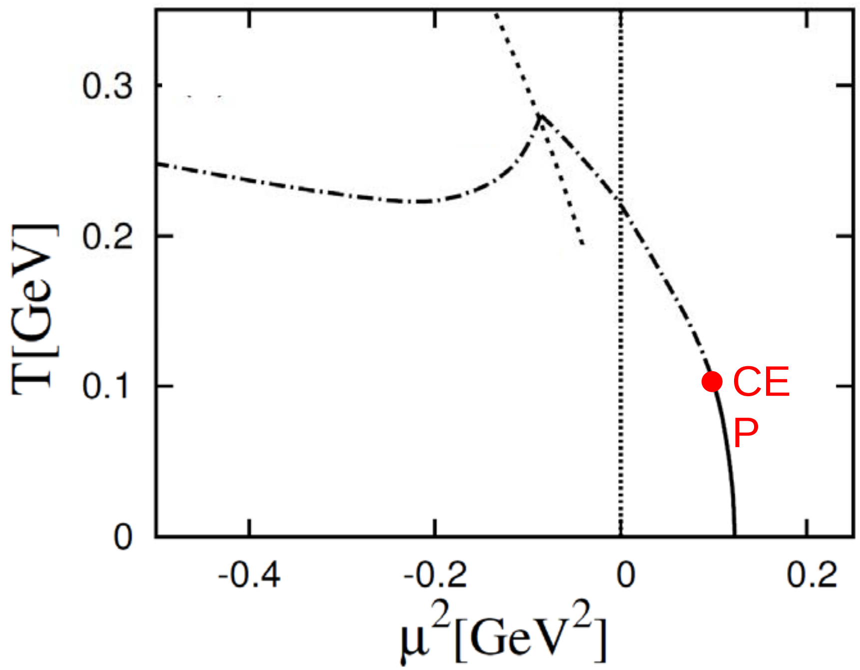
- There is a sign Problem at real μ .
- Extrapolation from results at imaginary μ to results at real μ .
- Determine the parameters of effective model at imaginary $\mu = i\theta T$.
- Study the properties of deconfinement!!

Deconfinement transition

- There is no exact symmetry at real μ .
- The dominance of Chiral transition at real μ .
  $\Rightarrow$Success of NJL model
- There is an exact symmetry at imaginary μ .
  Extended Z_3 symmetry
- There is a phase transition at imaginary μ .
  Roberge-Weiss (RW) transition = C violation
  Nucl. Phys. B275 (1986) 734.

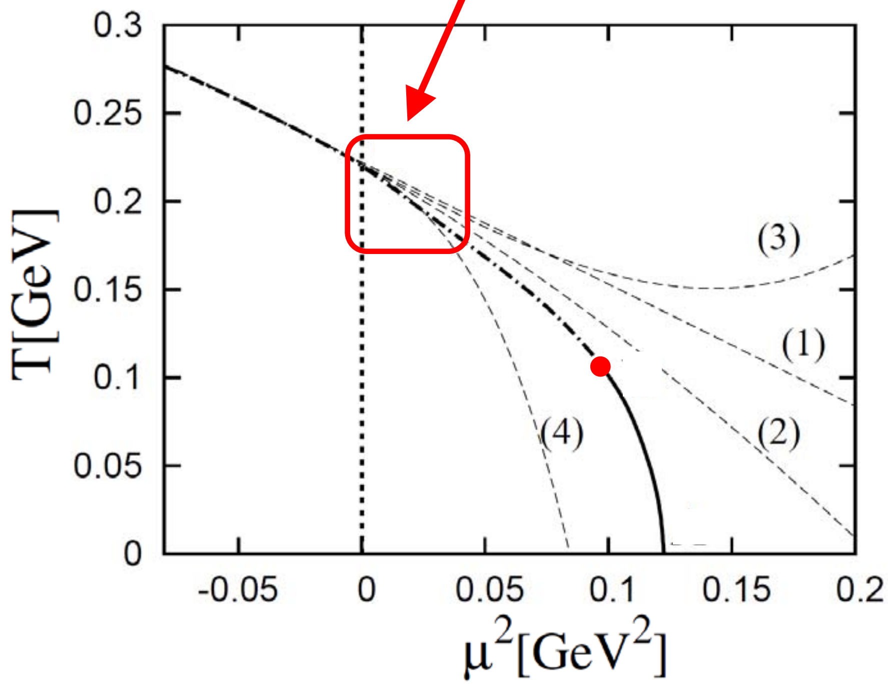
Phase diagram

PNJL model

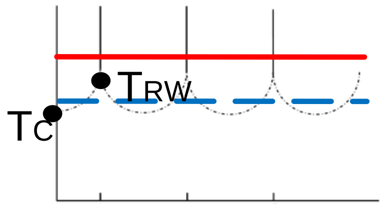


多項式近似による外挿

Lattice QCD



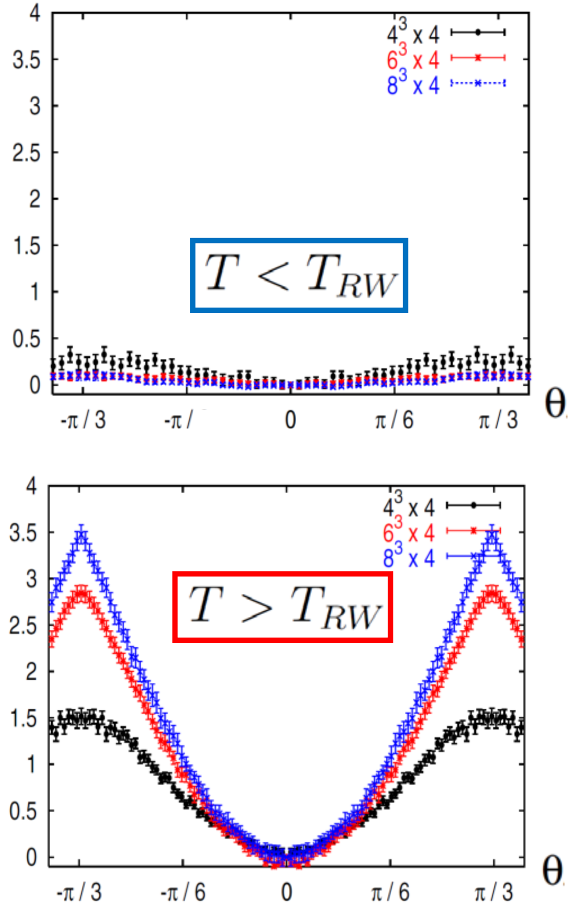
$$T = \sum_{n=0}^{m} a_n \mu^{2n} \quad \text{(m=1,2,3,4)}$$

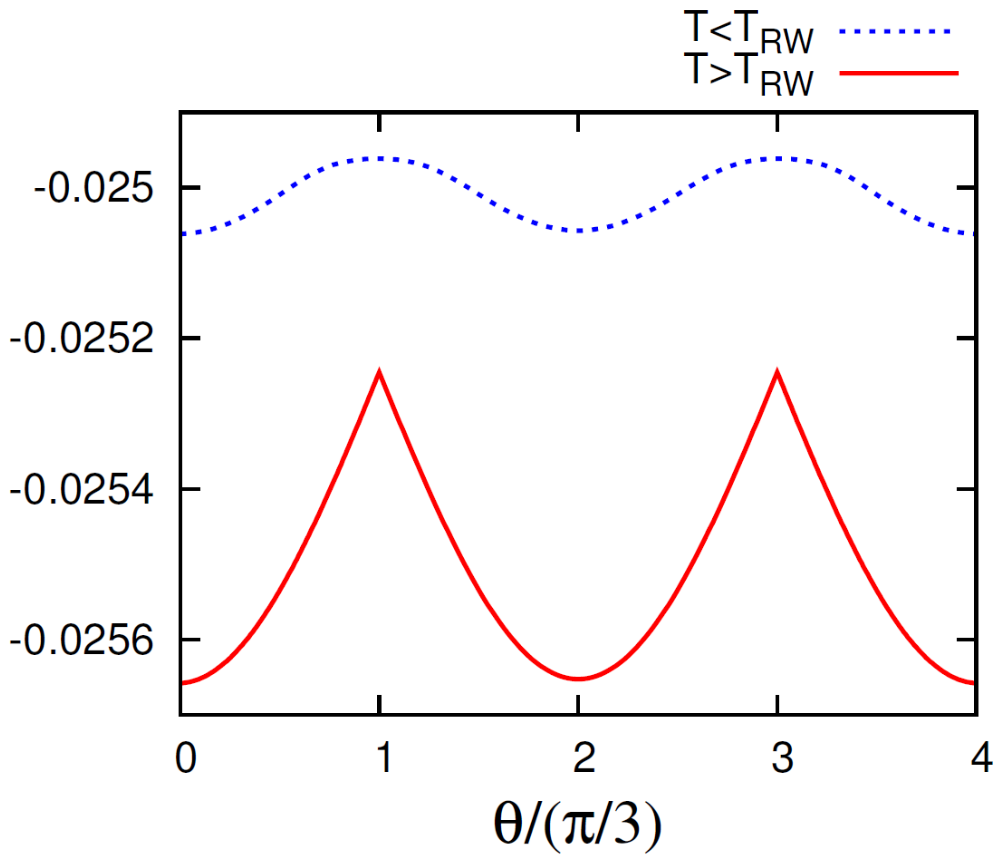


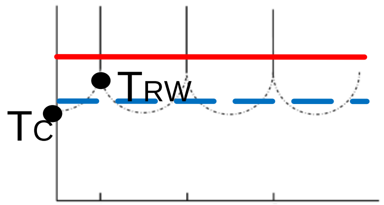
Roberg-Wess periodicity and transition

$$\Omega(\theta) = \Omega(\theta + 2\pi k/3) = \Omega(-\theta).$$

Kratochvila, Forcrand PRD**73**,114512(2006)



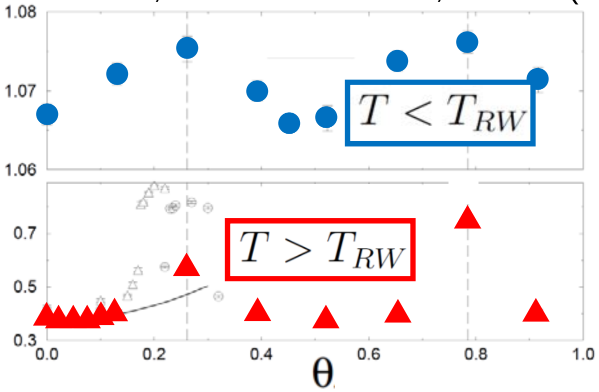


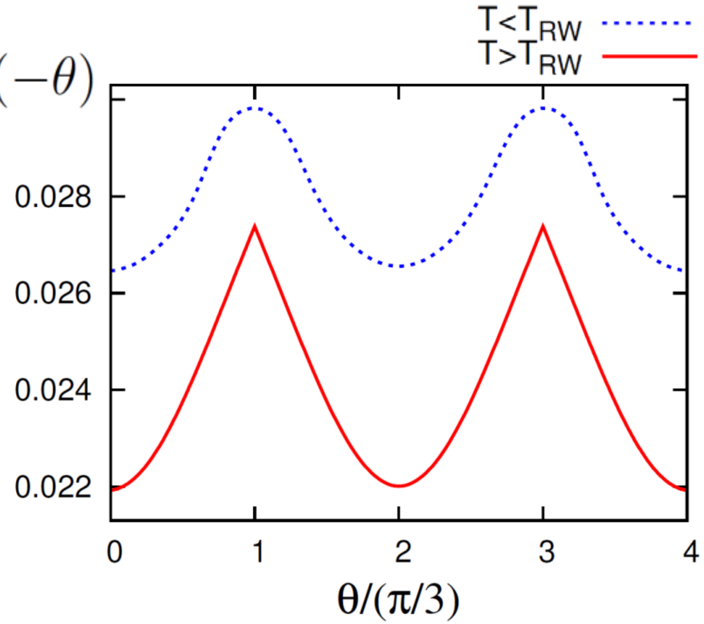


Chiral condensate and quark density

$$\sigma(\theta) = \sigma(\theta + 2\pi k/3) = \sigma(-\theta)$$

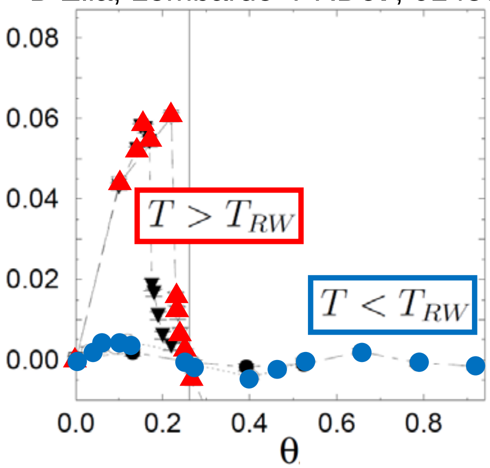
D'Elia, Lombardo PRD**67**, 014505 (2003)

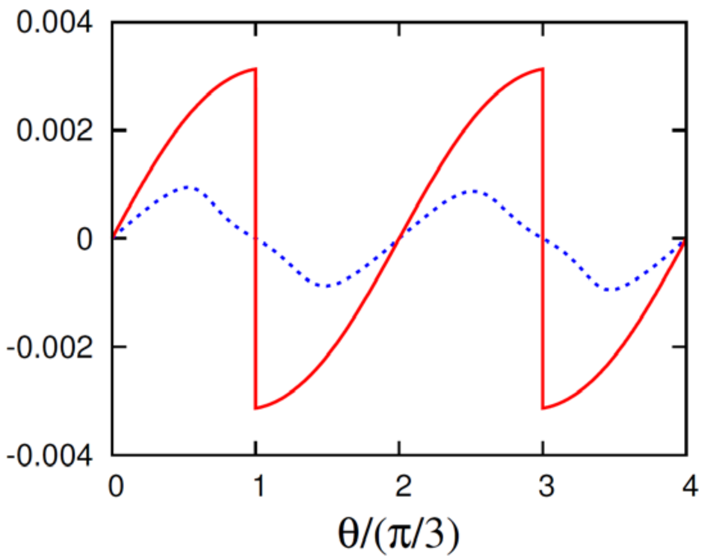


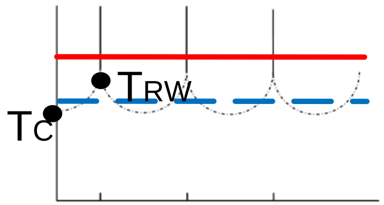


$$n(\theta) = n(\theta + 2\pi k/3) = -n(-\theta)$$

D'Elia, Lombardo PRD**67**, 014505 (2003)

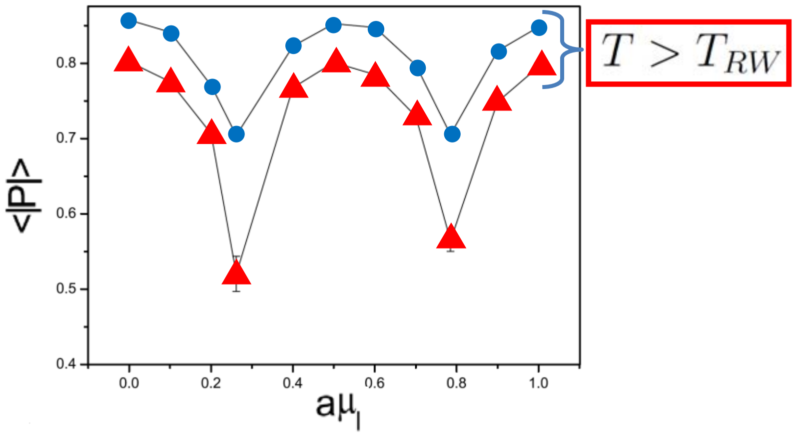


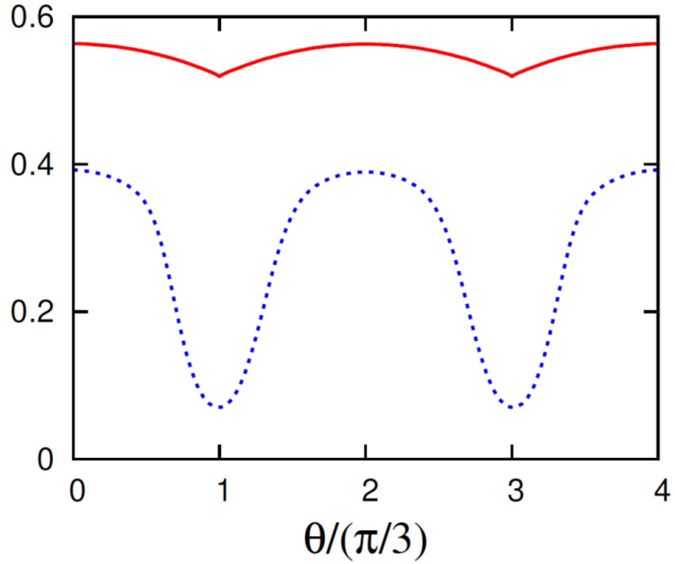




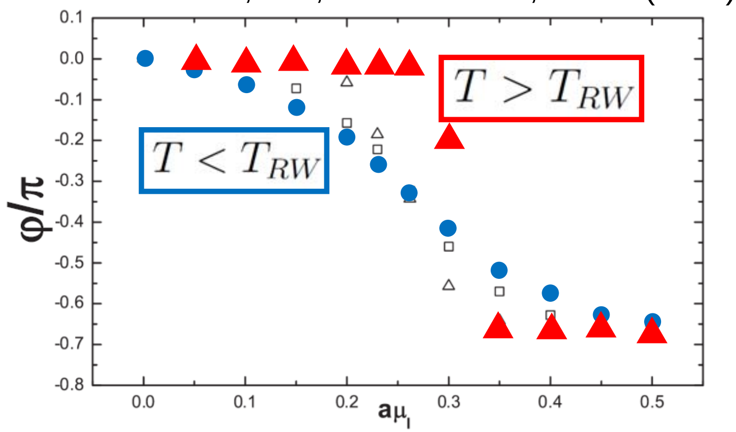
Polyakov Loop

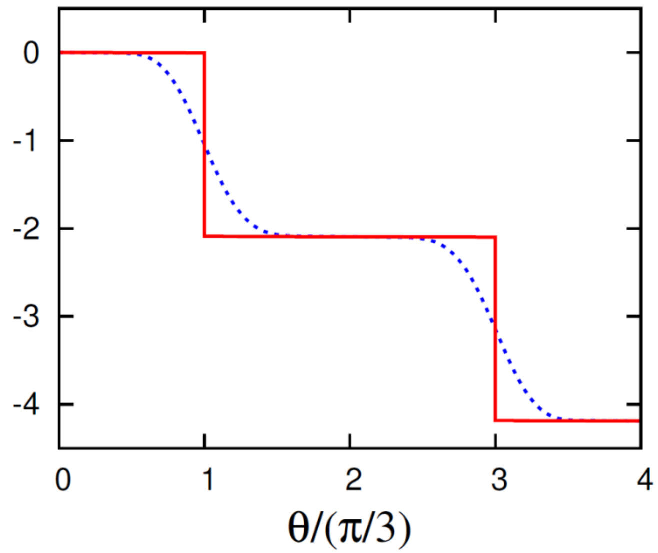
Chen, Luo PRD**72**, 034504 (2005)





Wu, Luo, Chen PRD**76**,034505(2007)





PNJL model

- K. Fukushima, Phys. Lett. B591, 277(2004)

  Polyakov loop+NJL

- Chiral symmetry

  extended Z_3symmetry

H. K., Y. Sakai, K. Kashiwa, M. Yahiro,

arXiv:0904.0925(hep-ph), to be published

in J. Phys. G

RW periodicity and extended Z_3 symmetry（EZ_3）

$$\theta = \mu_I/T$$

$$\Omega = -2N_f \int \frac{d^3p}{(2\pi)^3}\Big[3E(p) + \frac{1}{\beta}\ln\ [1 + 3(\Phi + \Phi^* e^{-\beta E^-(\mathbf{p})})e^{-\beta E^-(\mathbf{p})} + e^{-3\beta E^-(\mathbf{p})}] + \frac{1}{\beta}\ln\ [1 + 3(\Phi^* + \Phi e^{-\beta E^+(\mathbf{p})})e^{-\beta E^+(\mathbf{p})} + e^{-3\beta E^+(\mathbf{p})}]\Big] + U_M + \mathcal{U}$$

$$E(p) = \sqrt{\mathbf{p}^2 + M^2}\,,\ \sigma = \langle\bar{q}q\rangle, \quad \Sigma_s = -2G_s\sigma, \quad U_M = G_s\sigma^2,\ M = m_0 + \Sigma_s.$$

extended Z_3 trans.

$$\theta \to \theta + 2\pi k/3,\ \ \Phi(\theta) \to \Phi(\theta)e^{-i2\pi k/3} \qquad \theta = \mu_I/T$$

修正版Polyakovループ $\Psi \equiv e^{i\theta}\Phi$

$$\theta \to \theta + 2\pi k/3\,, \quad \Psi(\theta) \to \Psi(\theta), \quad \Psi(\theta)^* \to \Psi(\theta)^*$$

Thermodynamic potential

$$\theta = \mu_I/T$$

$$\Omega = -2N_f \int \frac{d^3p}{(2\pi)^3}\Big[3E(p) + \frac{1}{\beta}\ln\left[1 + 3\Psi e^{-\beta E(\mathbf{p})} + 3\Psi^* e^{-2\beta E(\mathbf{p})} e^{3i\theta} + e^{-3\beta E(\mathbf{p})} e^{3i\theta}\right]$$
$$+\frac{1}{\beta}\ln\left[1 + 3\Psi^* e^{-\beta E(\mathbf{p})} + 3\Psi e^{-2\beta E(\mathbf{p})} e^{-3i\theta} + e^{-3\beta E(\mathbf{p})} e^{-3i\theta}\right]\Big] + U_M + \mathcal{U}$$

$$E(p) = \sqrt{\mathbf{p}^2 + M^2}, \ \sigma = \langle\bar{q}q\rangle, \quad \Sigma_s = -2G_s\sigma, \quad U_M = G_s\sigma^2, \ M = m_0 + \Sigma_s.$$

extended Z_3 trans.

$$\theta \to \theta + 2\pi k/3, \ \ \Phi(\theta) \to \Phi(\theta)e^{-i2\pi k/3}$$

修正版Polyakovループ $\Psi \equiv e^{i\theta}\Phi$ $\quad \theta \to \theta + 2\pi k/3, \quad \Psi(\theta) \to \Psi(\theta)$

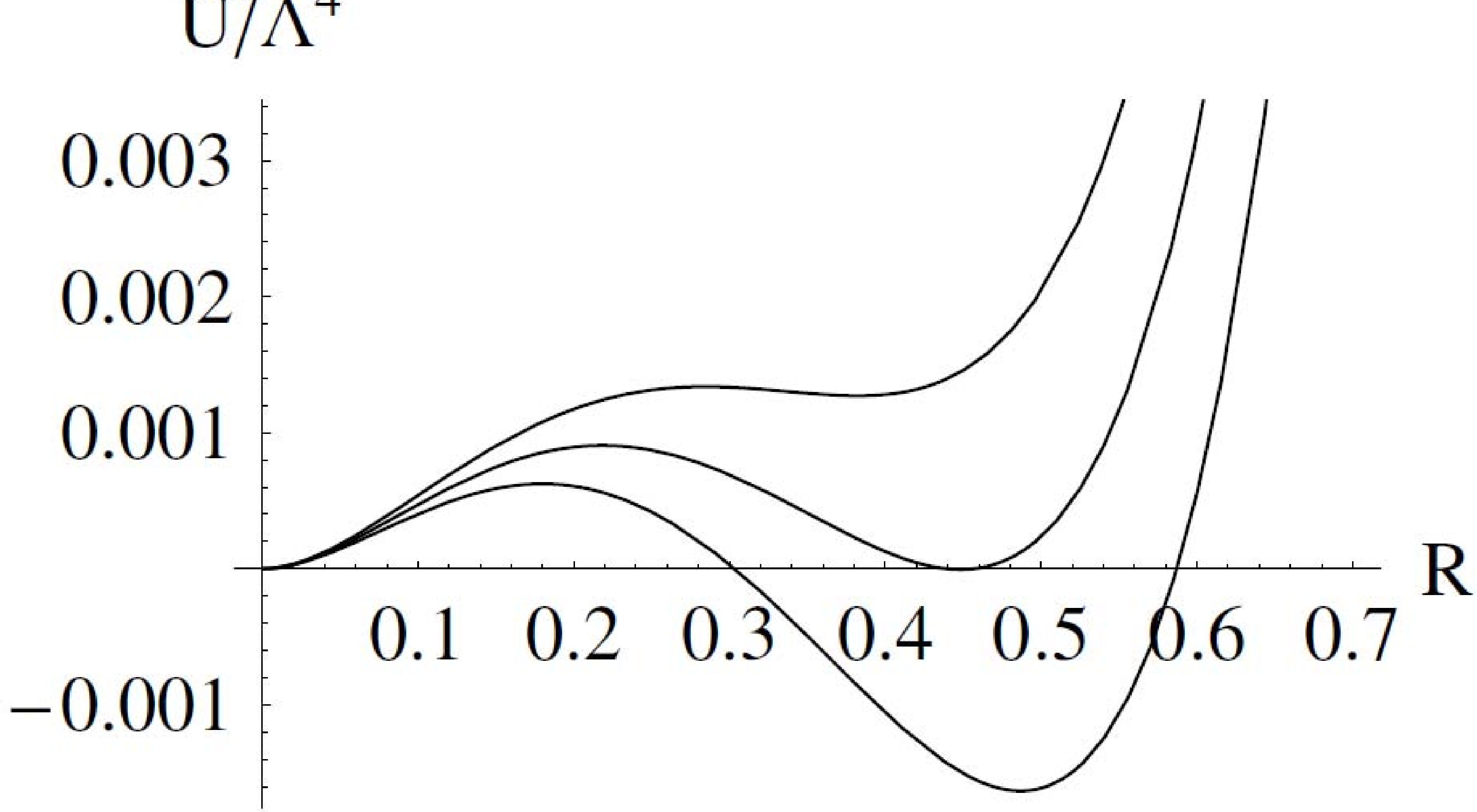
$$\Omega(\theta) = \Omega(\Psi(\theta), \Psi(\theta)^*, e^{3i\theta})$$

Extended Z_3 inv.

Deconfinement in pure gauge

- At $T=T_c$, $\Phi(T)$ jumps from 0 to finite value.
- At low temperature, phase of Φ can not be defined since $\Phi=0$.
- At high temperature, phase ϕ of Φ can be defined since $|\Phi|>0$.

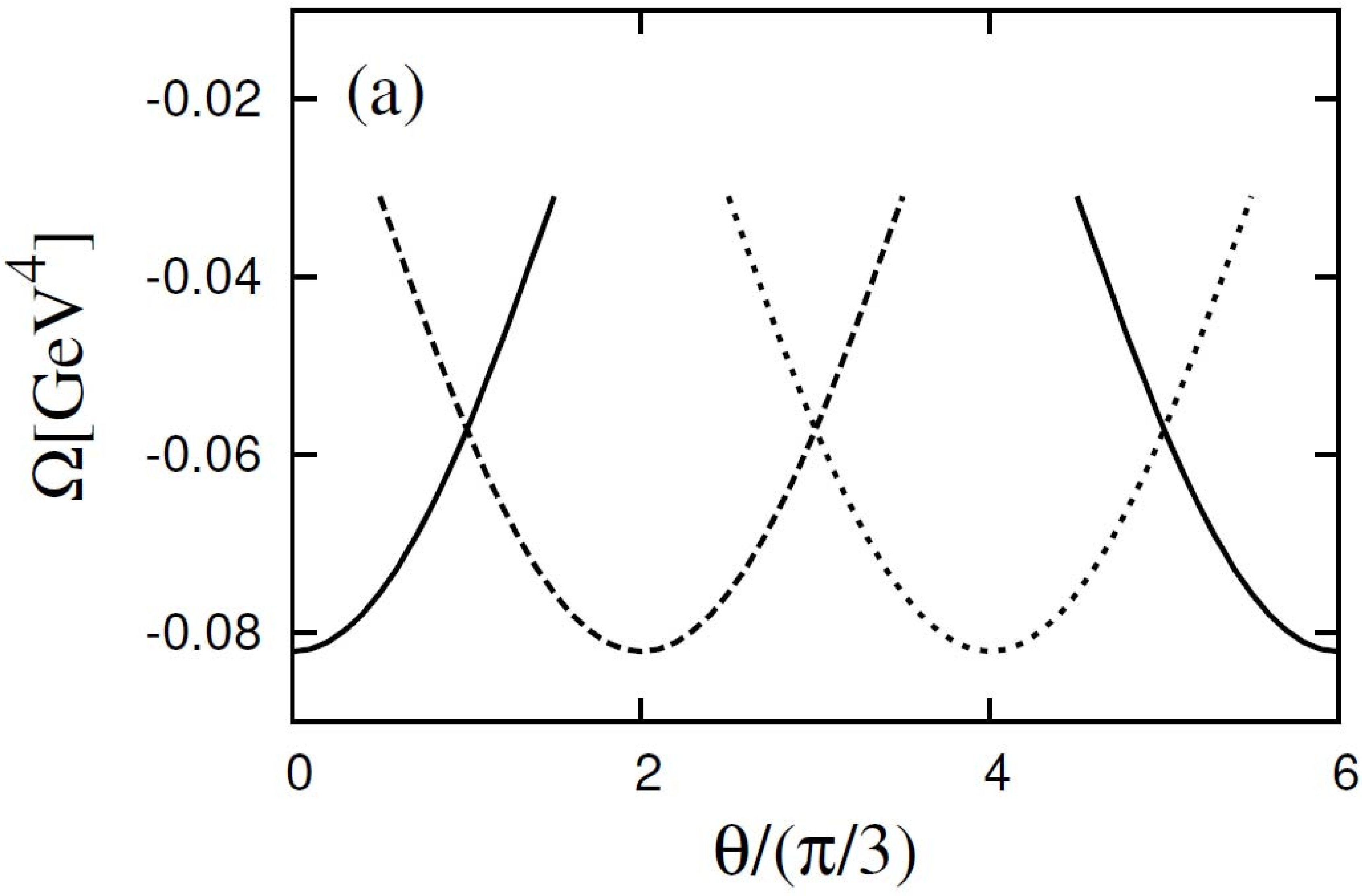
$$\phi = 2k/3 \quad (k=0,1,2)$$



U/Λ^4
0.003
0.002
0.001
−0.001
R
0.1
0.2
0.3
0.4
0.5
0.6
0.7

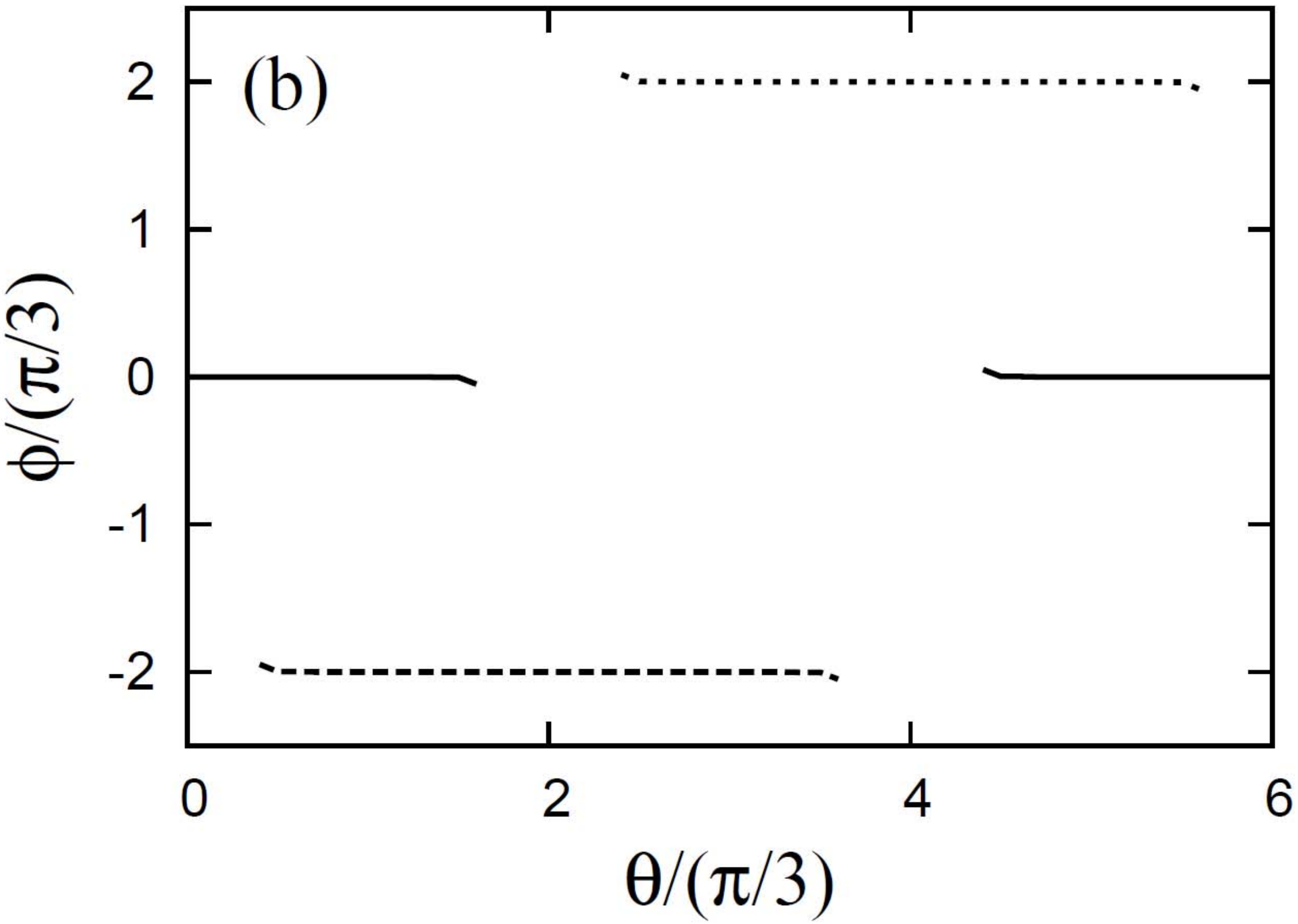
RW transition

- θ-even quantity has a cusp at $T>T_{RW}$, $\theta=(2k+1)\pi/3$ (k integer), while θ-odd quantity is discontinuous there.
- At high temperature, there are three continuous solutions with different phases of Polyakov-loop.
- One solution is transformed into the other solutions by Z_3 transformation.

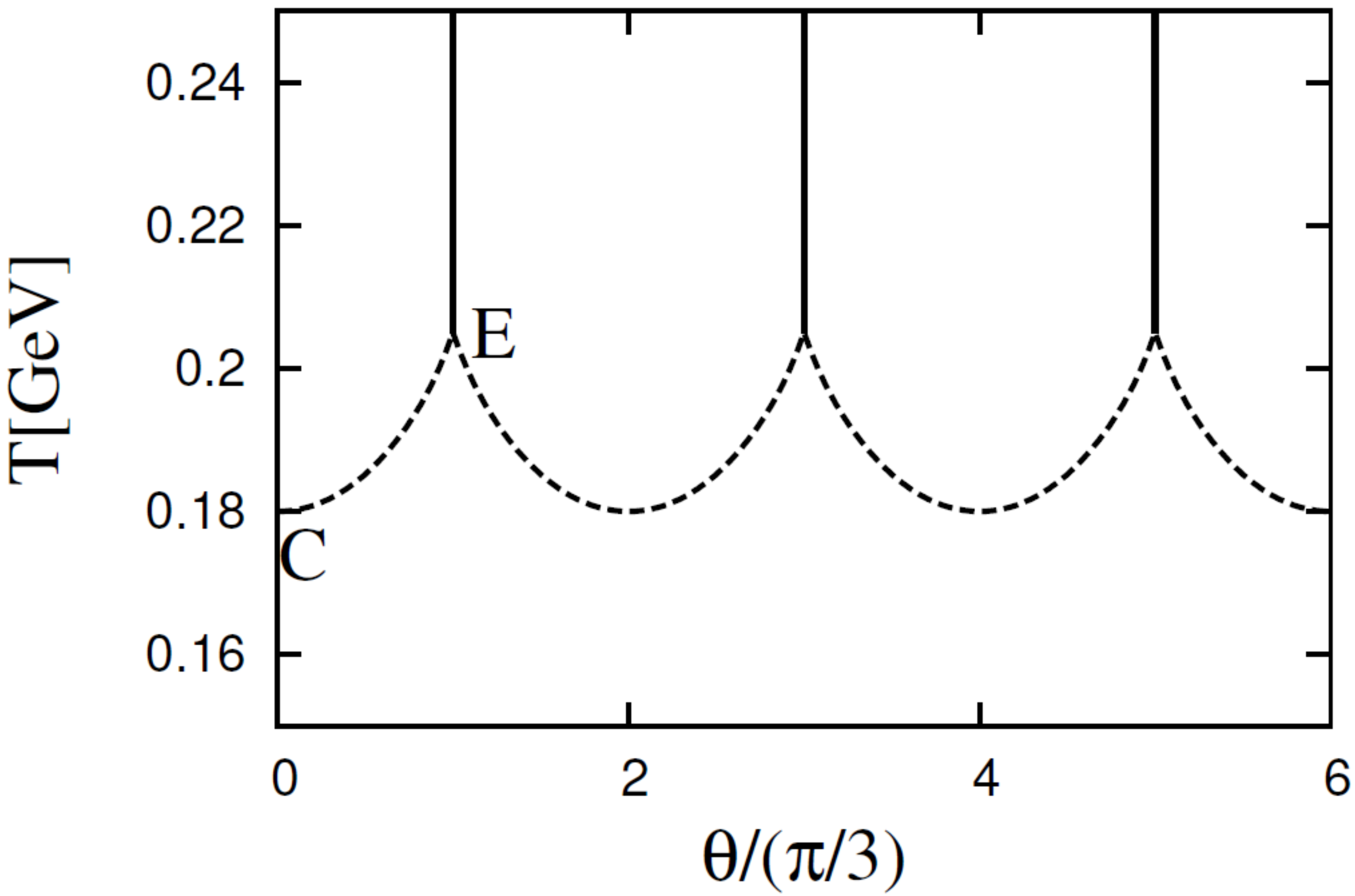


(a)
-0.02
-0.04
-0.06
-0.08
Ω[GeV^4]
0
2
4
6
θ/(π/3)

ＲＷ論文 Fig. 8-(b)

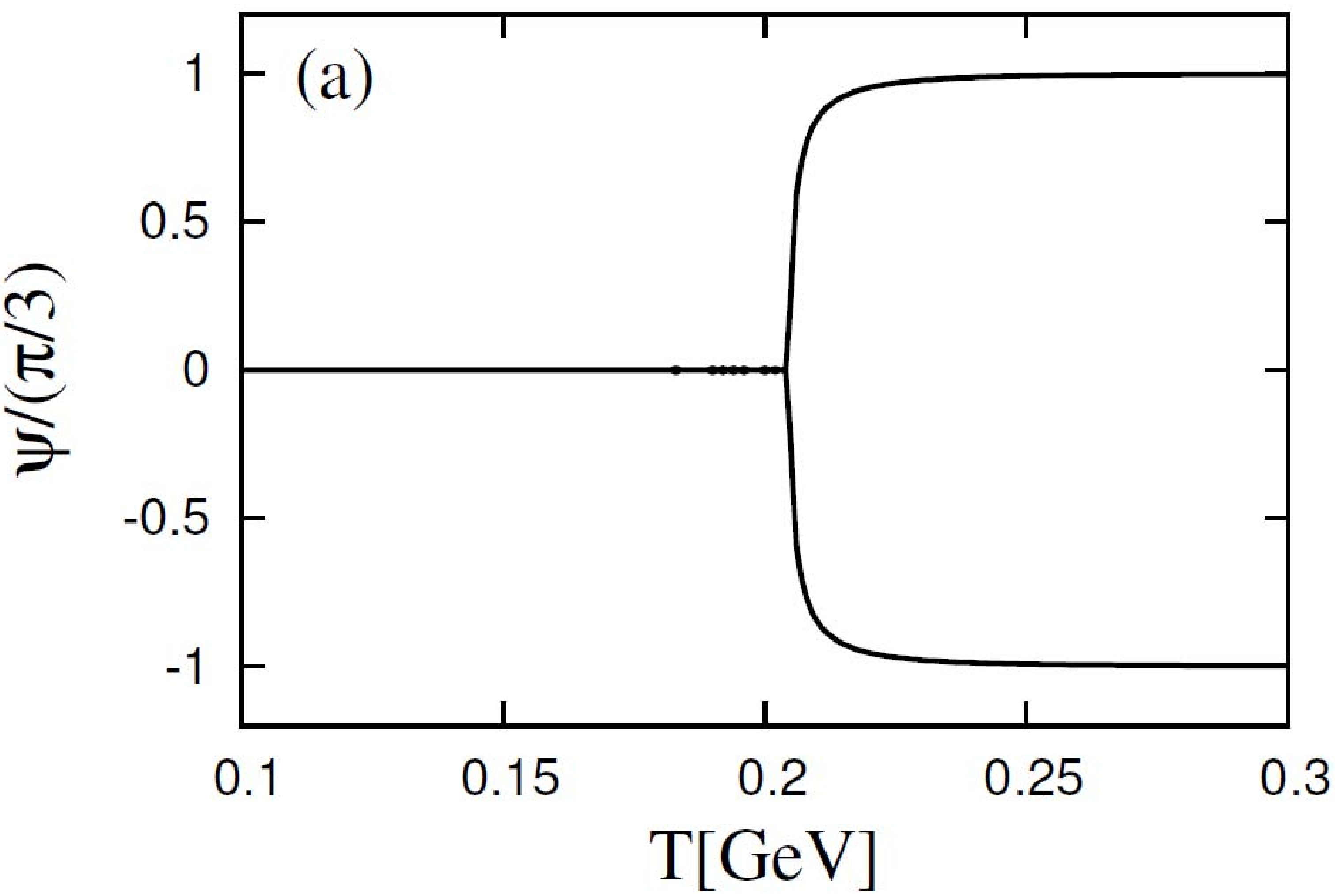


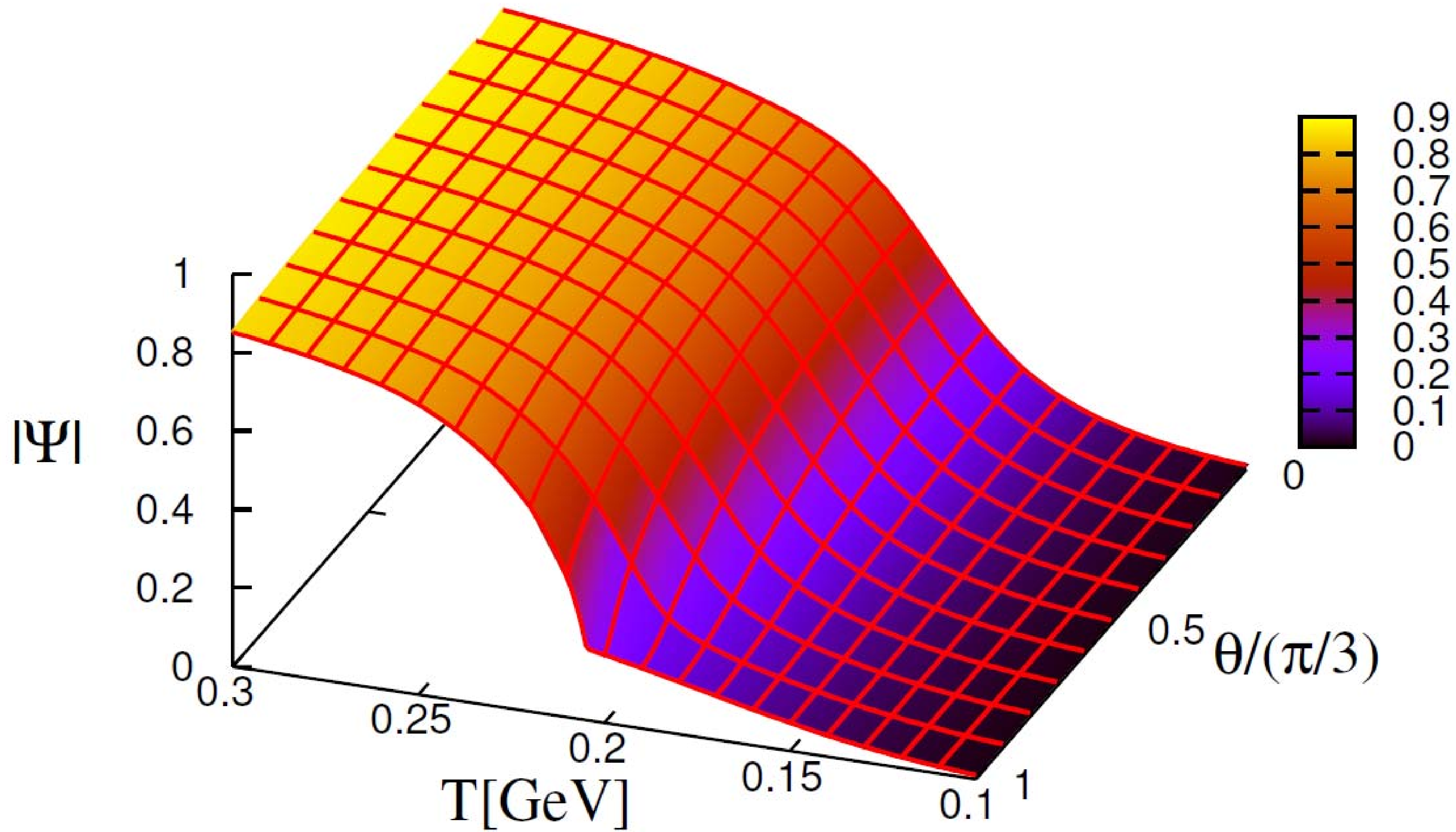
Phase diagram



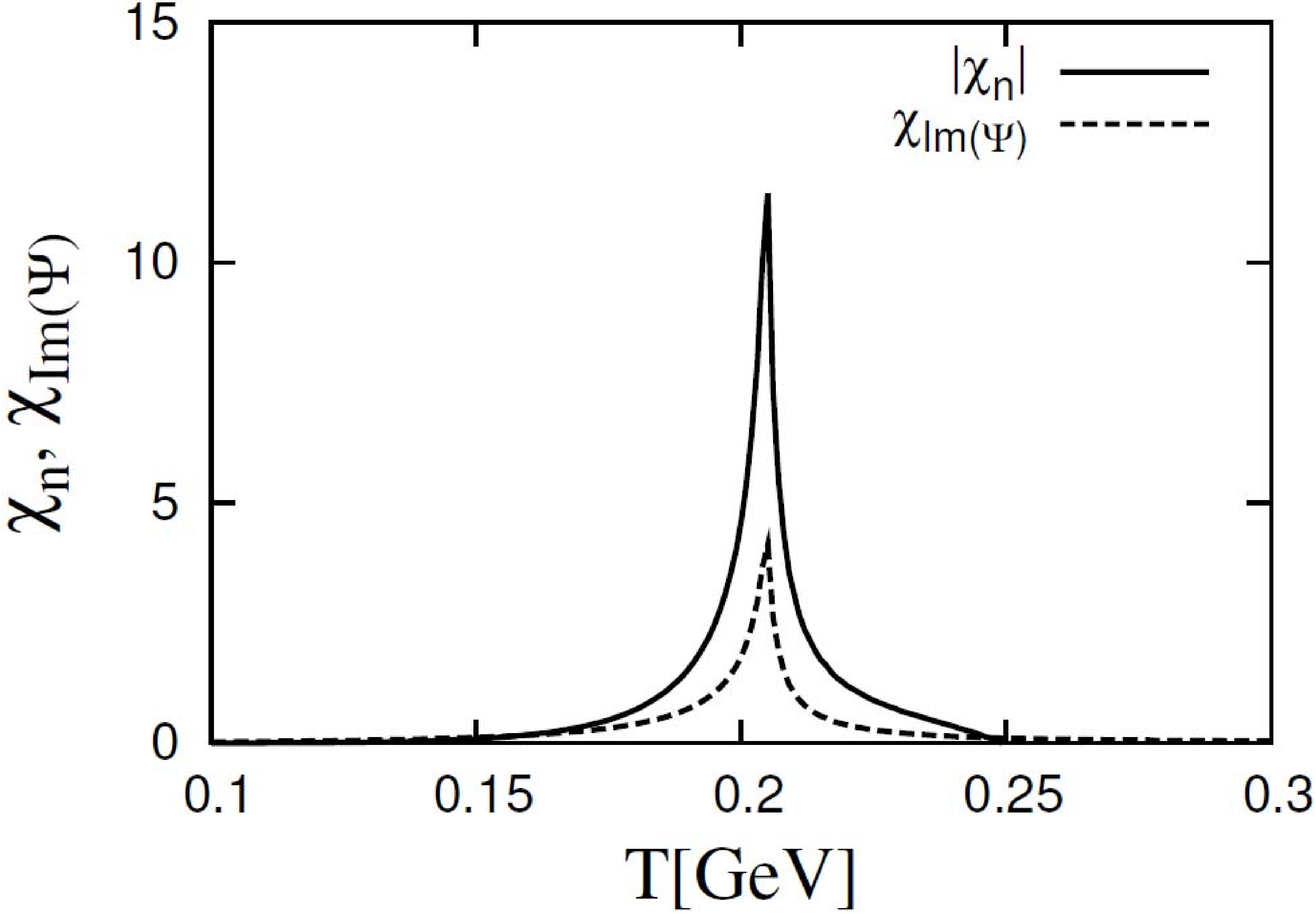
Charge conjugation symmety

- At $\Theta = k\pi/3$, Ω is invariant under $\Psi \Leftrightarrow \Psi^*$ (or $\psi \Leftrightarrow -\psi$).
- C-symmetry is preserved if k is even, but broken if k is odd (RW).
- Θ-odd quantities such as ψ or n are order parameters.
- Θ-even quantities has a cusp.

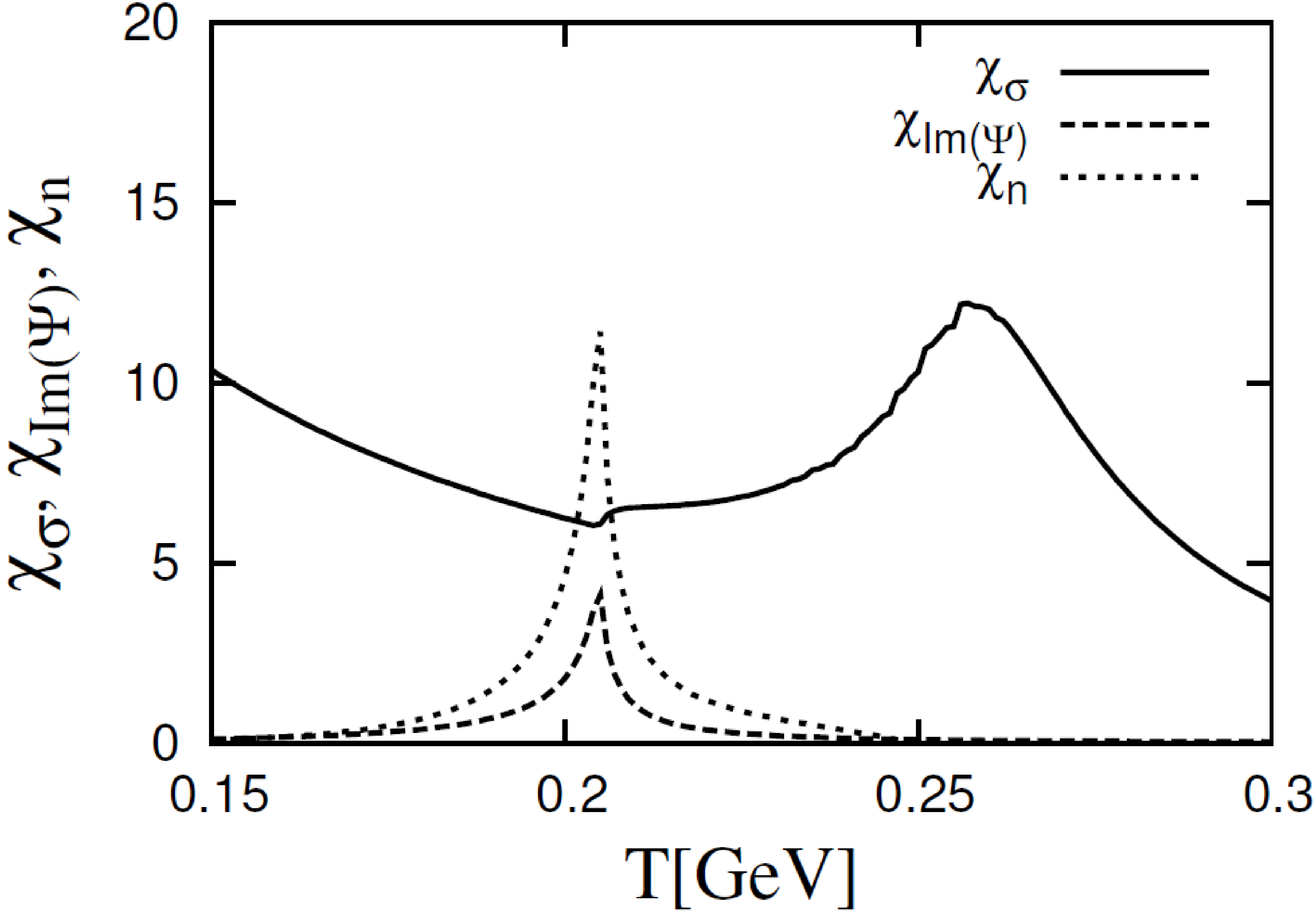

(a)
1
0.5
0
-0.5
-1
ψ/(π/3)
0.1
0.15
0.2
0.25
0.3
T[GeV]



|Ψ|
1
0.8
0.6
0.4
0.2
0
0.3
0.25
0.2
0.15
0.1
T[GeV]
0
0.5
1
θ/(π/3)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

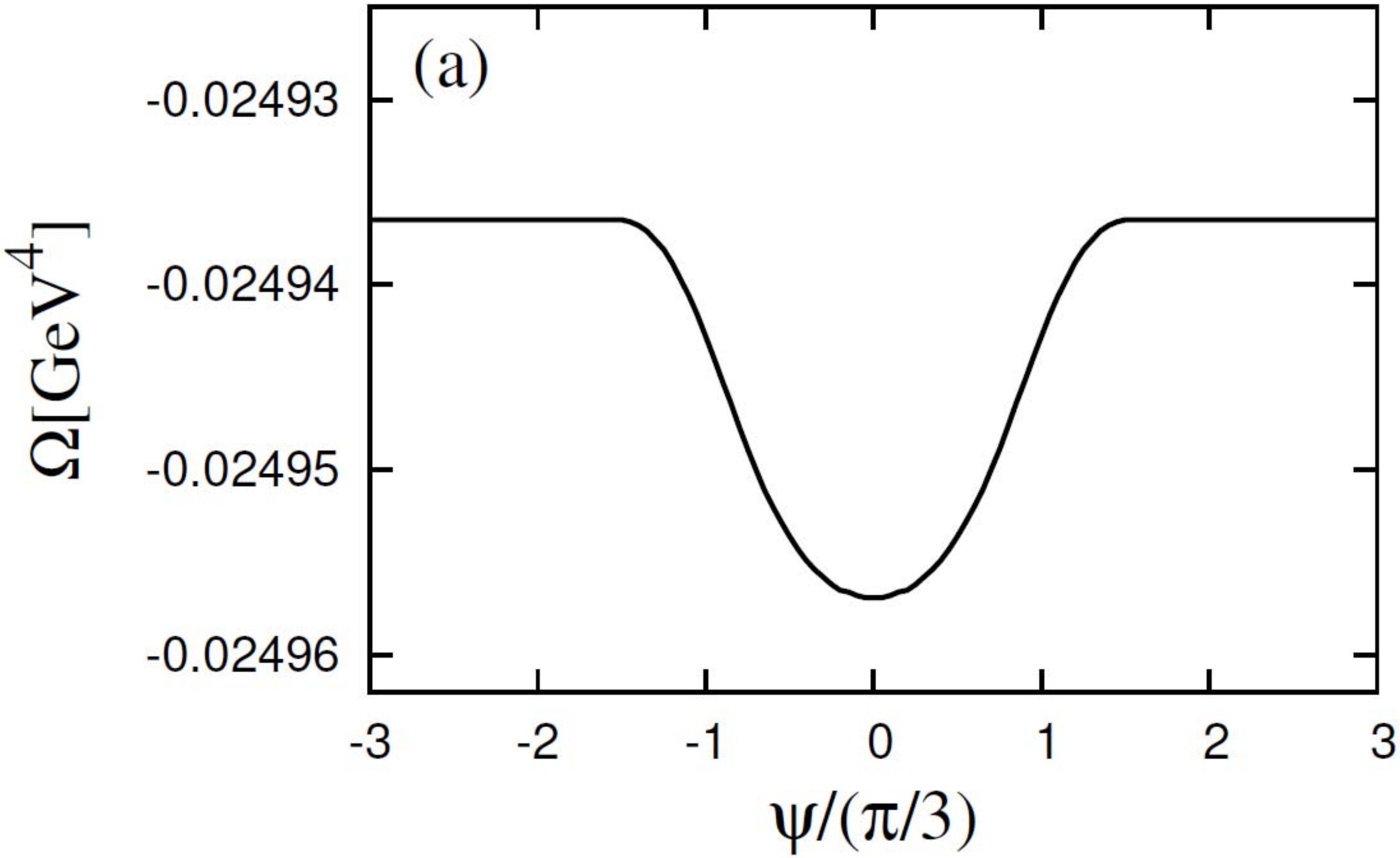


15
10
5
0
0.1
0.15
0.2
0.25
0.3
T[GeV]
χn, χIm(Ψ)
|χn|
χIm(Ψ)

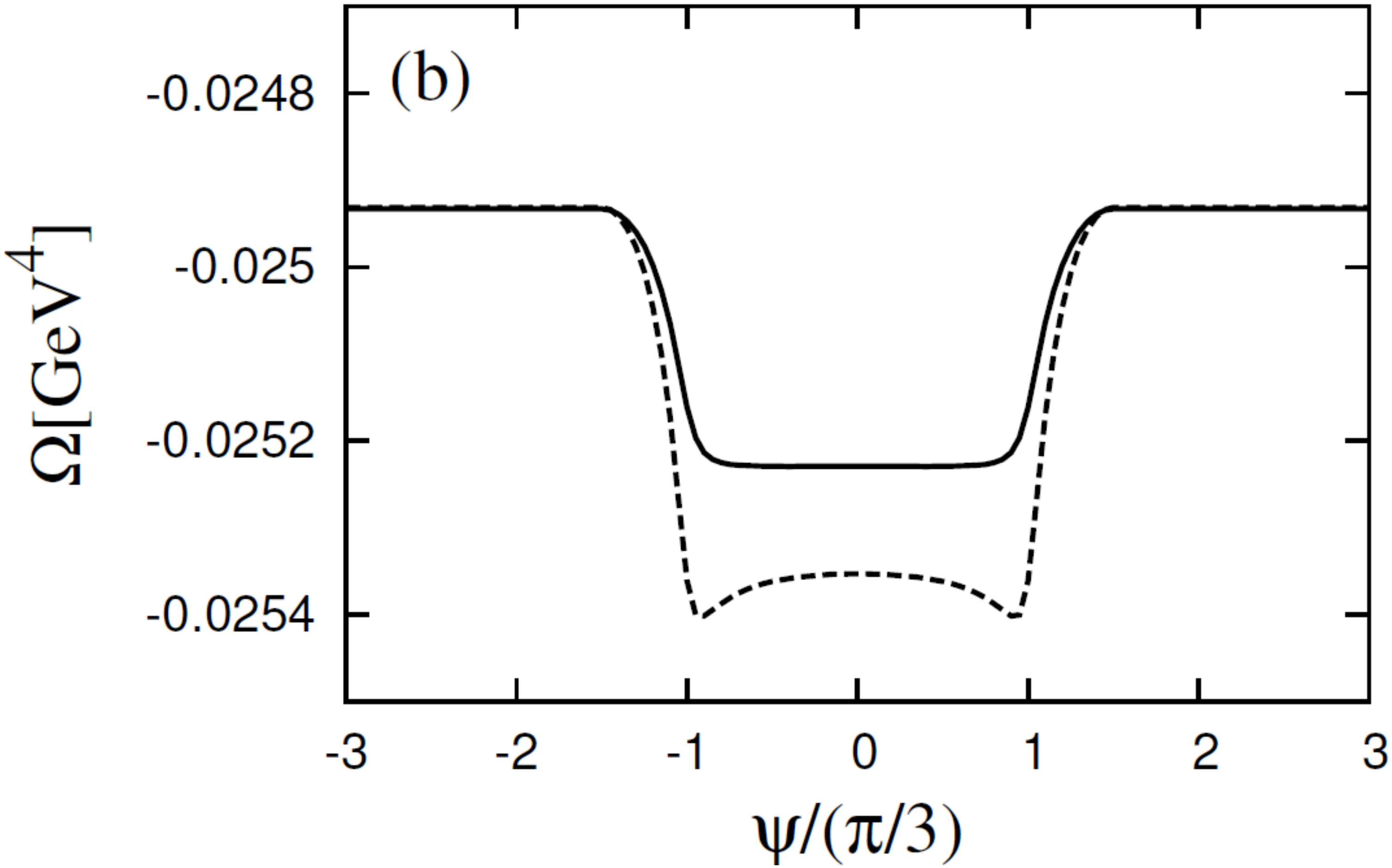


20
15
10
5
0
0.15
0.2
0.25
0.3
T[GeV]
χ_σ, $\chi_{\mathrm{Im}(\Psi)}$, χ_n
χ_σ
$\chi_{\mathrm{Im}(\Psi)}$
χ_n

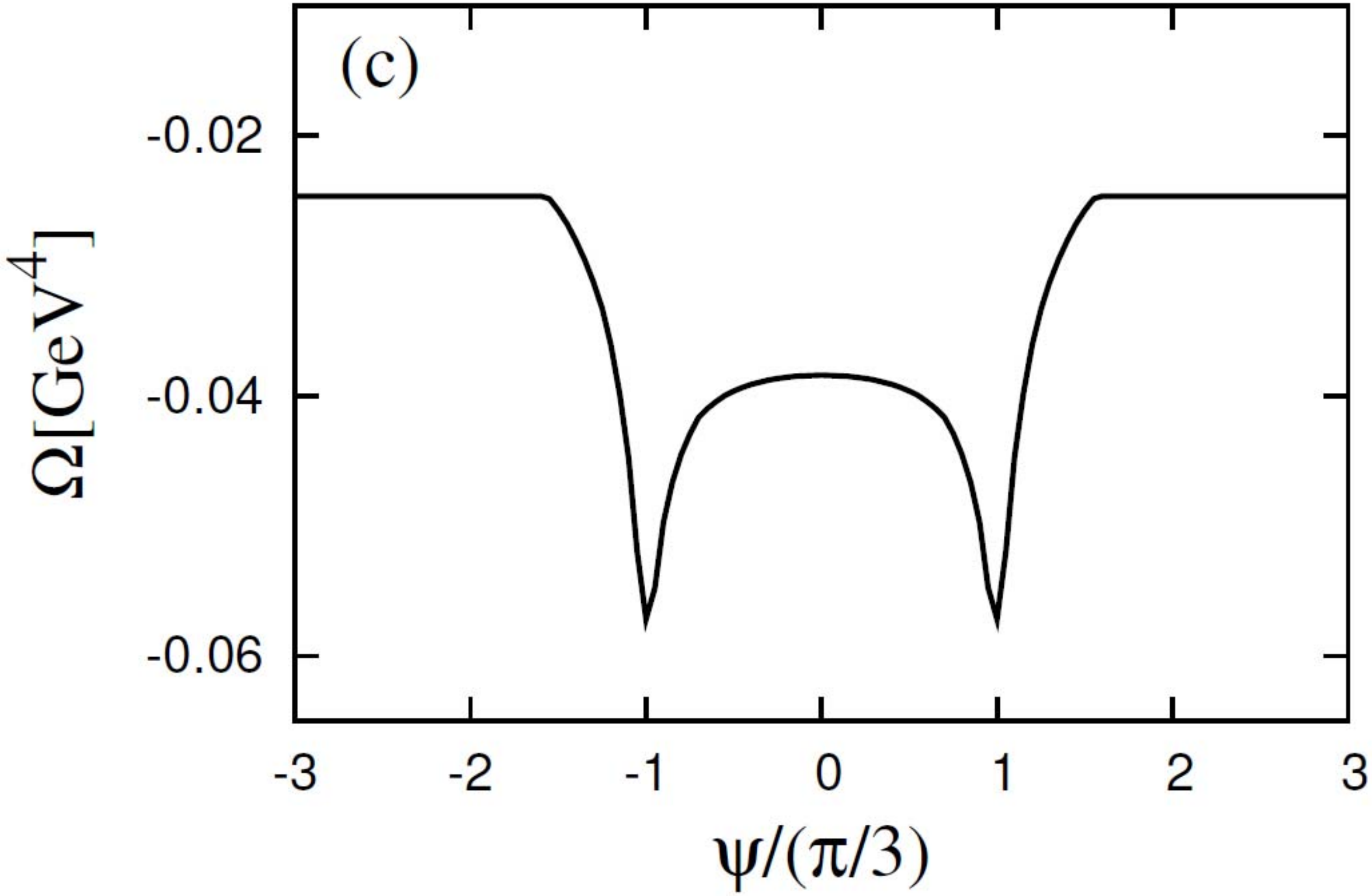
On RW line, low T



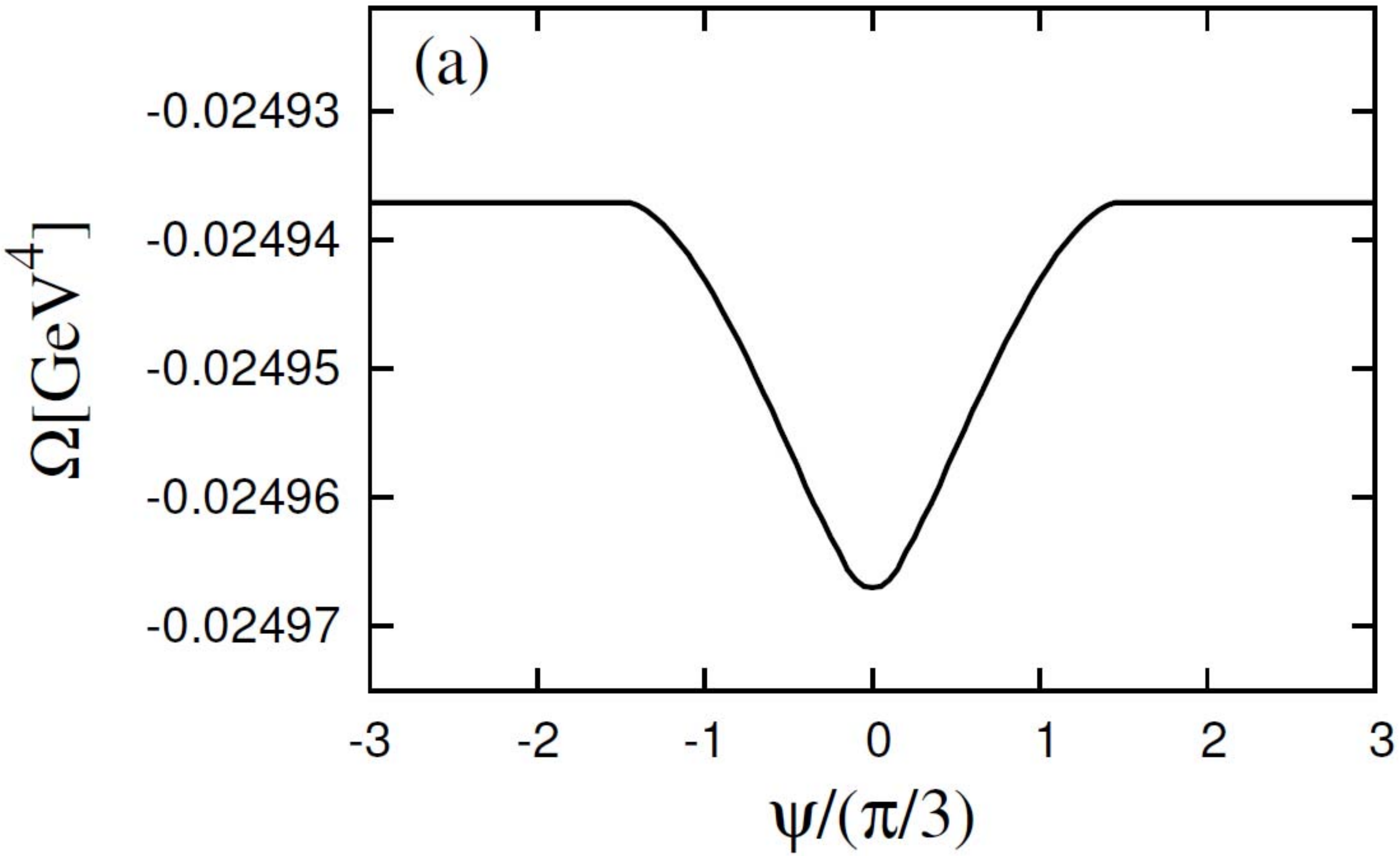
On RW line, near T_E



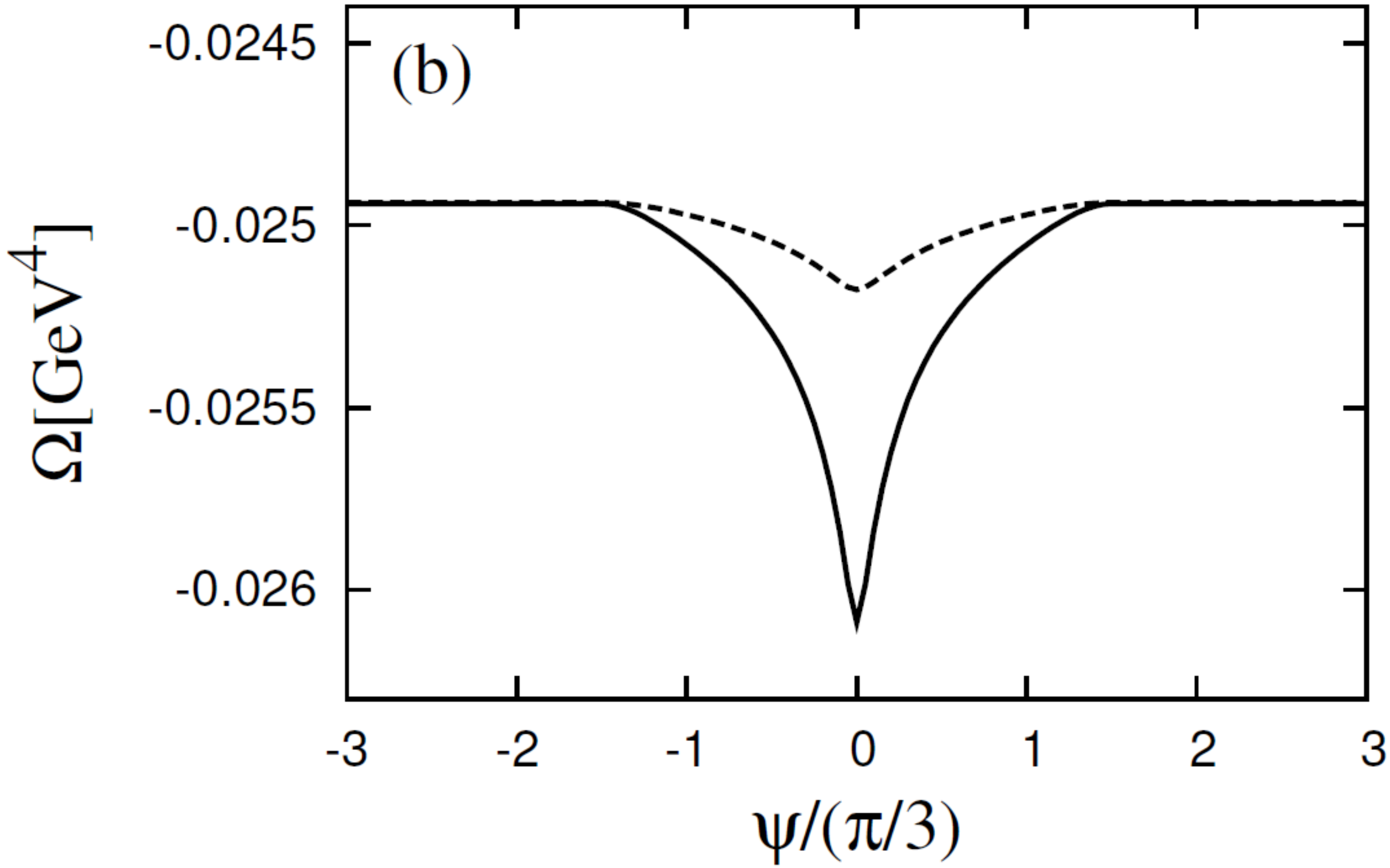
On RW line, high temperature



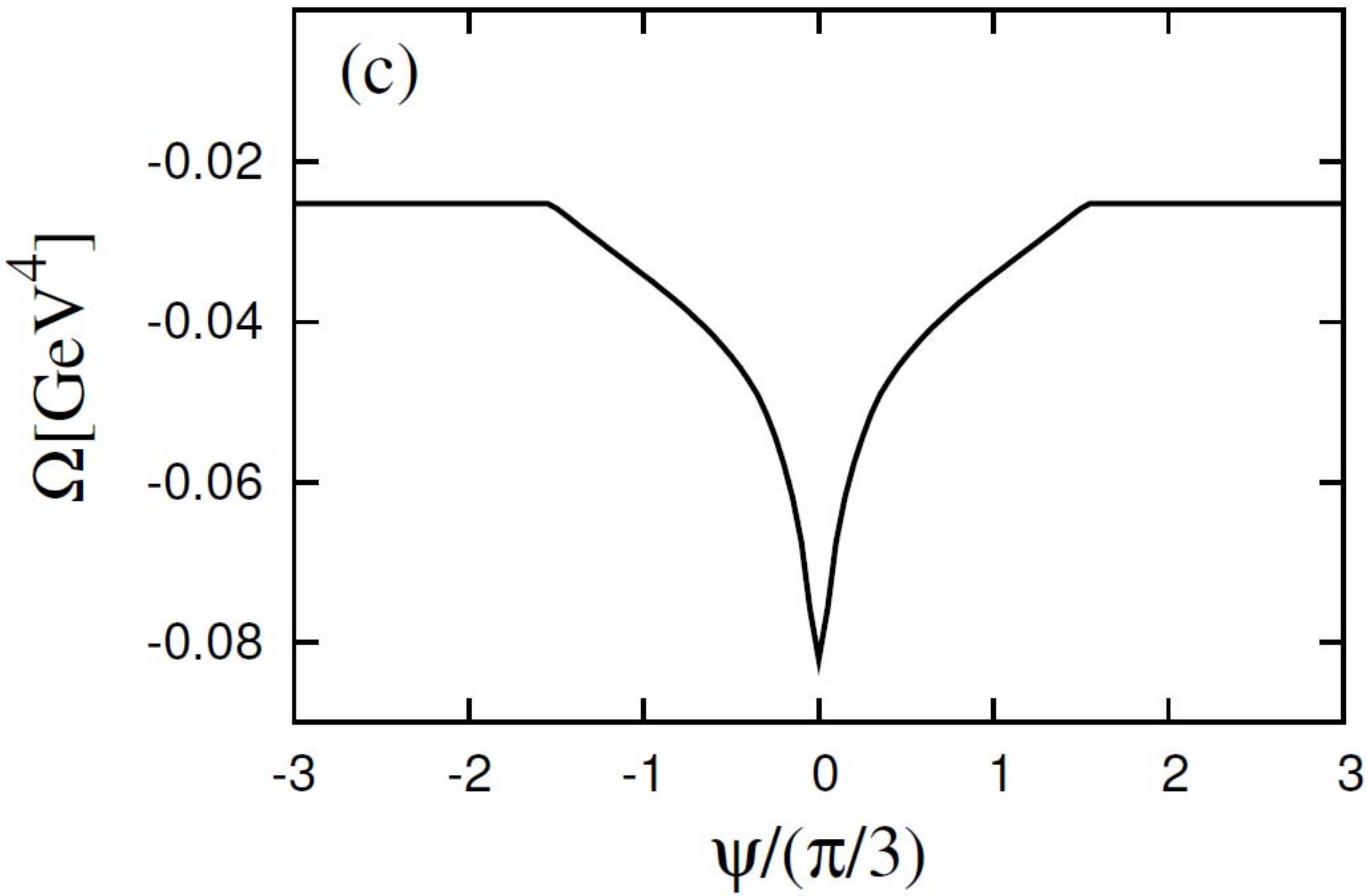
At μ =0, low temperature



At μ =0, near T_E and T_C



μ =0, high temperature



Dashen mechanism

- CP violation in θ vacuum.

Dashen, Phys. Rev. D3, 1879(1971)

NJL version

Boer and Boomsma,

Phys. Rev. D78,054027 (2008)

Θ -term in two-flavor NJL

$$L_{NJL} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m_0 + \gamma_0\mu)\psi + L_4 + L_{\theta}$$

$$L_4 = (1-c)G[(\overline{\psi}\tau_a\psi)^2 + (\overline{\psi}\tau_a i\gamma_5\psi)^2]$$

$$L_{\theta} = cG\det(\overline{\psi}_R\psi_L) + H.c.$$

$$\times e^{i\theta}$$

P violation at large c

- At $\theta = \pi$, there is P symmetry. However, If c is greater than c_{criti}, the P symmetry is spontaneously broken. An order parameter, η, is finite and discontinuous there.

$$\eta = \langle \bar{\psi} i \gamma_5 \psi \rangle \neq 0$$

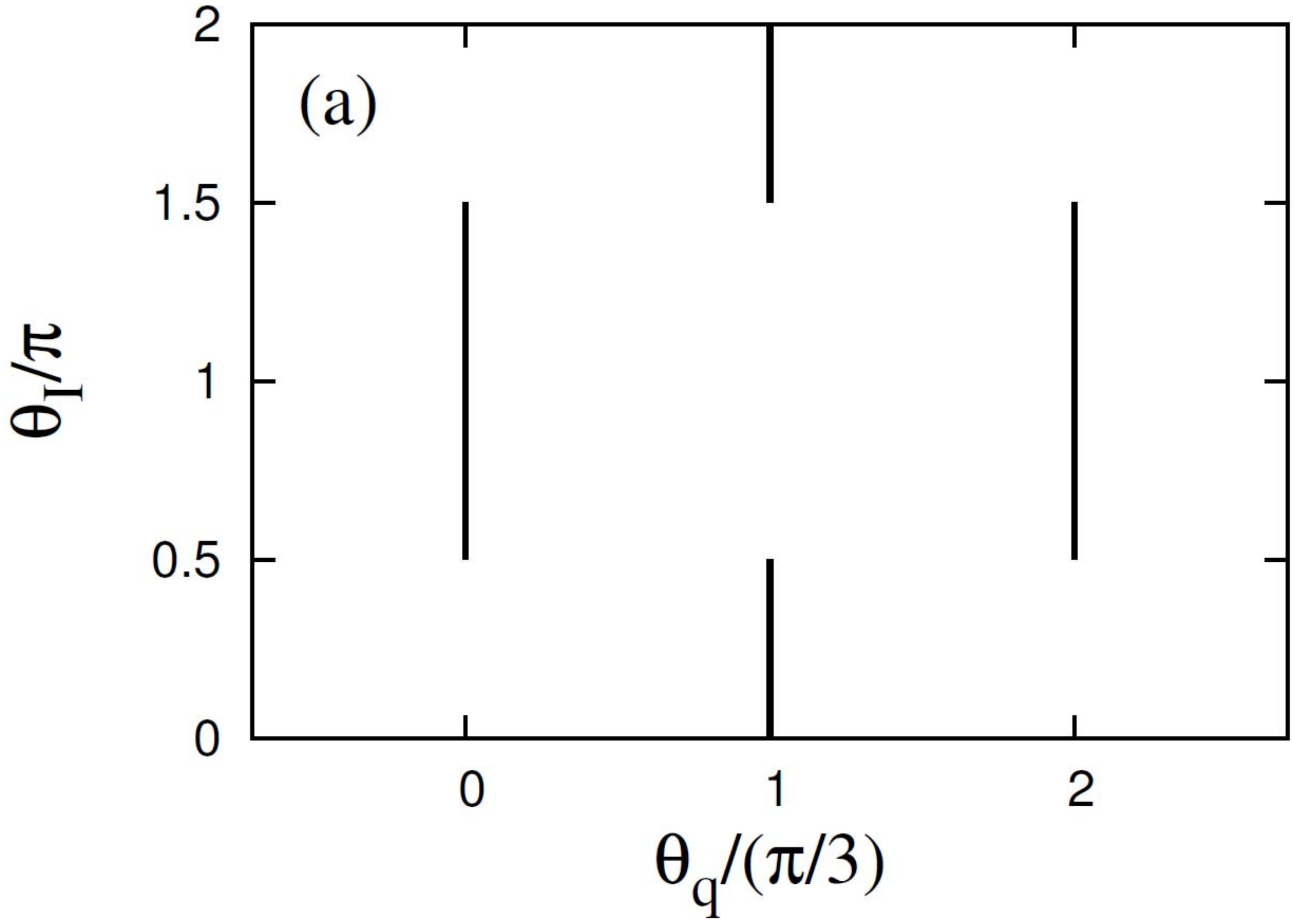
Dashen vs. RW

	Dashen	RW
Symmetry	P	C
Condition	At large c	At high T
Order parameter	η	n_q
Ω	has a cusp	has a cusp
energy	low	high
origin	anomaly instanton	term with $e^{3i\theta}$

Summary

- At high temperature, there are three continuous solutions.
- At low temperature, only one continuous solutions.
- RW transition: transition among solutions with different phases of Polyakov loop.
- On RW line, susceptibilities of θ-odd quantities diverge, while those of θ-even quantities do not.
- At μ=0, there is no phase transiton, but, the rapid change of Polyakov loop appears as a remnant of RW transition.

isospin論文 Fig. 4-(a)



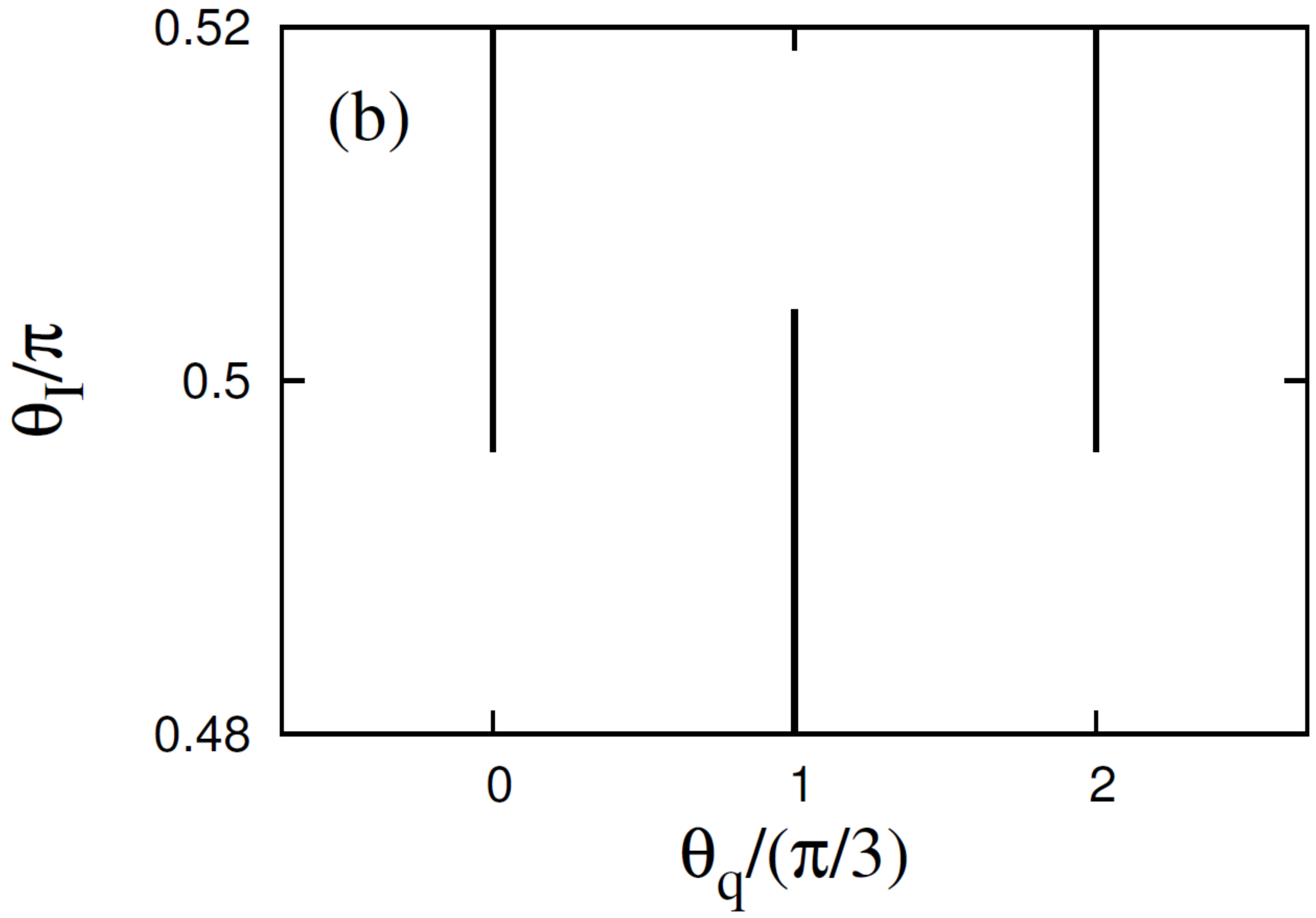
See

Y. Sakai, H.K., M. Yahiro

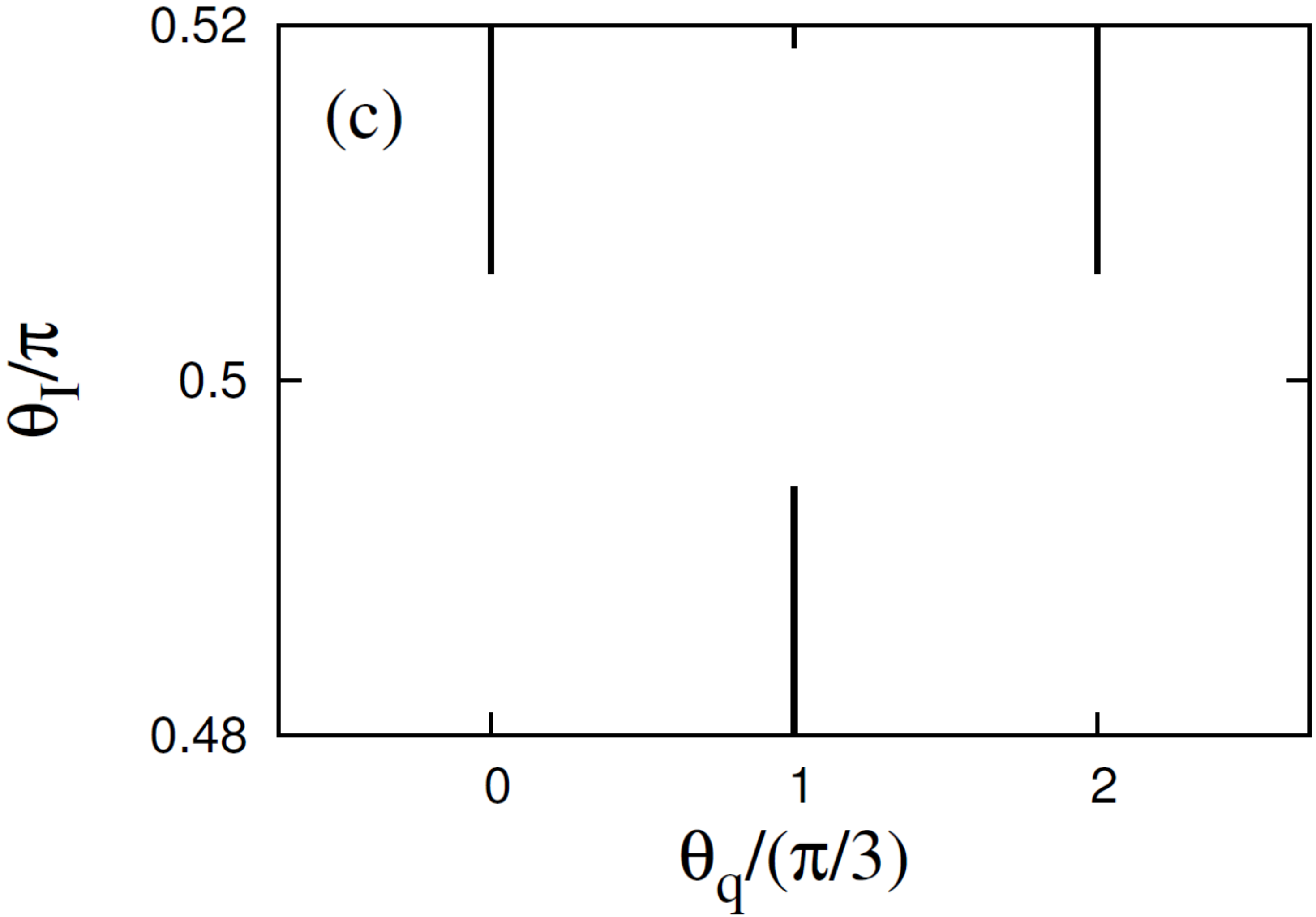
arXiv:0908.3088

QCD phase diagram at imaginary baryon and isospin chemical potentials

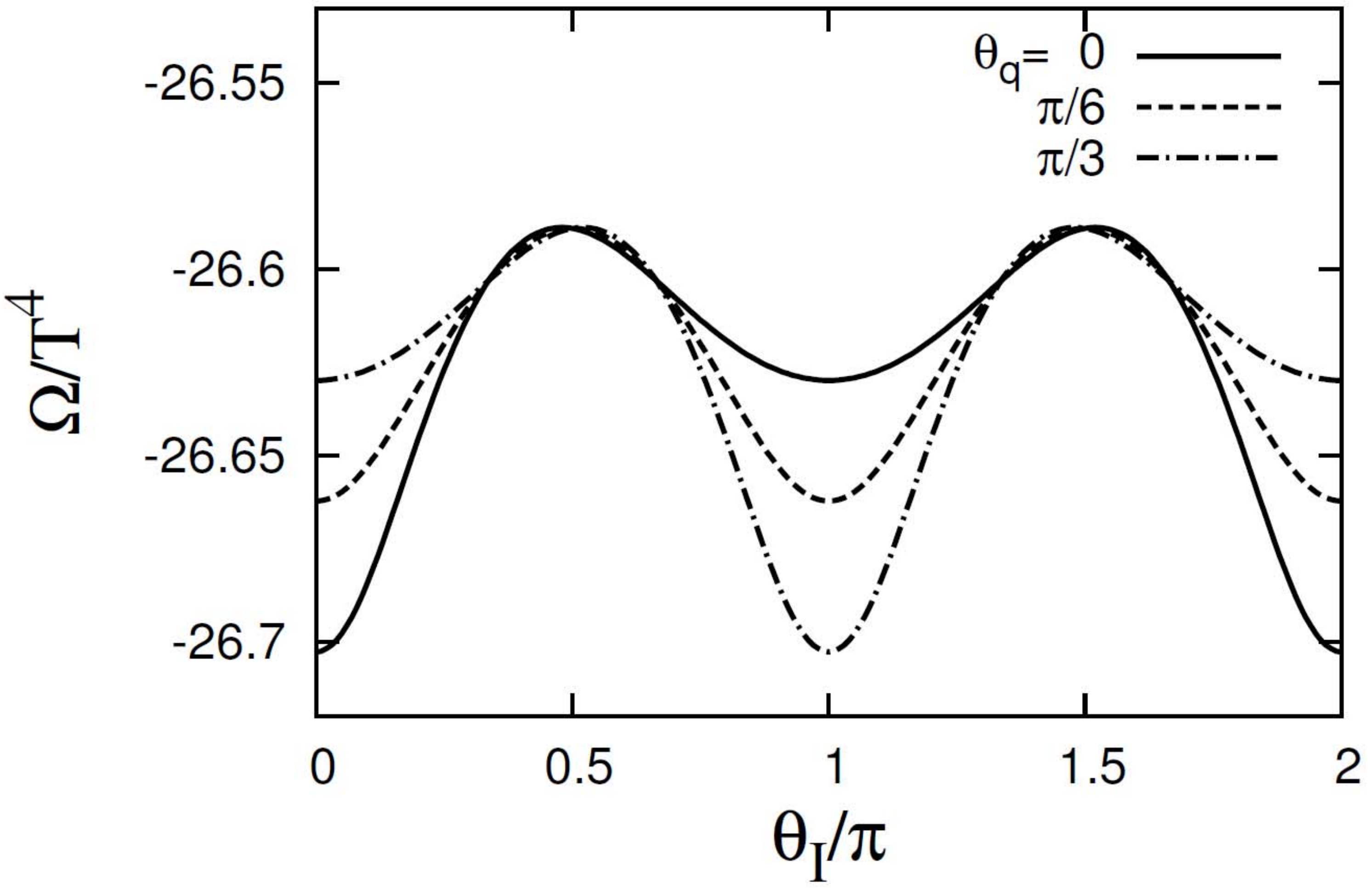
isospin論文 Fig. 4-(b)



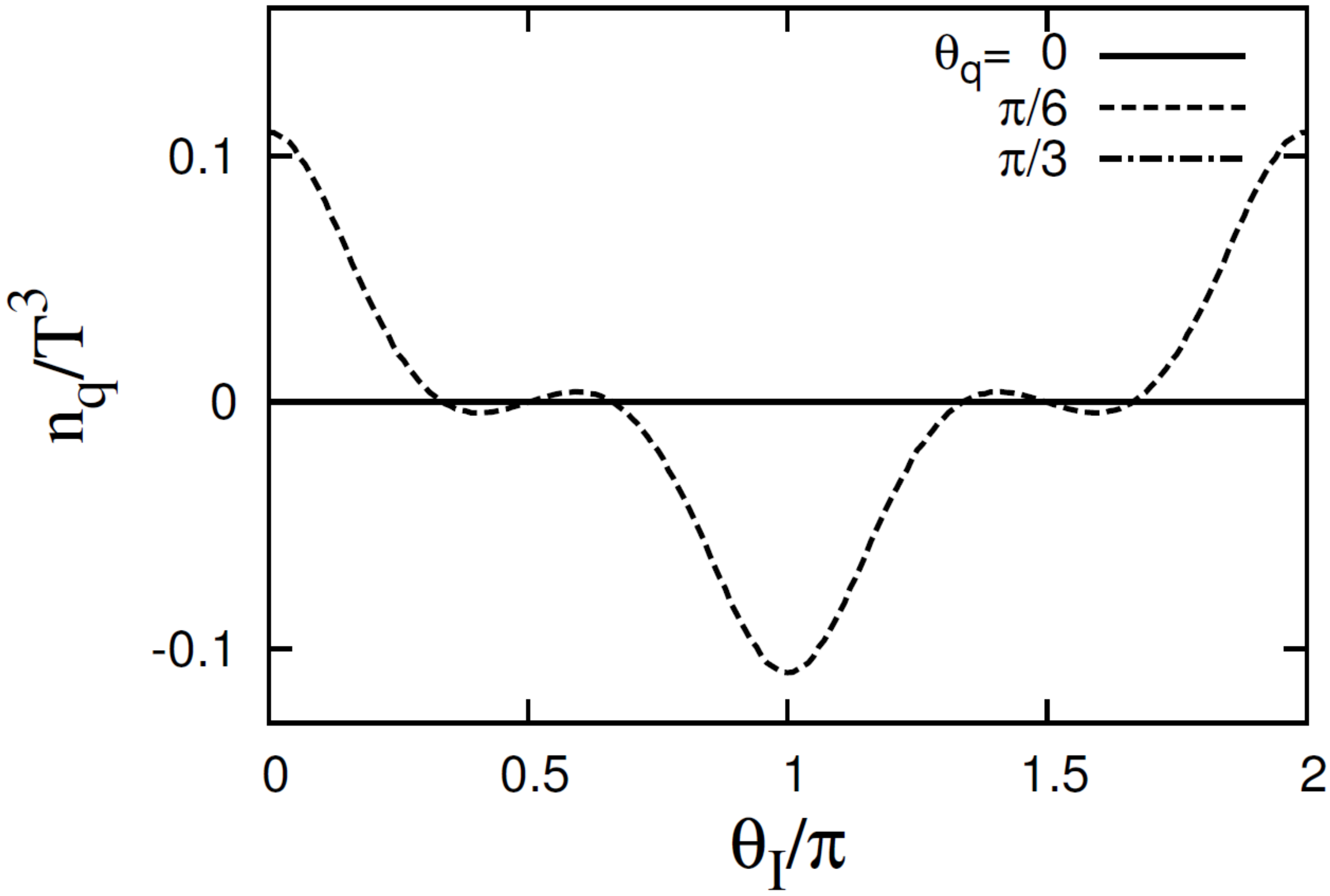
isospin論文 Fig. 4-(c)



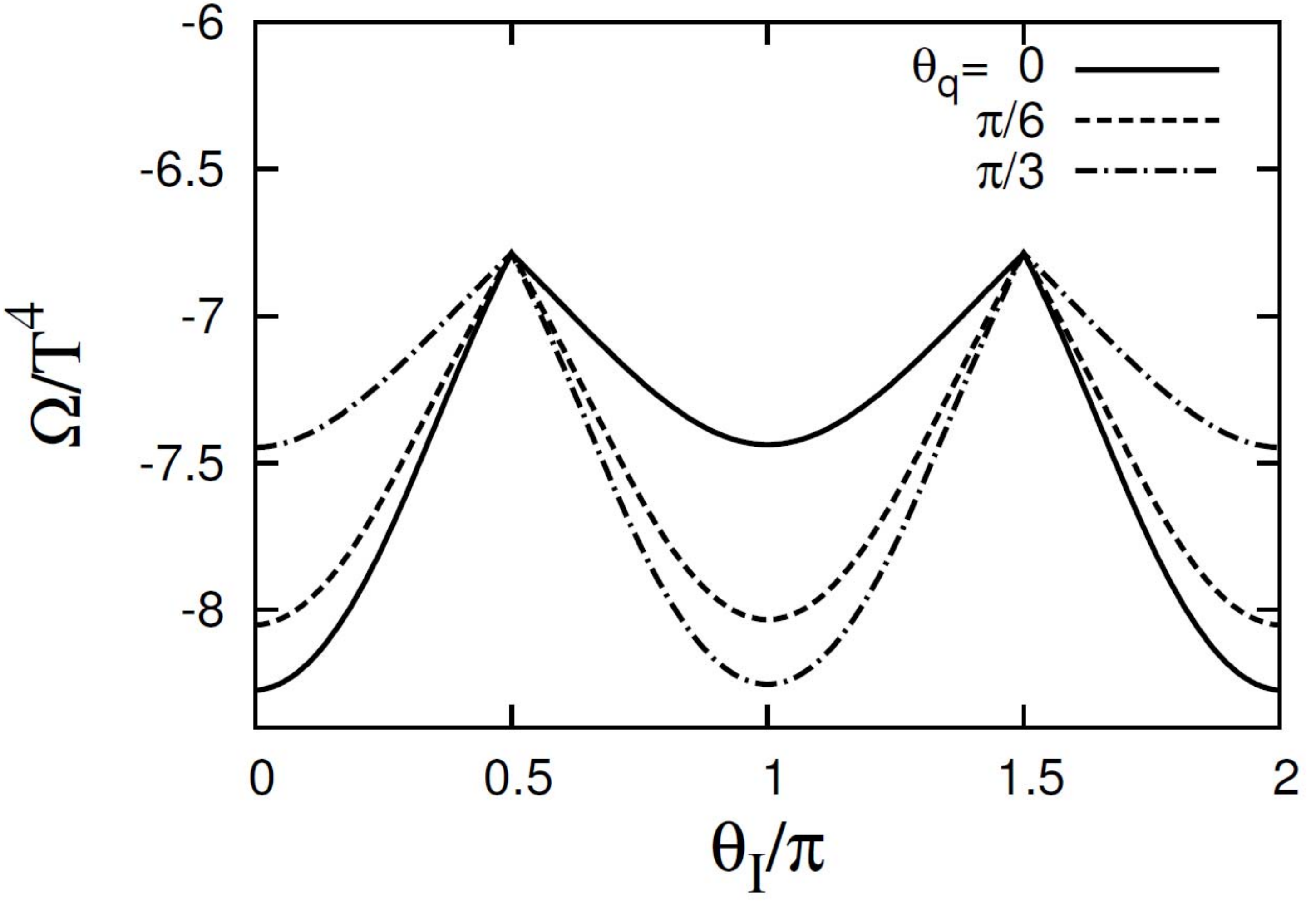
isospin論文 Fig. 5-(a)



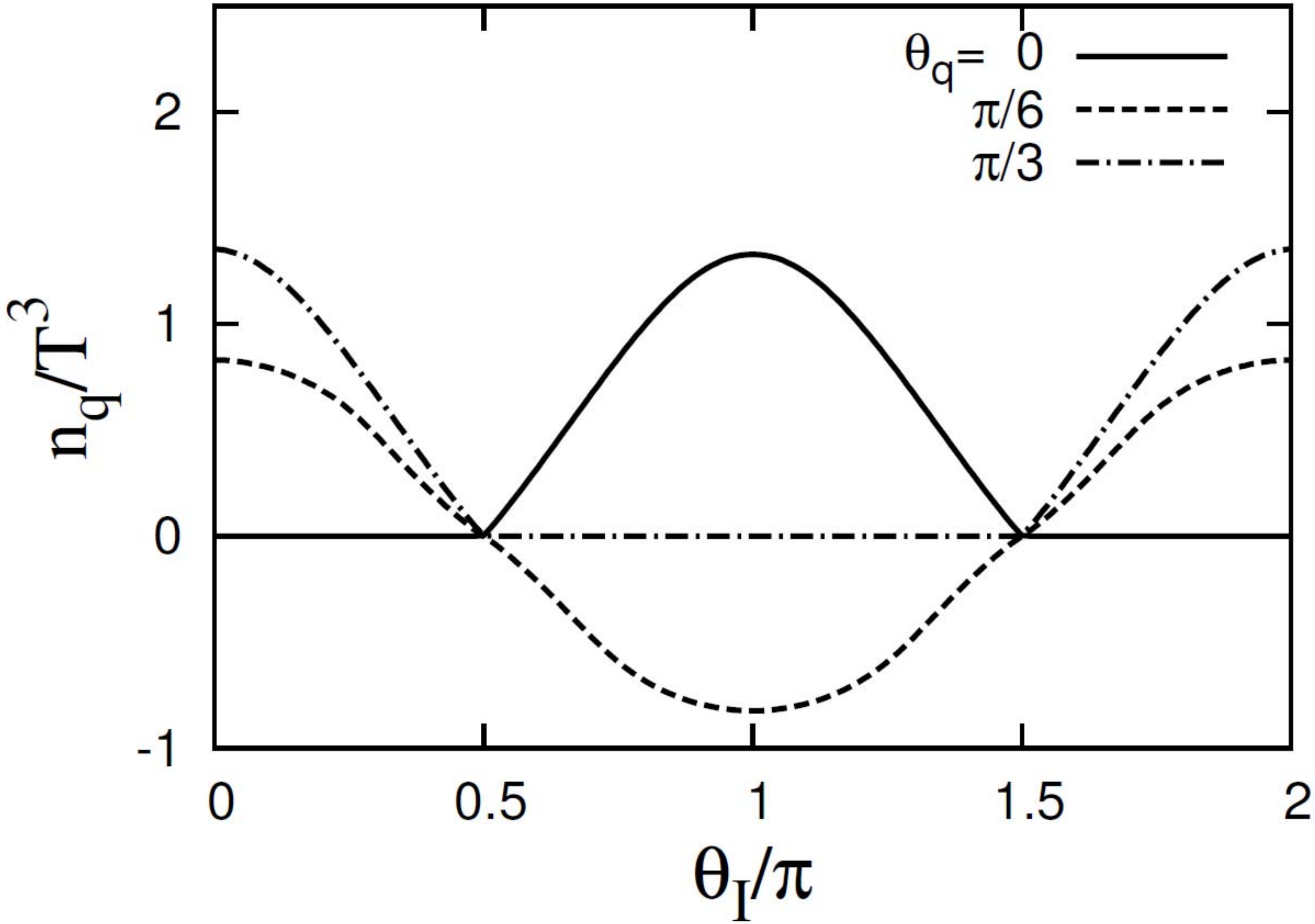
isospin論文 Fig. 5-(b)



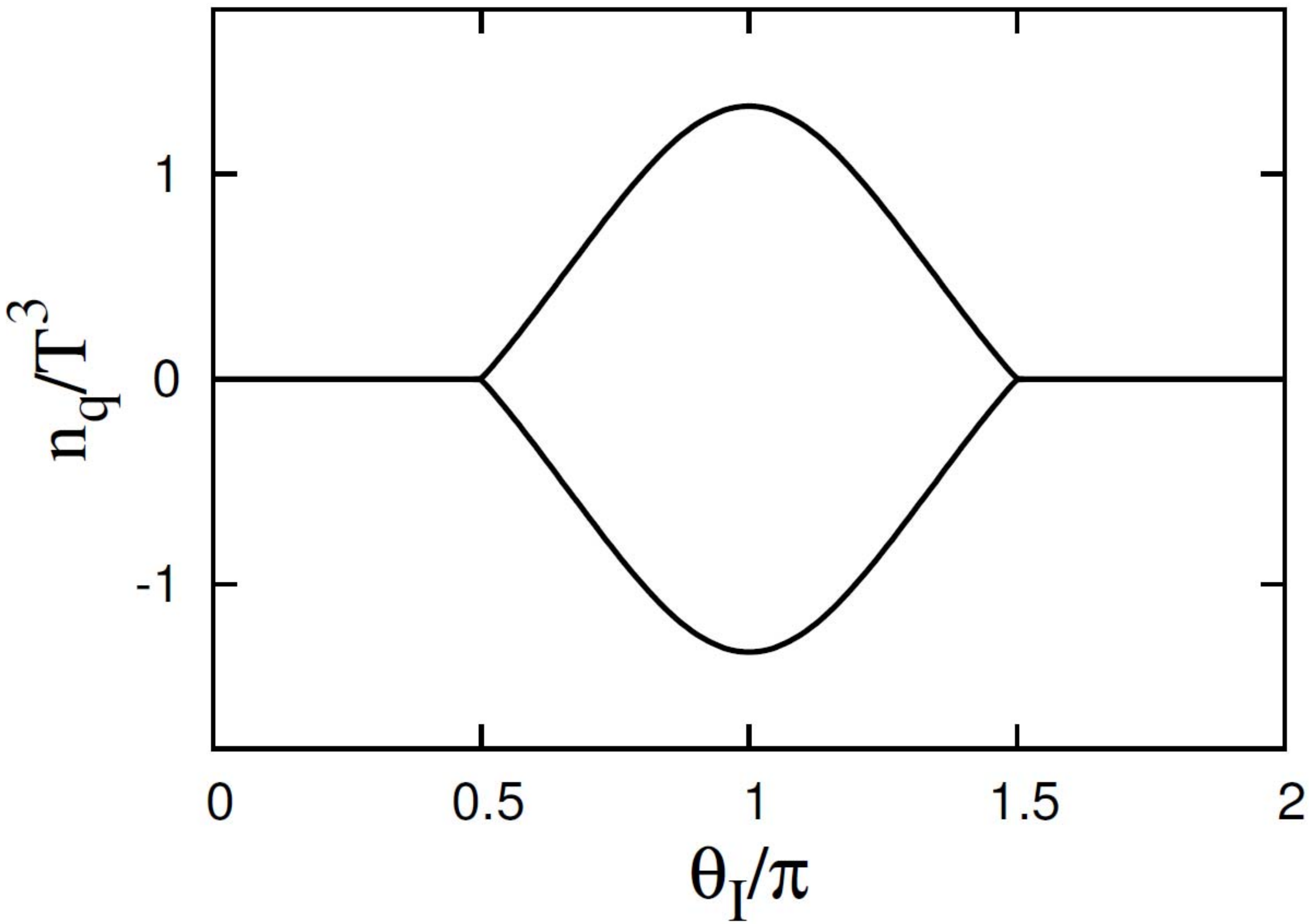
isospin論文 Fig. 6-(a)



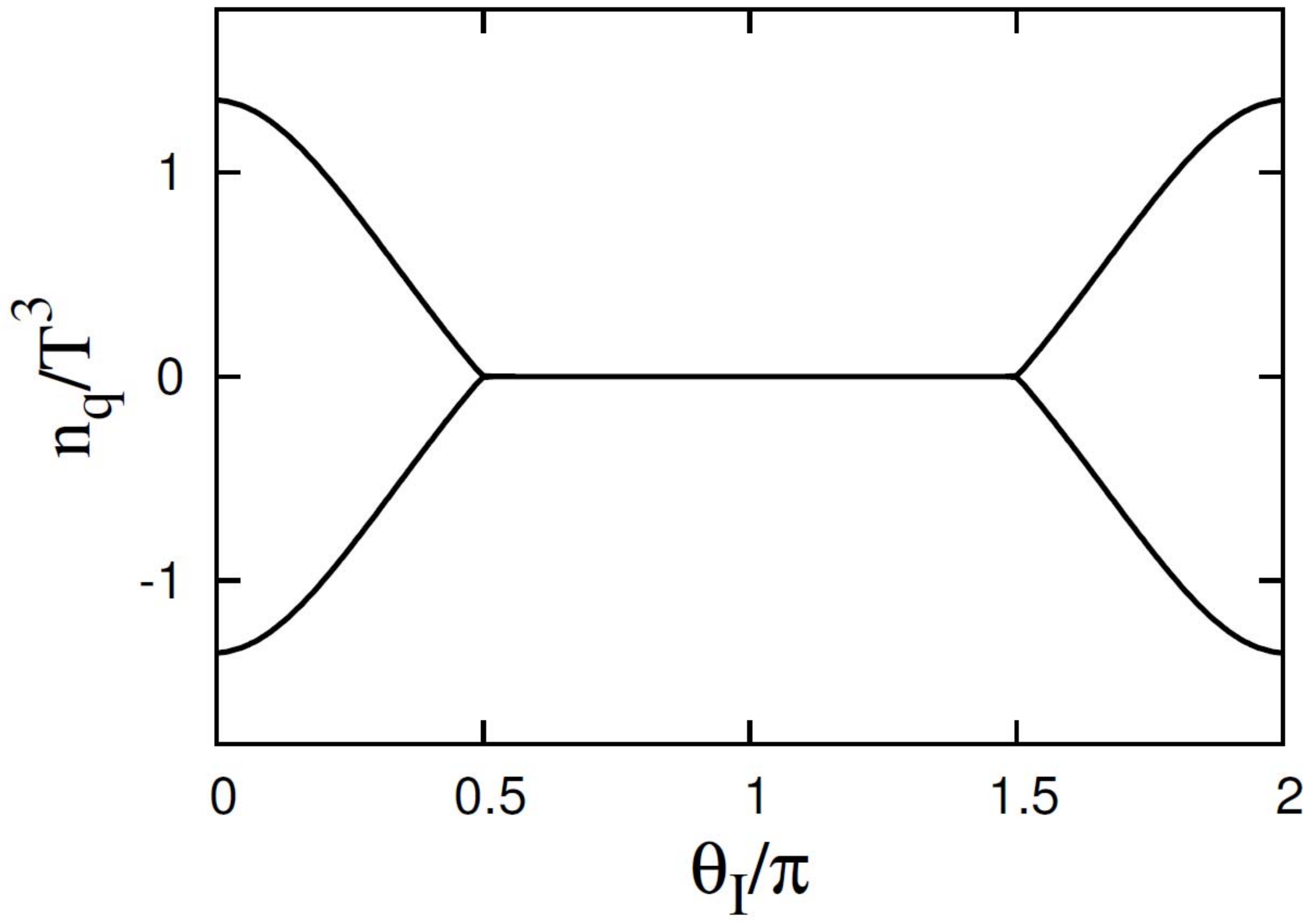
isospin論文 Fig. 6-(b)



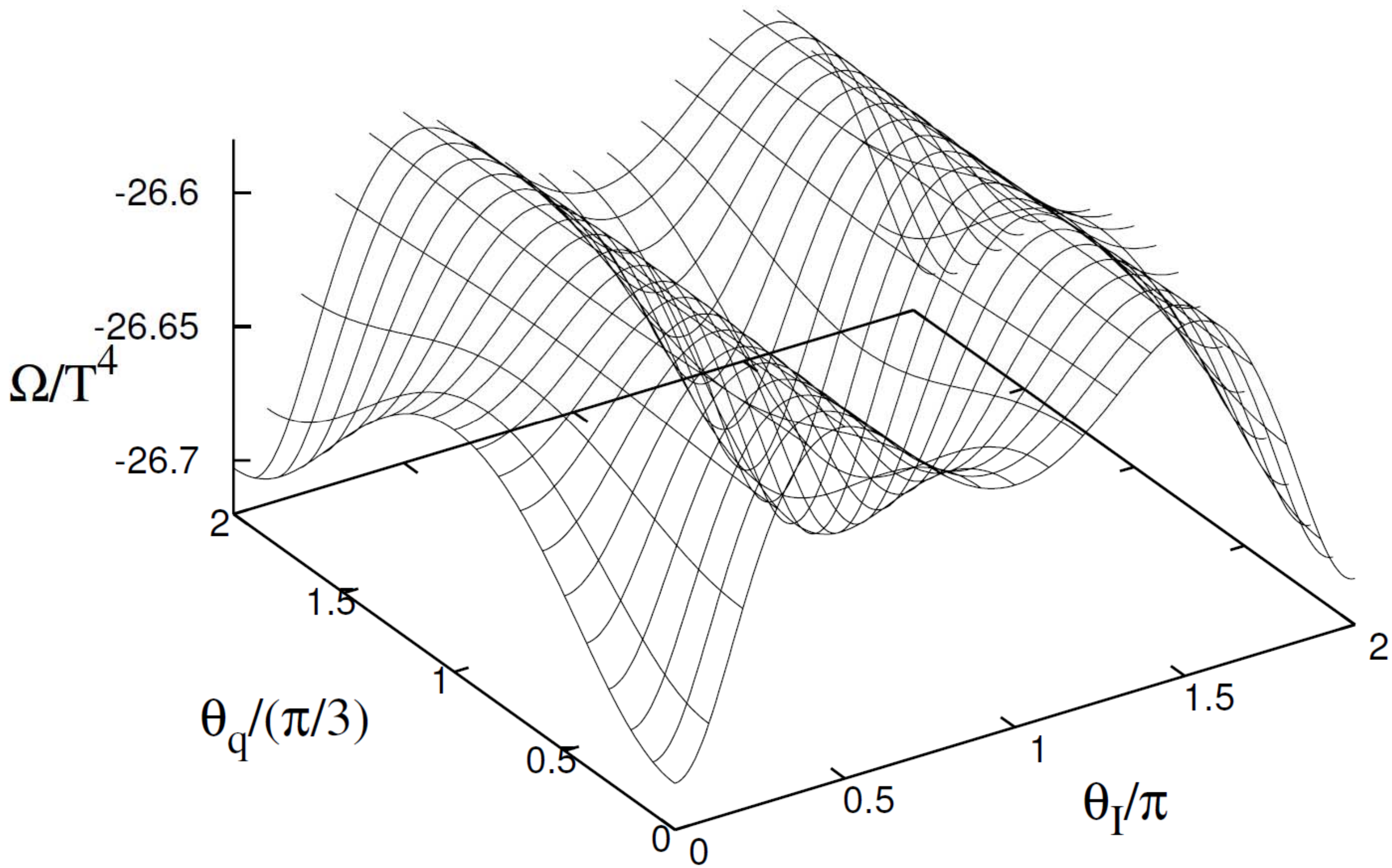
isospin論文 Fig. 7-(a)



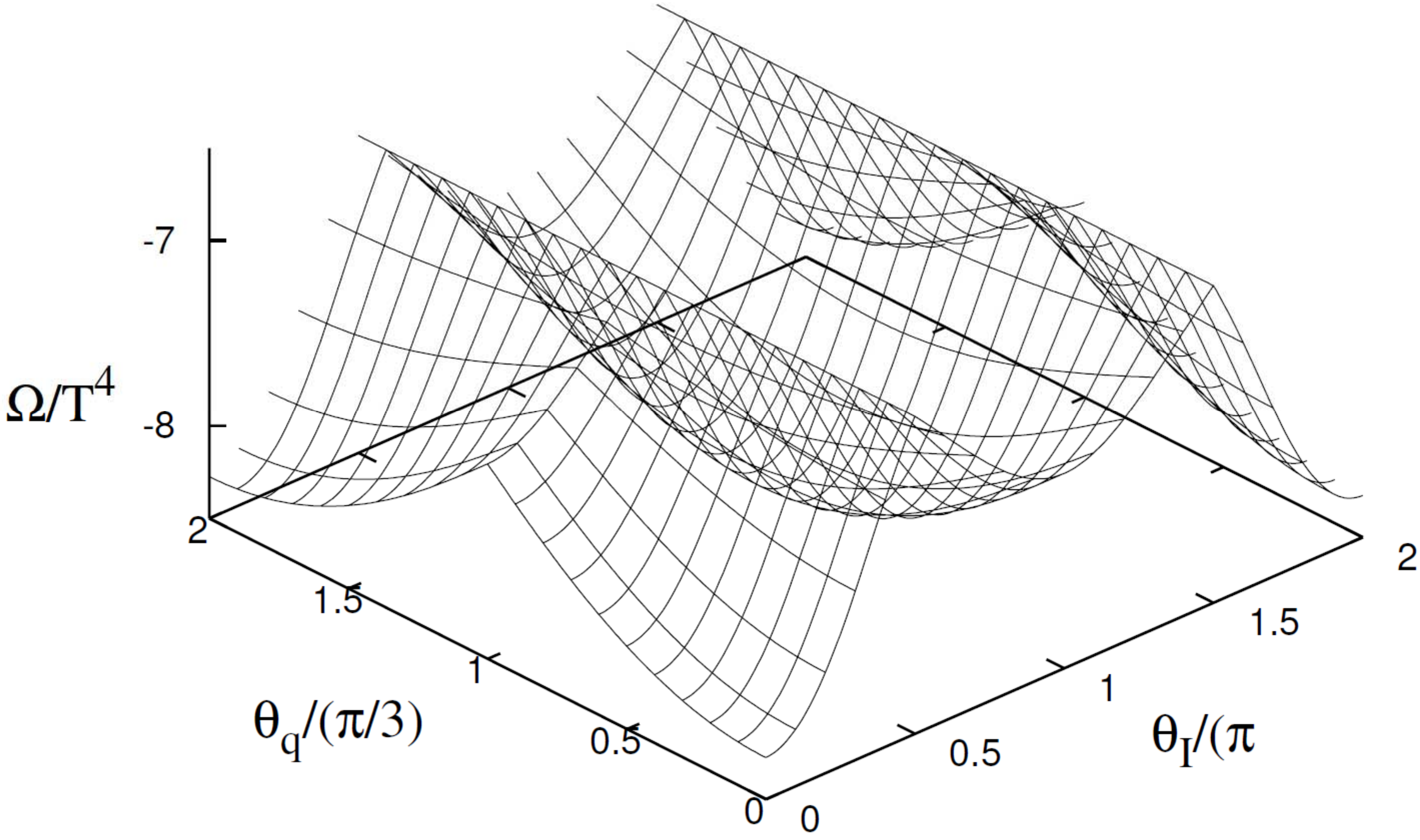
isospin論文 Fig. 7-(b)



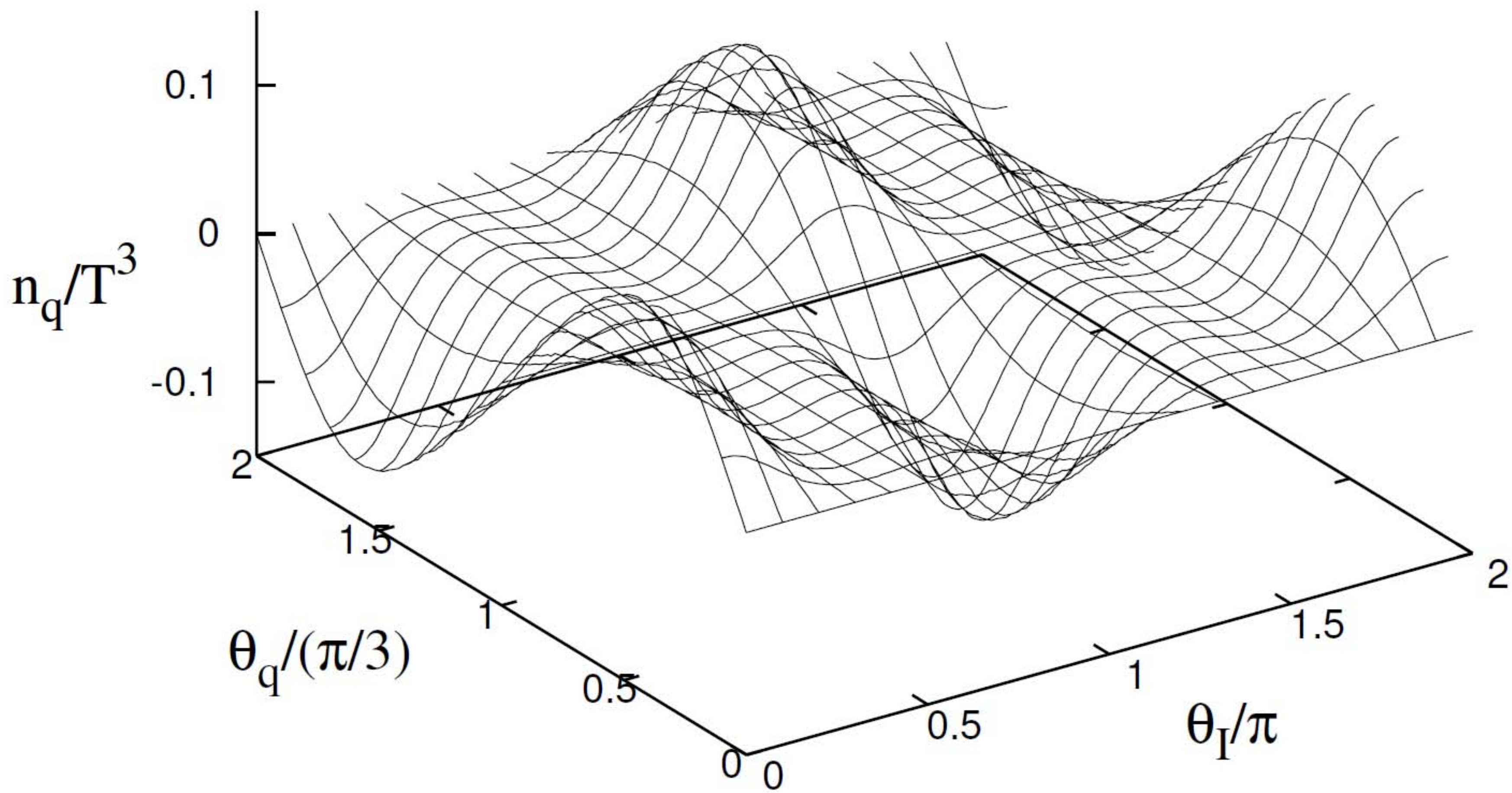
isospin論文 Fig. 8-(a)



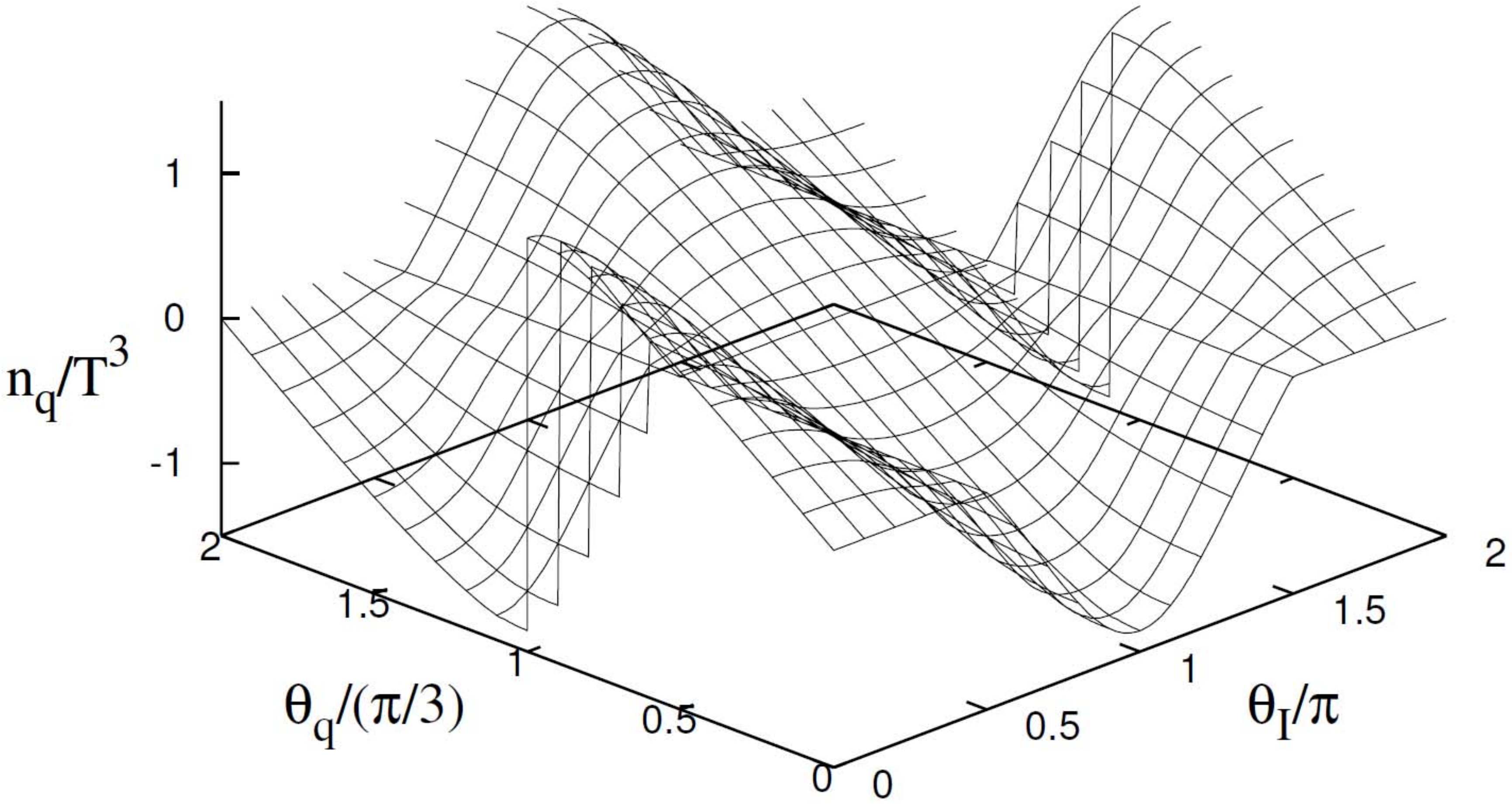
isospin論文 Fig. 8-(b)



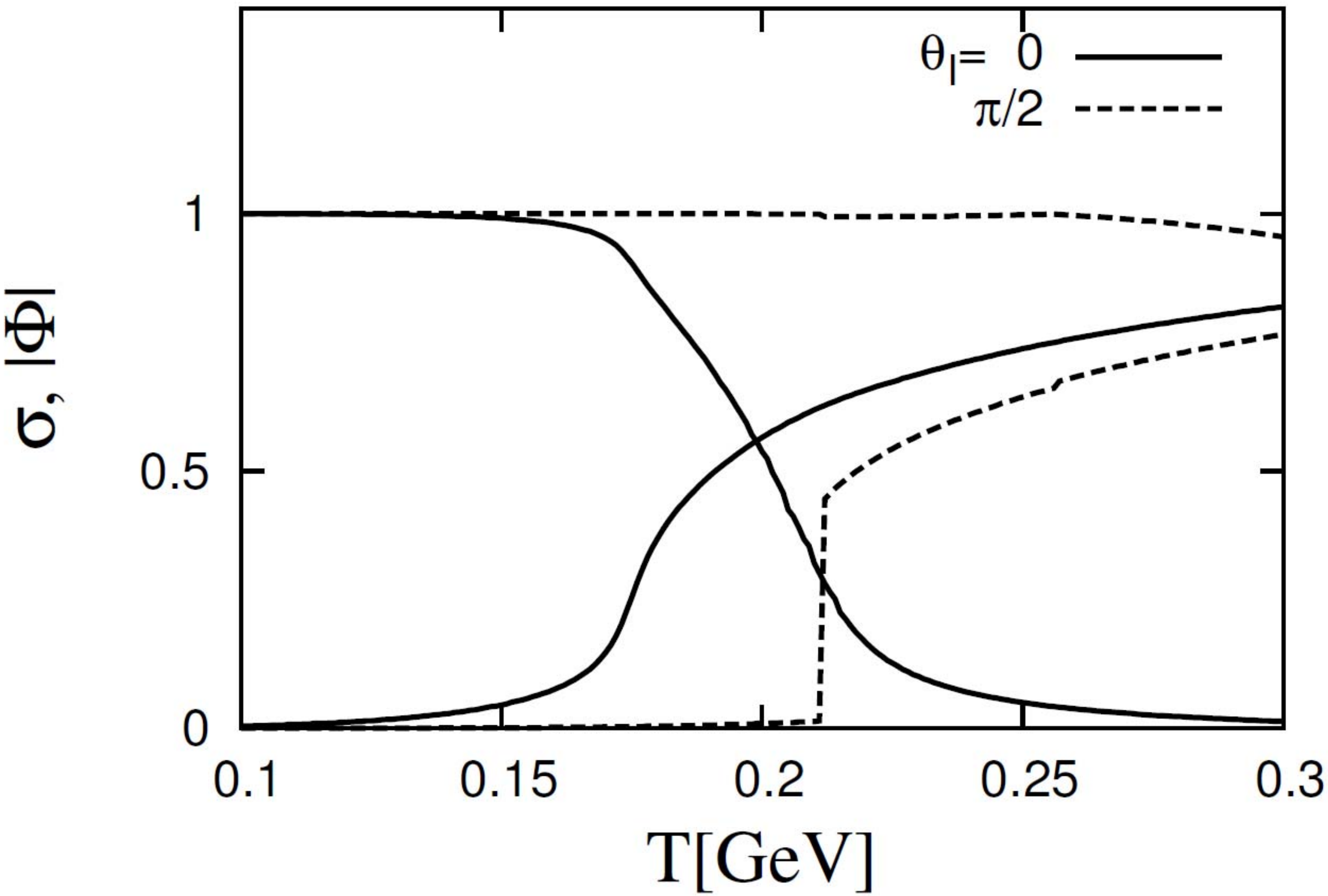
isospin論文 Fig. 8-(c)



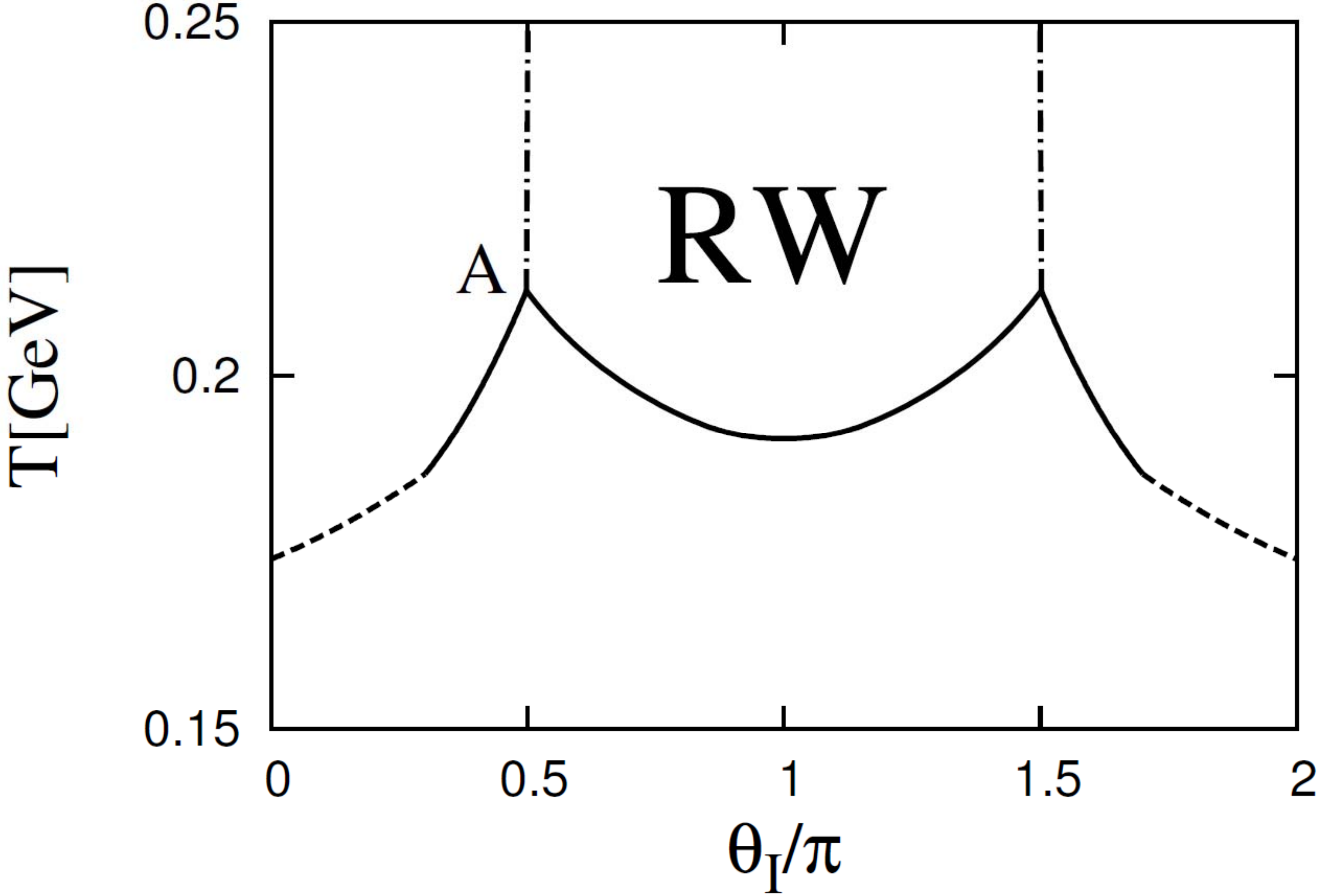
isospin論文 Fig. 8-(d)

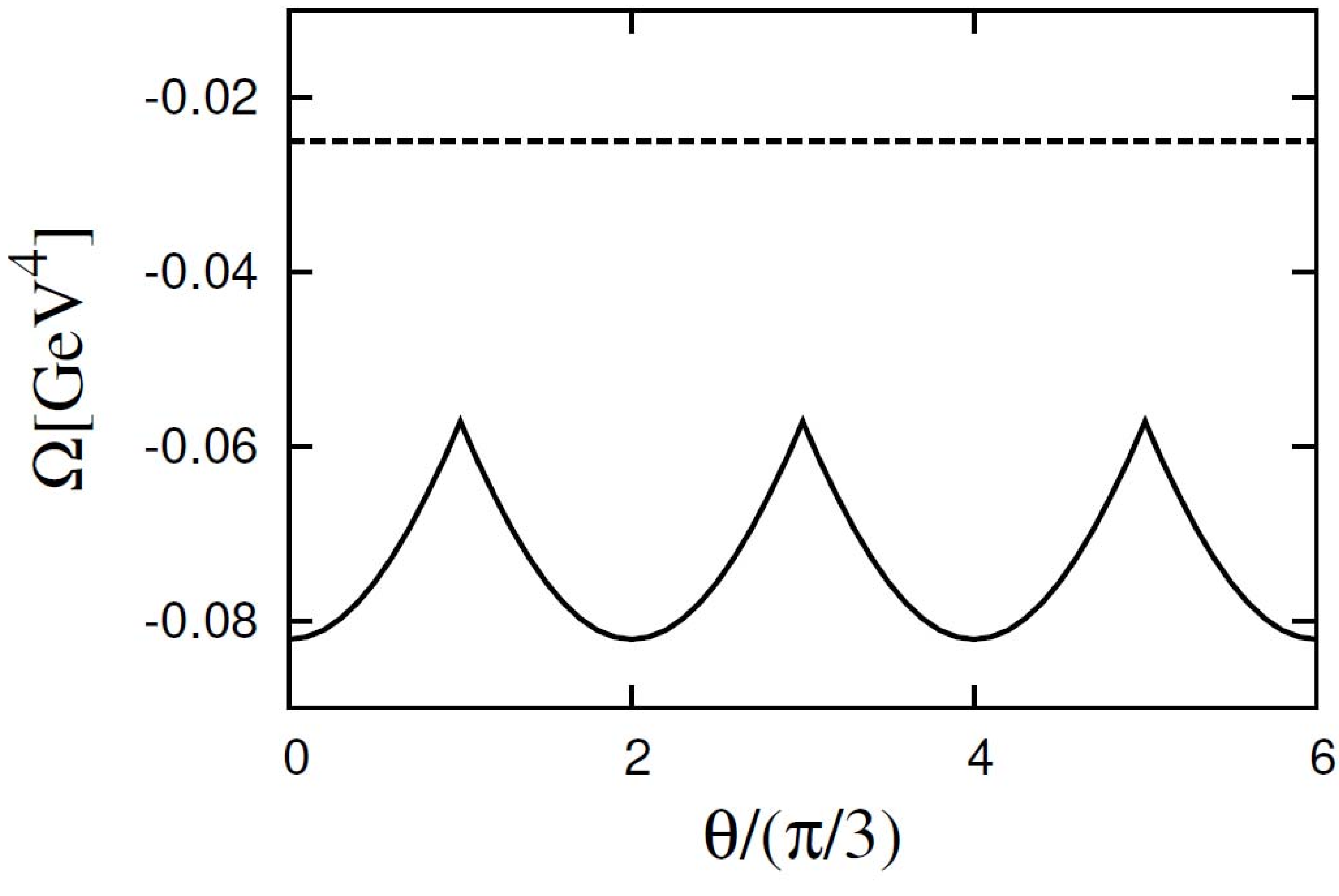


isospin論文 Fig. 9-(a)

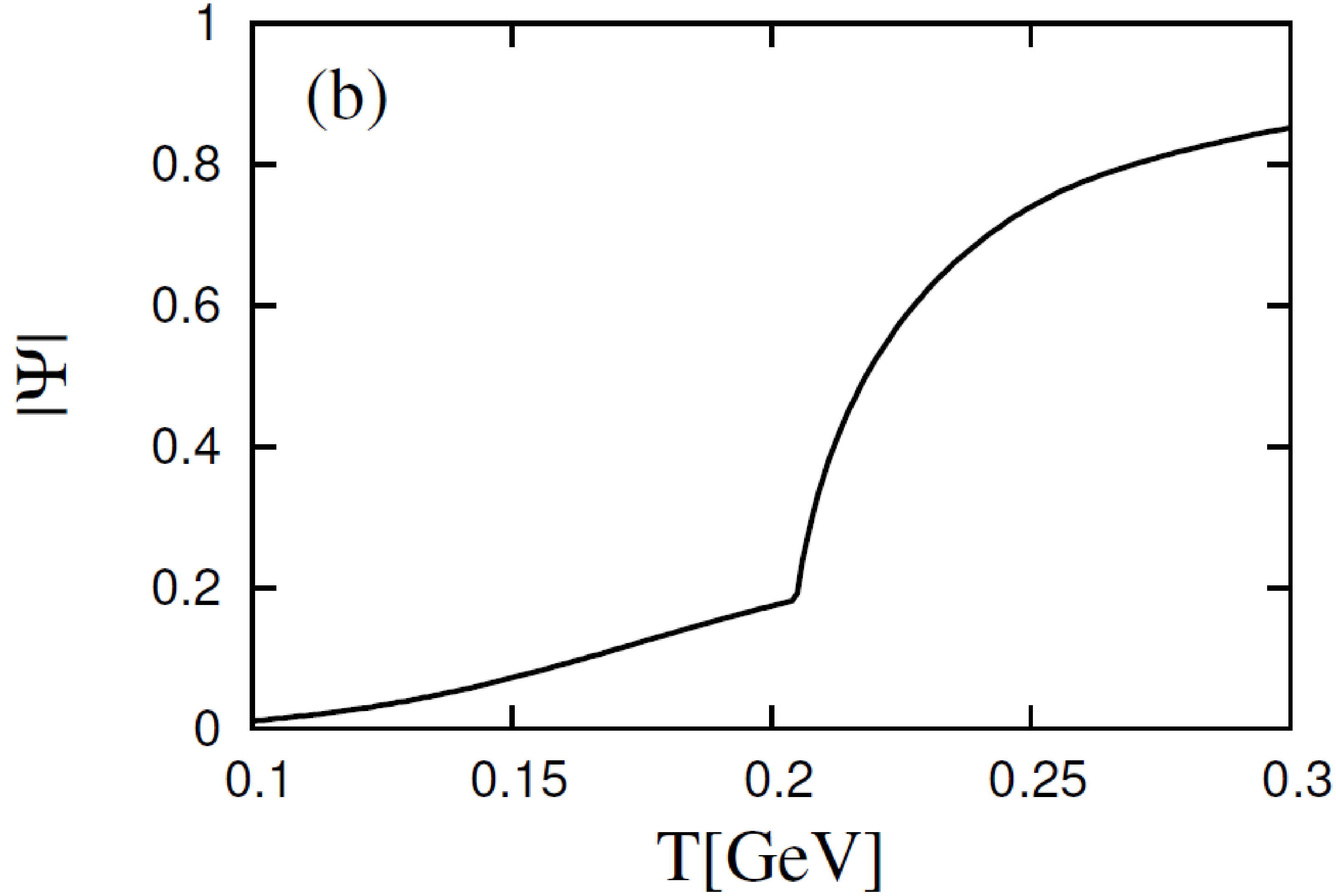


isospin論文 Fig. 9-(b)

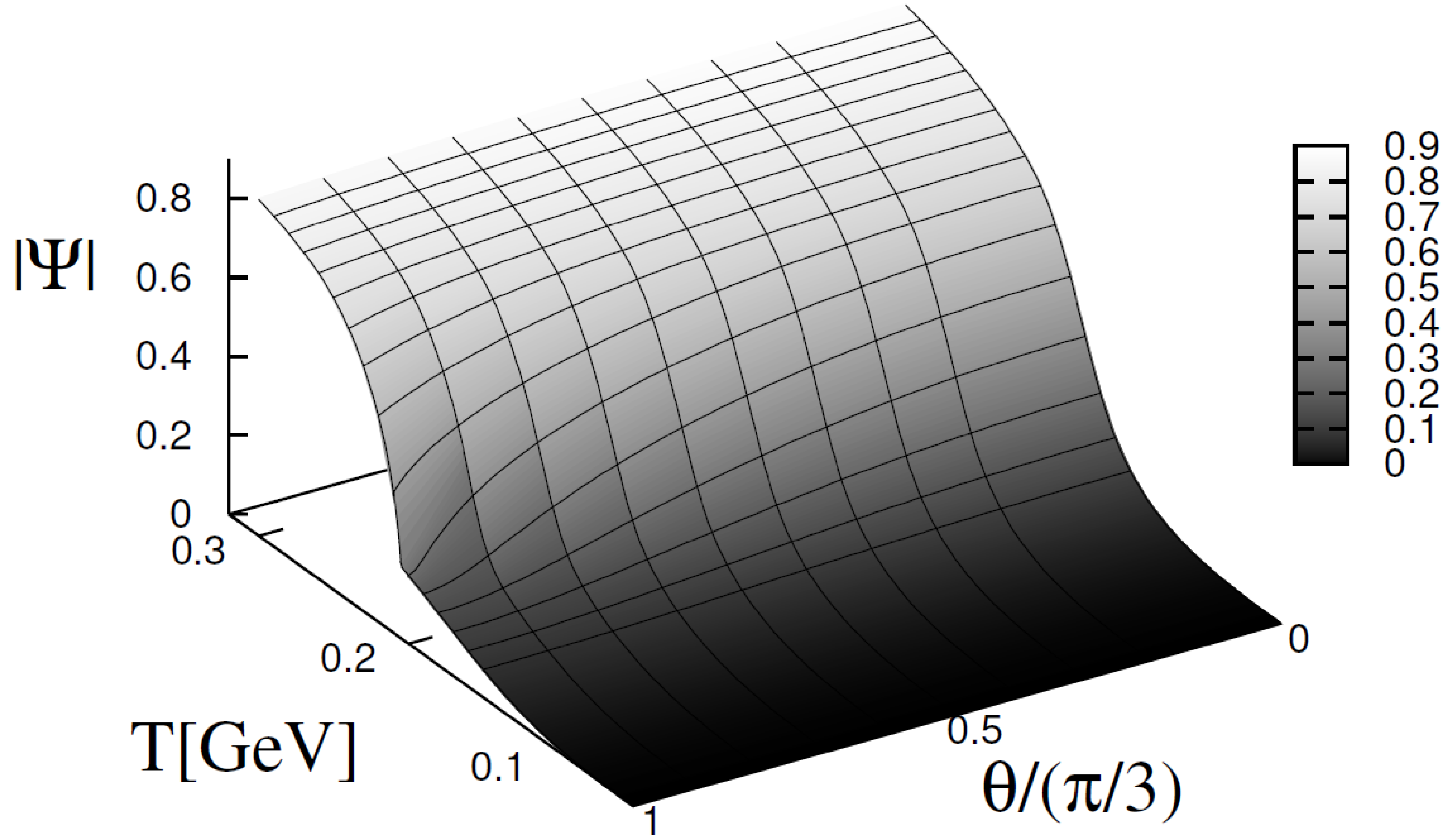


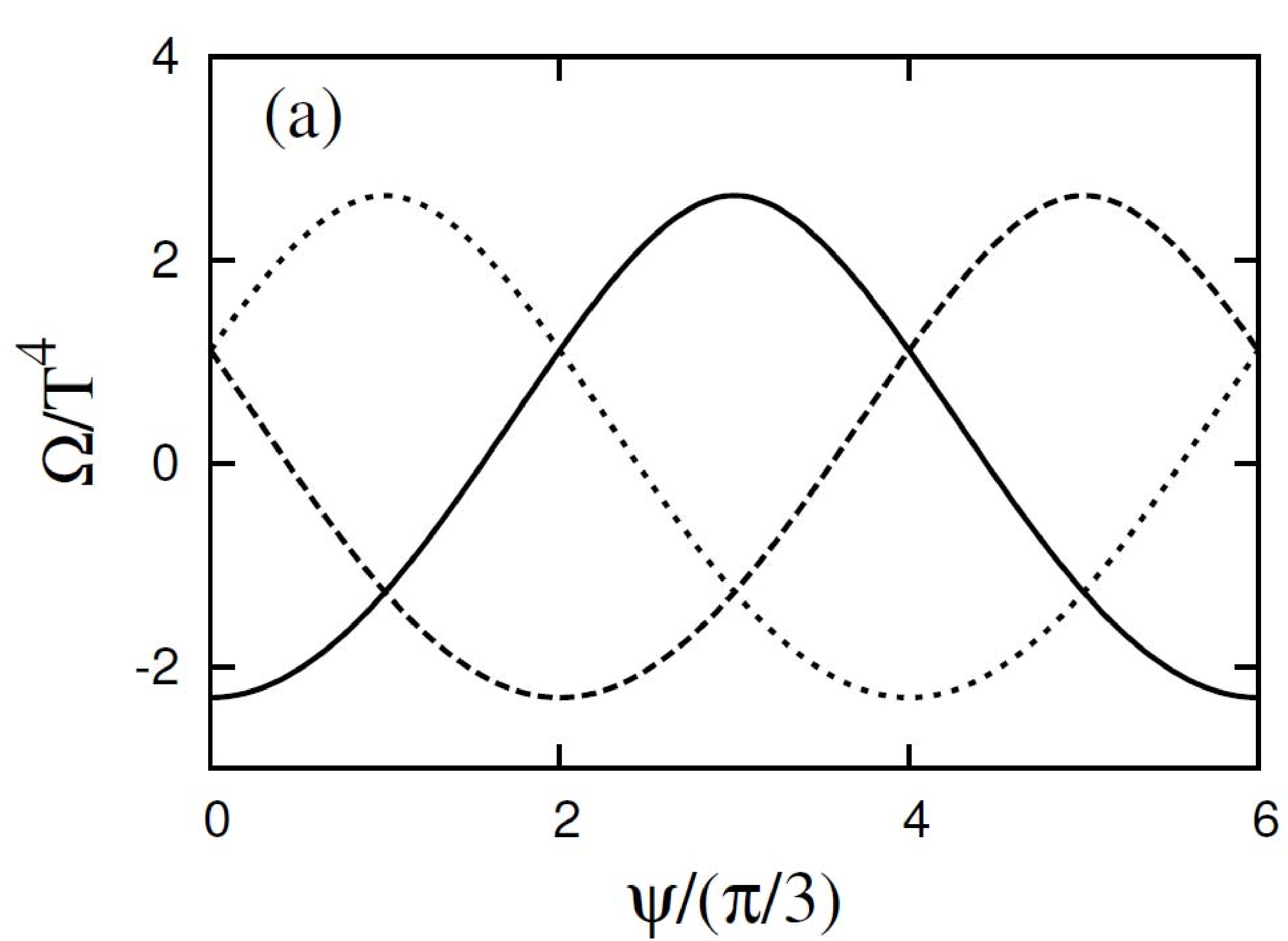


Ω[GeV^4]
-0.02
-0.04
-0.06
-0.08
0
2
4
6
θ/(π/3)

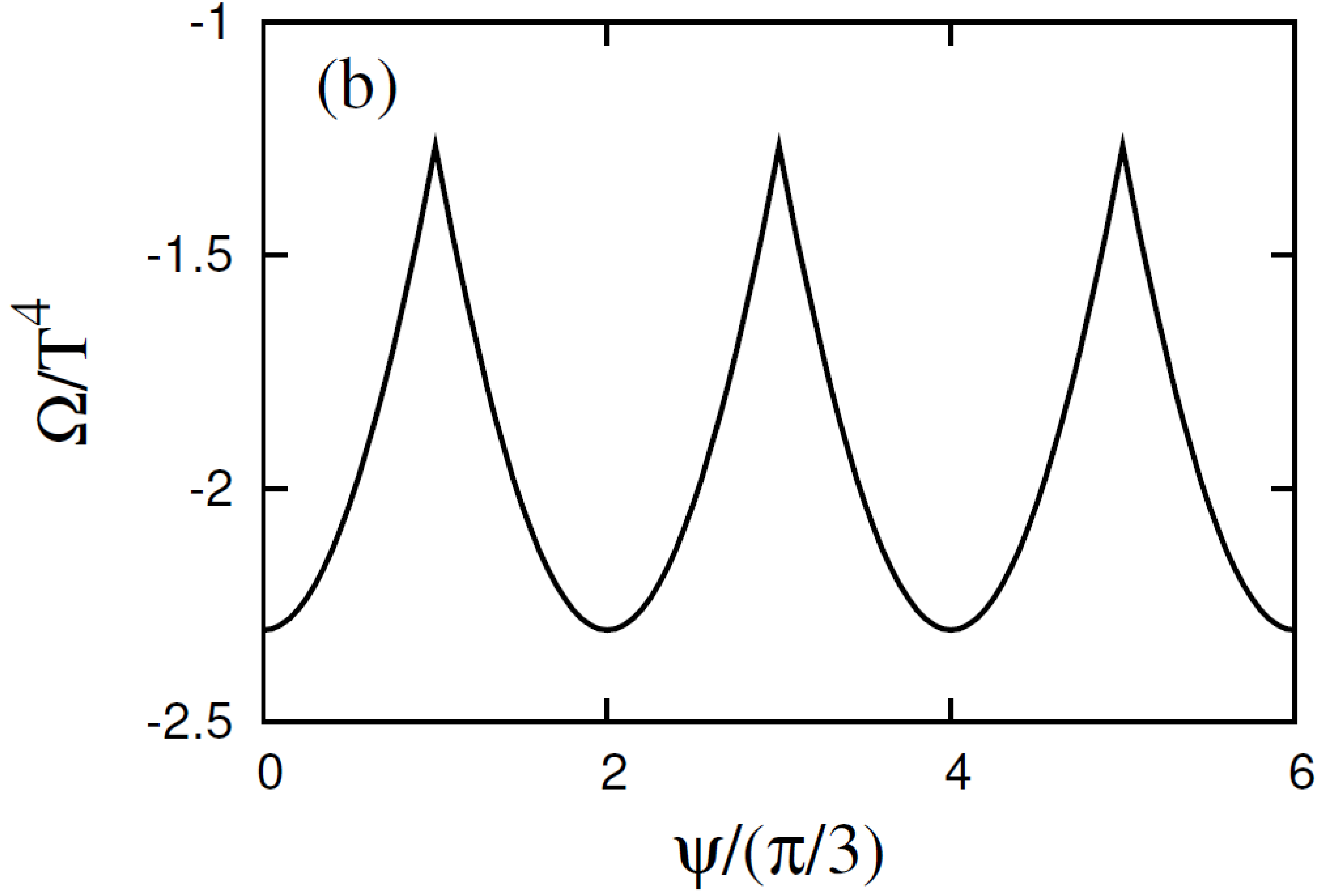


(b)
|Ψ|
T[GeV]
0
0.2
0.4
0.6
0.8
1
0.1
0.15
0.2
0.25
0.3


|Ψ|
0.8
0.6
0.4
0.2
0
0.3
0.2
0.1
T[GeV]
1
0.5
0
θ/(π/3)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

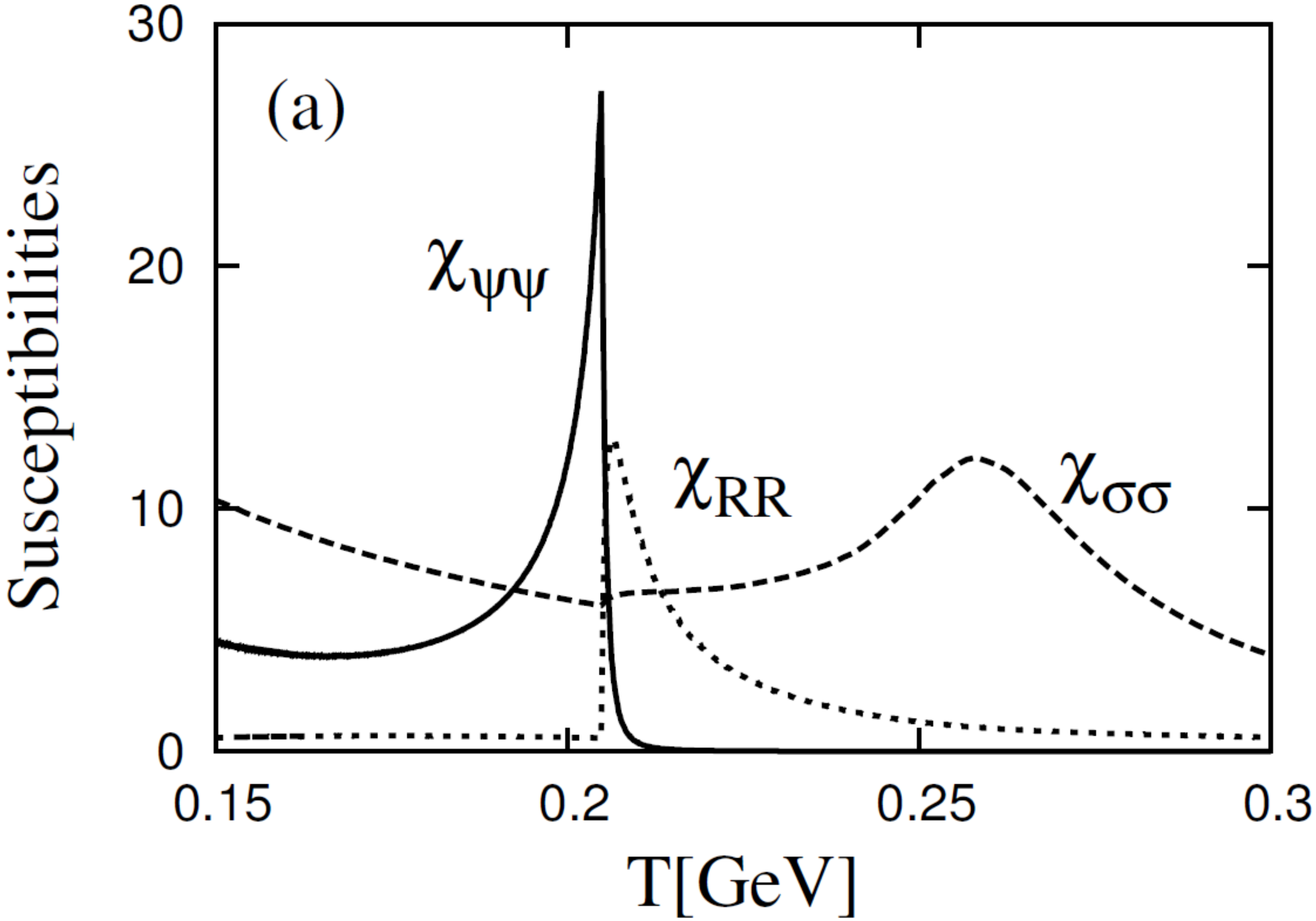


(a)
Ω/T^4
ψ/(π/3)
4
2
0
-2
0
2
4
6

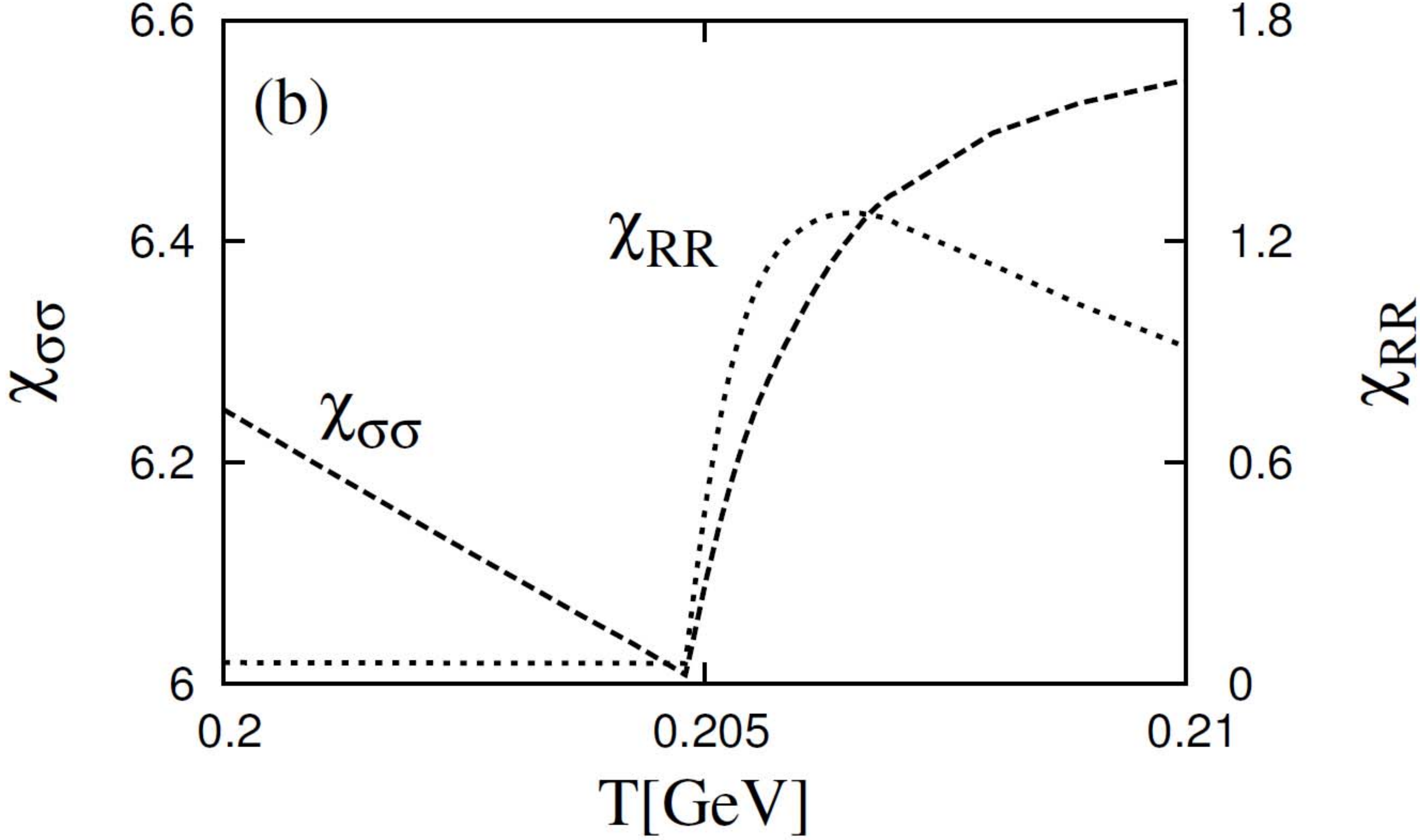


(b)
-1
-1.5
-2
-2.5
0
2
4
6
Ω/T^4
ψ/(π/3)

ＲＷ論文 Fig. 9-(a)



RW論文 Fig. 9-(a)



6.6
6.4
6.2
6
1.8
1.2
0.6
0
(b)
χ_{RR}
$\chi_{\sigma\sigma}$
$\chi_{\sigma\sigma}$
χ_{RR}
0.2
0.205
0.21
T[GeV]

ＲＷ論文 Fig. 9-(c)