熱場の量子論, Sep. 4, 2009

励起モードの性質から探る有限温度QCD

～クォーク励起スペクトルの解析を中心に～

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（阪大）

Outline:

1, Overview of (lattice) QCD @ *T*>0

2, Perturbative study of fermion spectrum at high *T*

3, Lattice study of quarks at nonzero *T*

MK, Kunihiro, Nemoto, PLB**631**,157(05); PLB**633**,269(06); PTP**117**,103(07); NJL
Karsch, MK, PLB**658**,45(07); MK, *et al.* PRD**77**,045034(08); Yukawa
Karsch, MK, PRD**80**,056001(09); MK, *et al.* in progress. Lattice

Quantum Chromodynamics (QCD)

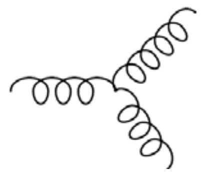
$$L = \bar{\psi}_\alpha (i\not{D} - m)\psi_\alpha - \frac{1}{4} F_{\mu\nu,a} F_a^{\mu\nu}$$

quarks

Dirac fermions
4-component spinor
3 colors
flavor dof: u, d, s, …

gluons

SU(3) non-Abelian gauge field

self-interaction: 

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asymptotic freedom

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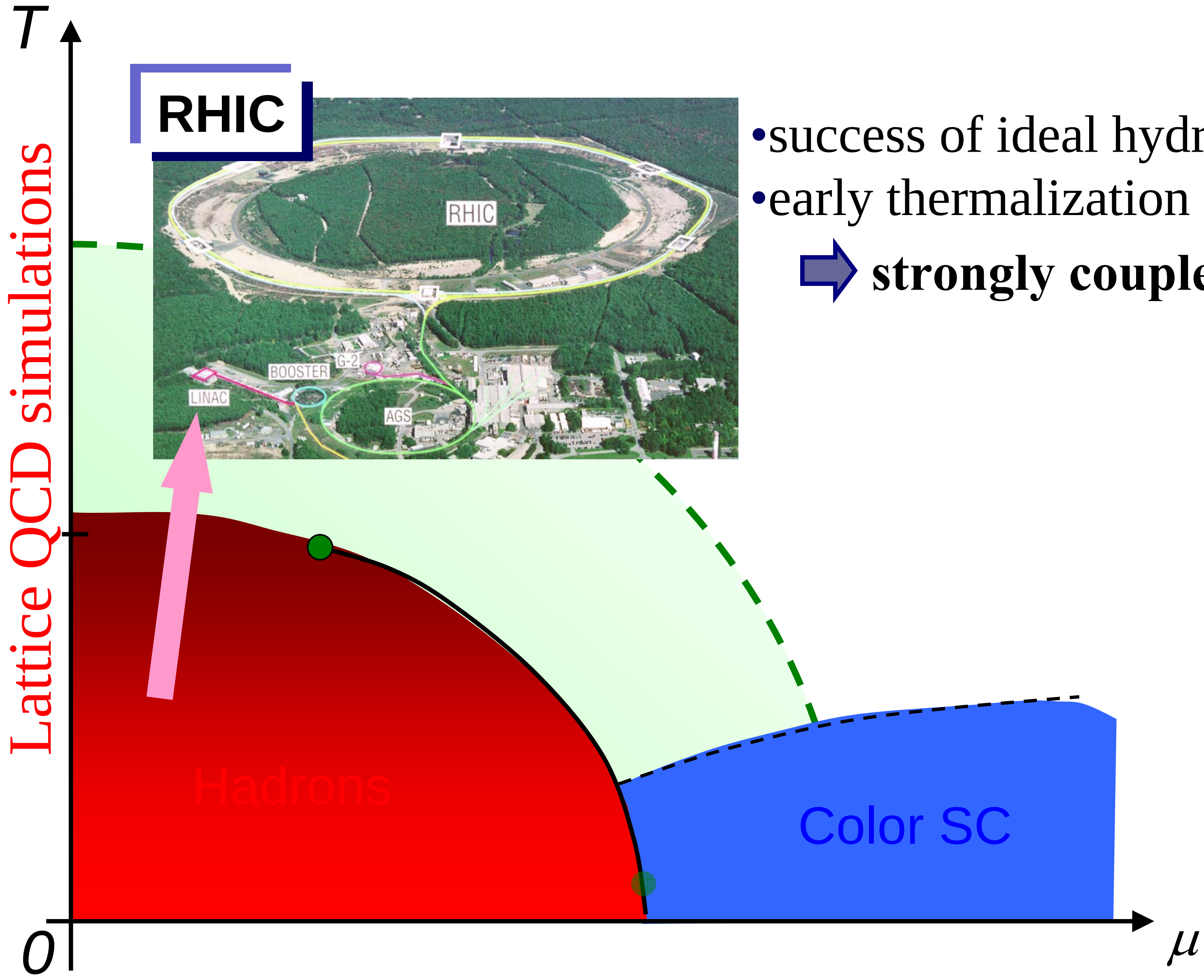
- vacuum : confinement, chiral symm. breaking, …
- high T : deconfinement, quark-gluon plasma (QGP)

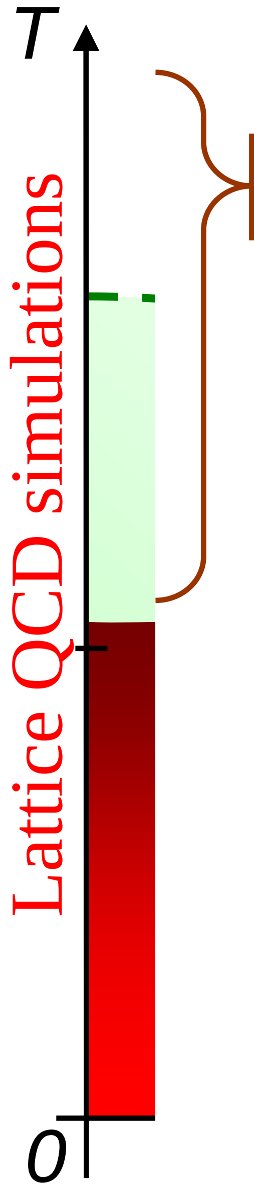
Phase Diagram of QCD



- success of ideal hydro. models
- early thermalization

⇨ **strongly coupled QGP** near T_c

Phase Diagram of QCD



Region explored in this talk

- fate of hadronic modes above T_c
  - charmonia: signal of realization of QGP phase
  - soft modes associated with chiral transition
- Other modes : diquarks, glueball, …
- Fundamental DoF in QCD: quarks and gluons

Lattice QCD at Nonzero *T*

- basic formulae:

$$Z = \mathrm{Tr}\,e^{-\beta H} = \sum_n \langle \phi_n | e^{-\beta H} | \phi_n \rangle = \int DU\, D\psi\, D\bar{\psi}\, \exp(-S_G - S_F)$$

$$\langle O \rangle = \frac{1}{Z} \mathrm{Tr}\left[O e^{-\beta H} \right]$$

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•Bulk thermodynamics: $\varepsilon = \frac{T^2}{V} \frac{\partial \ln Z}{\partial T}$, etc. $\langle S_E \rangle = -\partial \ln Z / \partial \beta$

•Expectation values: fluctuations, heavy quark potential, etc.

•Euclidean correlators:

$$D(\tau, \mathbf{x}) = \frac{1}{Z} \mathrm{Tr}\left[\mathrm{T}_\tau O(\tau, x) O(0,0) e^{-\beta H} \right]$$

⇒ $D(i\omega_n, \mathbf{p})$ ⇒ $D^R(\omega, \mathbf{p})$ ⇒ spectral function (dynamical properties)

F.T.   解析接続

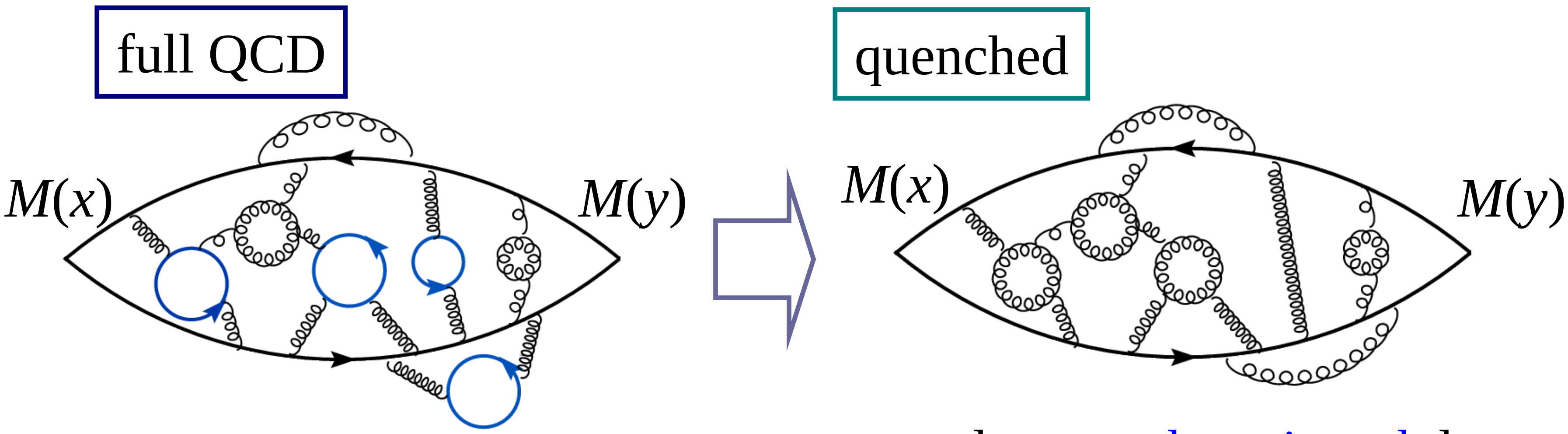
Quenched Approximation

- Meson propagator

$$\left\langle M(x)M(0)\right\rangle = \frac{1}{Z}\int DU\,D\psi\,D\bar{\psi}\,M(x)M(0)\exp(-S_G - S_F)$$

$$= \frac{1}{Z}\int DU\,\mathrm{cntr}[M(x)M(0)]\,\det K\,\exp(-S_G)$$

quenched approx.



•neglect quark-antiquark loops

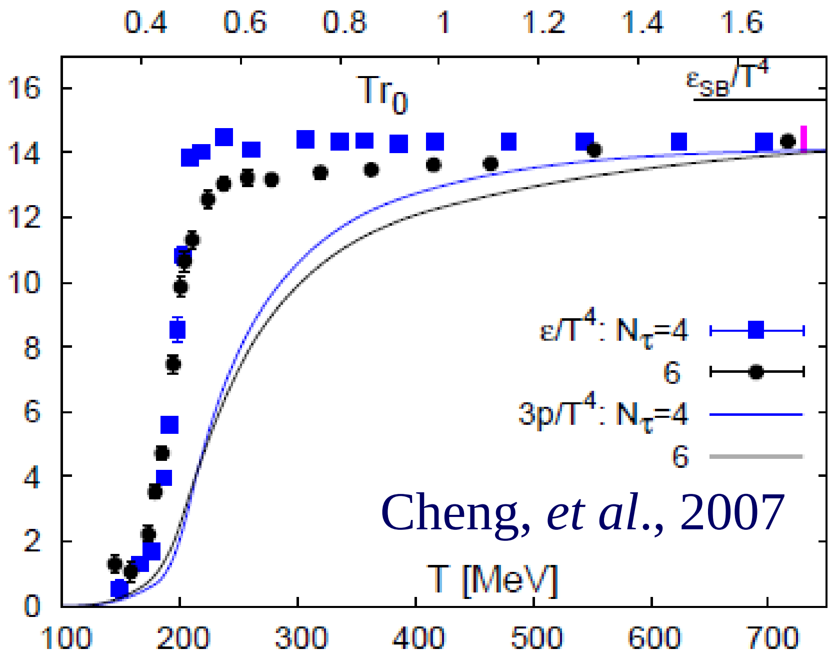
•Quenched approx. in thermodynamics → pure glue —— 100^3x25

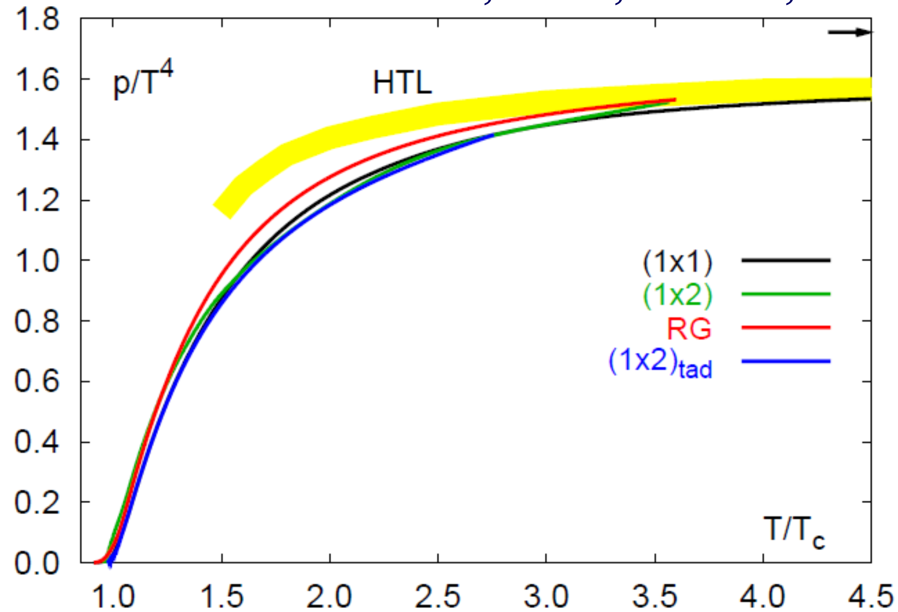
•Smaller m_q, heavier calculation.

•Simulations with physical mass have just started.

} 40^3x10

Bulk Thermodynamics





rapid increase of ε at T~190MeV
but no discontinuity → crossover

⟺ 1st order for massless 3-flavor

Pressure is well reproduced by perturbation theory for T>3~4T_c

deviation from ideal gas behavior ($\varepsilon = 3p$)

Fluctuations & Higher Order Moments

- Expectation values of operator O: $\langle O \rangle = \frac{1}{Z} \mathrm{Tr}\left[O e^{-\beta H} \right]$

ex.) $c_2 \equiv \left\langle \delta N_B^2 \right\rangle = \left\langle N_B^2 \right\rangle - \left\langle N_B \right\rangle^2 = -T\partial^2\Omega / \partial\mu_B^2$ baryon # fluct.

$c_4 \equiv \left\langle \delta N_B^4 \right\rangle - 3\left\langle \delta N_B^2 \right\rangle^2 = -T\partial^4\Omega / \partial\mu_B^4$, …

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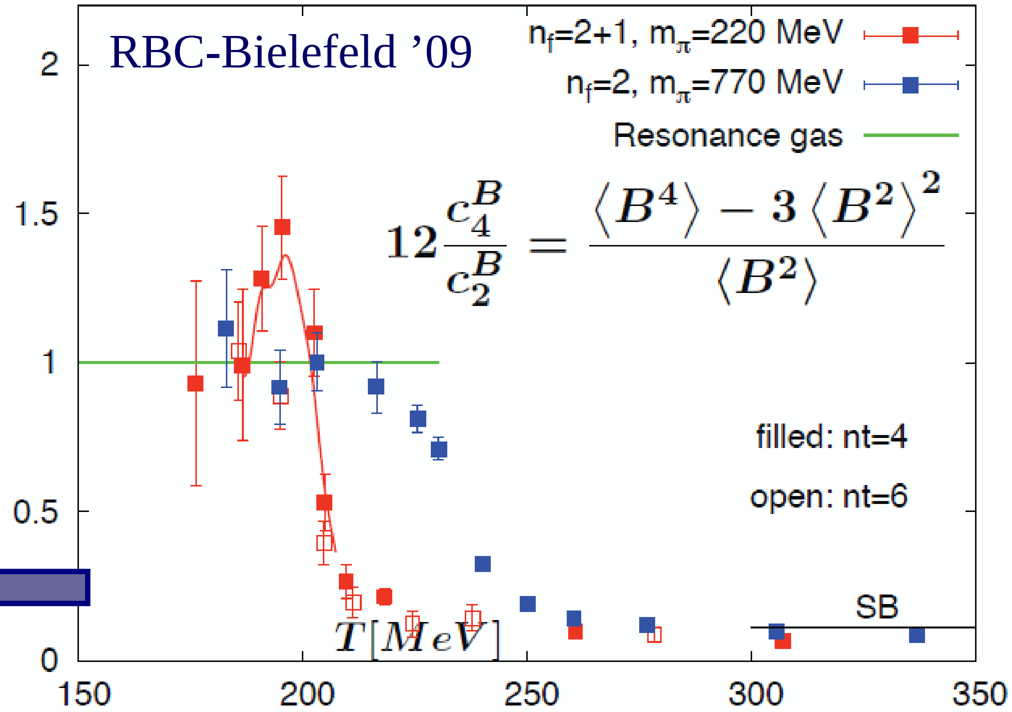
When $m_i \gg T$

$c_4 / c_2 \sim B_i^2$

charges of particles

Ejiri, Karsch, Redlich, '05

hadron-quark transition?



cf.) 3rd moments also serves as good exp. signals. Asakawa, Ejiri, MK, '09

Extracting Spectral Functions

$$D(\tau) = \int_{-\infty}^{+\infty} d\omega K(\omega,\tau)\rho(\omega)$$

$$K(\omega,\tau) = \frac{e^{(\beta/2-\tau)\omega}}{e^{\beta\omega/2} + e^{-\beta\omega/2}}$$ for fermions

$$D(\tau) = \langle T_\tau O(\tau) O(0) \rangle$$

lattice observable
discrete and noisy

spectral function:
dynamical information

Ill-posed problem

Extracting Spectral Functions

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spectral function:
dynamical information

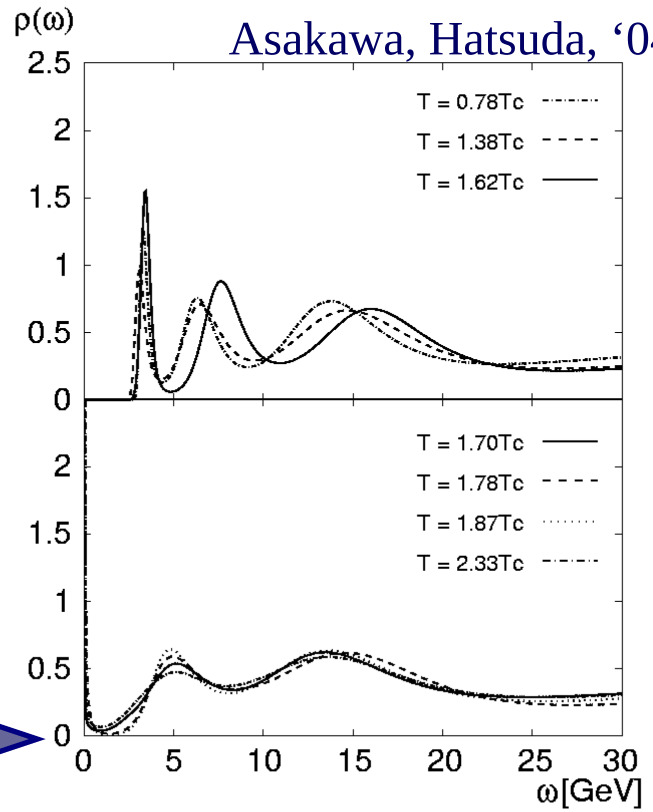
Ill-posed problem

Maximum Entropy Method (MEM)

Nakahara, Asakawa, Hatsuda, 1999~

- method to infer the most probable image
- Bayes' theorem + prior information

Asakawa, Hatsuda, '04



J/ψ spectral function

Lattice Studies on Spectral Properties for $T>T_c$

- Hadronic channels:
  - •charmonia — Asakawa, Hatsuda, ’04; Datta, et al., Umeda, et al., Aarts, et al., Petreczky et al., …
  - •light quarks — Asakawa, et al.,’03; Bielefeld group
  - •baryons — Asakawa, ’08

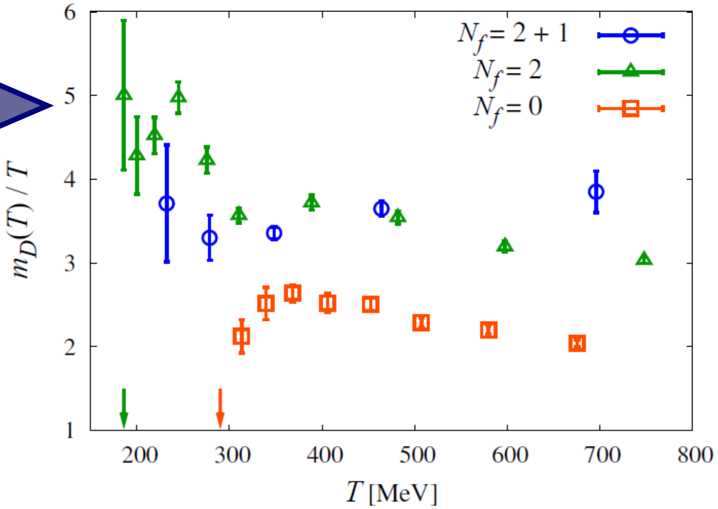
- correlators of EM tensor

$$\rho_{12,12}(\omega,\mathbf{p}) = \mathrm{Im}\,\mathrm{F.T.}\left\langle \left[T_{12}(x), T_{12}(0) \right] \right\rangle \vartheta(t) \;\Rightarrow\; \text{viscosity:}\ \eta = \lim_{\omega\to 0} \frac{\rho_{12,12}(\omega,\mathbf{0})}{\omega}$$

with fitting ansatze   Karsch, Wyld, ’87; Nakamura, Sakai, ’97,’04; Mayer, ’07,’08

- gluon: screening mass

  Nakamura, Saito, ’04
  Kaczmarek, Zantow, ’05
  Maezawa, et al. ’08



- quark: present study

2, Perturbative Study of Fermion Spectrum at high T

Free Dirac Propagator

$$S(\omega,\mathbf{p}) \equiv \text{F.T.}\langle \psi(\mathbf{x})\bar{\psi}(0)\rangle = \frac{1}{\not{p}-m} = \frac{1}{\omega\gamma^0 - \mathbf{p}\cdot\boldsymbol{\gamma} - m}$$

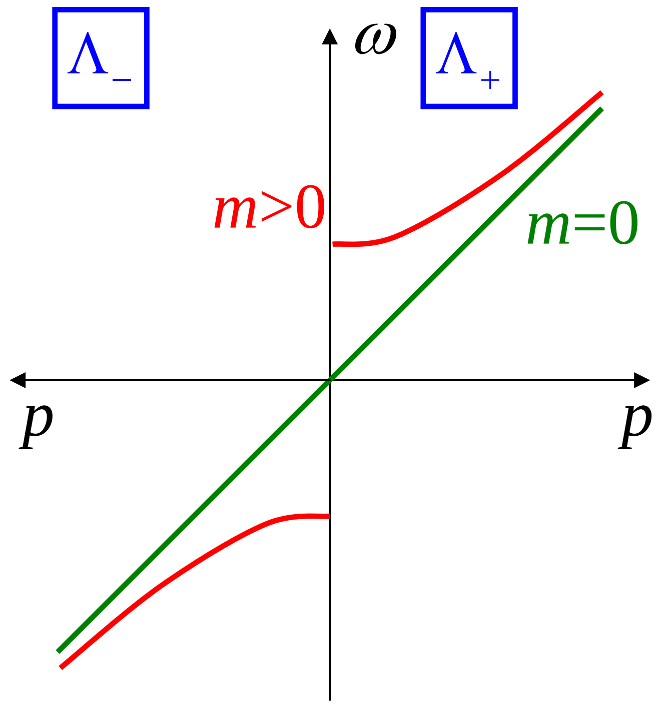
$\psi(\mathbf{x})$: 4-component spinor
γ^μ : 4x4 matrices in spinor space

Projector: $\Lambda_\pm(\mathbf{p}) = \dfrac{E_\mathbf{p} \pm \gamma_0(\mathbf{p}\cdot\vec{\gamma}+m)}{2E_\mathbf{p}}$

$$E_\mathbf{p} = \sqrt{\mathbf{p}^2+m^2}$$

$$S(\omega,\mathbf{p})\gamma^0 = \frac{\Lambda_+(\mathbf{p})}{\omega - E_\mathbf{p}} + \frac{\Lambda_-(\mathbf{p})}{\omega + E_\mathbf{p}}$$

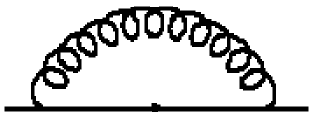
particles | antiparticles



Quarks at Extremely High T

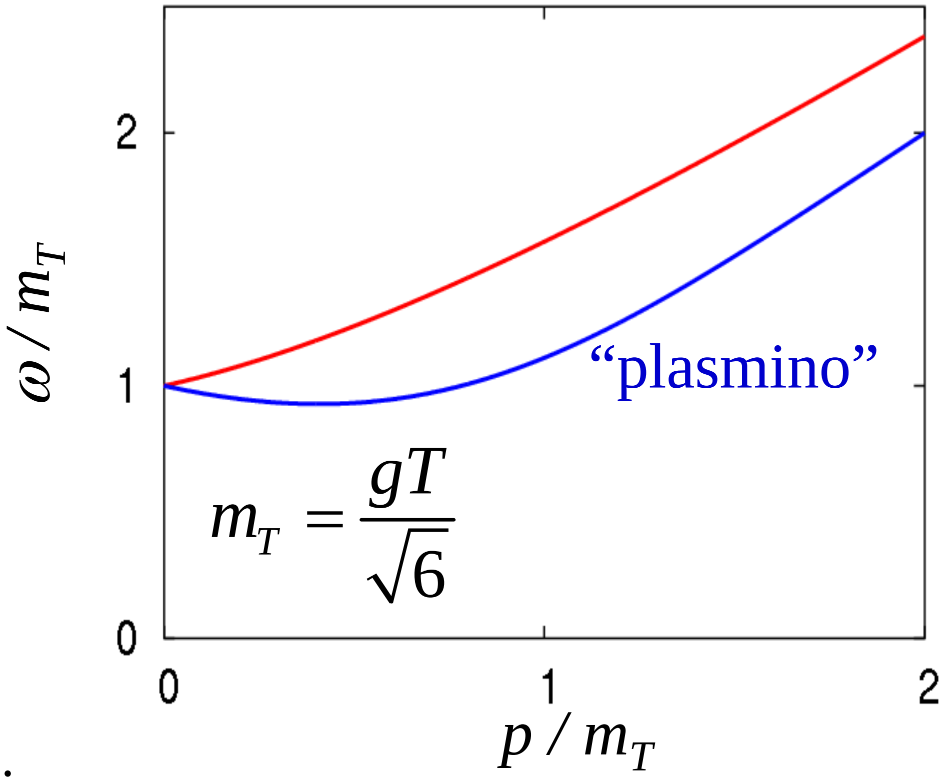
Klimov '82, Weldon '83
Braaten, Pisarski '89

- 1-loop (g<<1)
- Hard Thermal Loop approx. ($\boldsymbol{p}$, ω, m_q<<T)

$\Sigma(\omega,\mathbf{p}) =$ 

$$S(\omega,\mathbf{p}) = \frac{1}{\omega\gamma_0 - \mathbf{p}\cdot\boldsymbol{\gamma} - \Sigma(\omega,\mathbf{p})}$$

- Gauge independent spectrum
- 2 collective excitations having a "thermal mass" ~ gT
- width ~g^2T
- The plasmino mode has a minimum at finite $\boldsymbol{p}$.



- Similar mass gap in gluon dispersion.
- Thermal mass emerges with any bosons.

Quarks at Extremely High T

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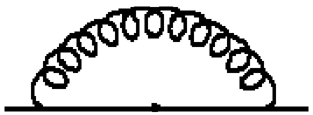
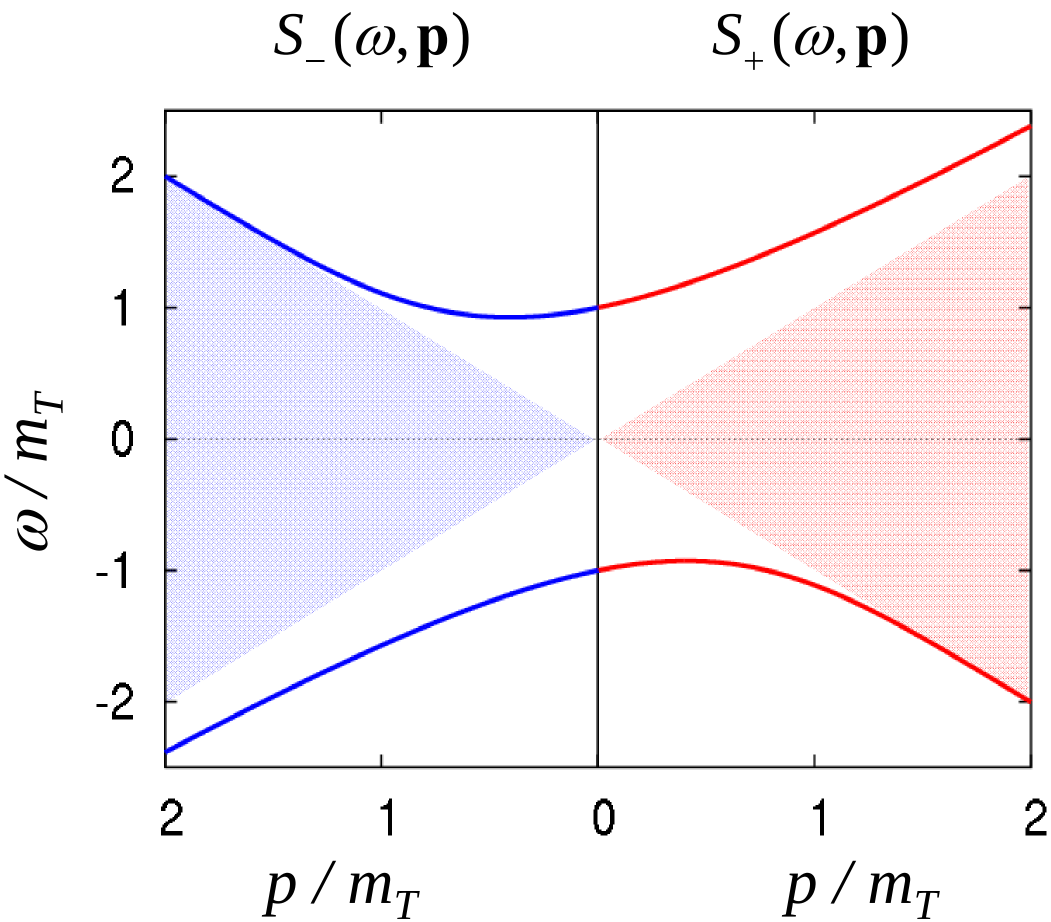
$\Sigma(\omega,\mathbf{p}) =$ 

$$S(\omega,\mathbf{p}) = \frac{1}{\omega\gamma_0 - \mathbf{p}\cdot\boldsymbol{\gamma} - \Sigma(\omega,\mathbf{p})}$$

$$= S_+(\omega,\mathbf{p})\Lambda_+(\vec{\mathbf{p}})\gamma^0$$

$$+ S_-(\omega,\mathbf{p})\Lambda_-(\vec{\mathbf{p}})\gamma^0$$

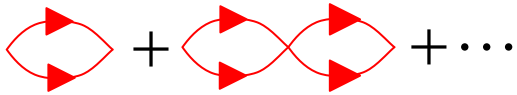
$$\Lambda_\pm(\mathbf{p}) = \frac{1 \pm \gamma_0 \mathbf{p}\cdot\vec{\gamma}}{2}$$



Physical Origin of m_T & Plasmino

analogy to **Electron & Soft mode in SC**

Effect of **precursory fluctuations** of SC on electrons above T_c

soft mode 

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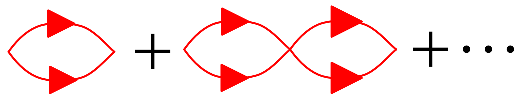
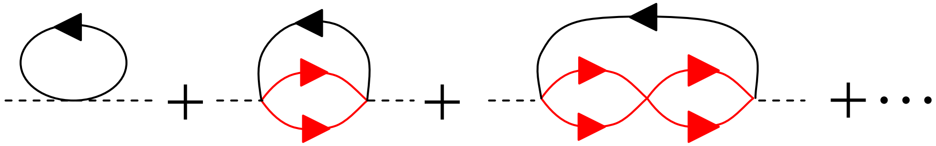
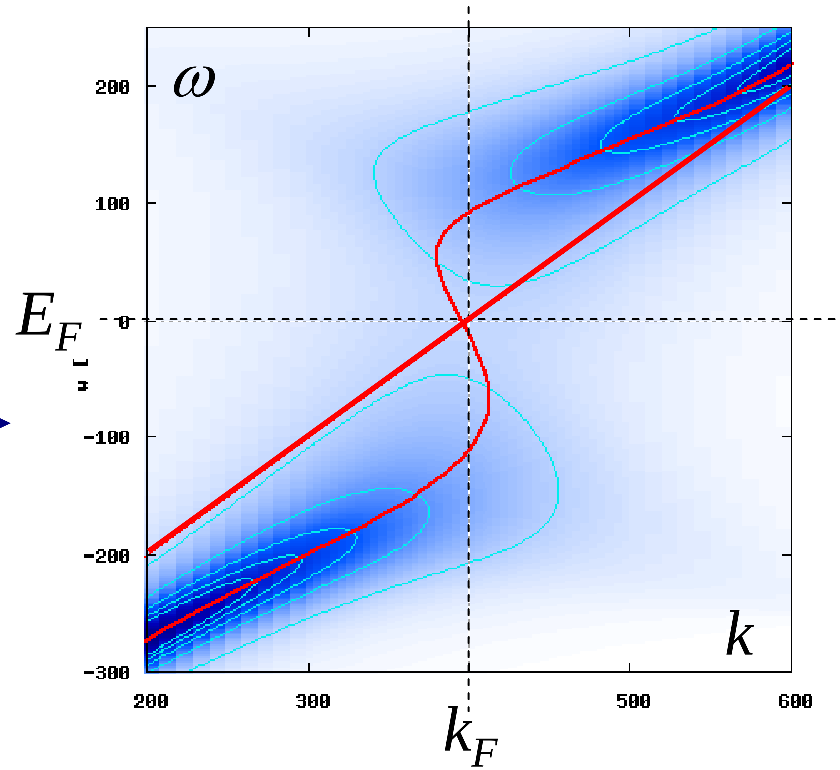


T-matrix approximation



“Pseudogap” spectrum
around the Fermi energy

particle-hole mixing



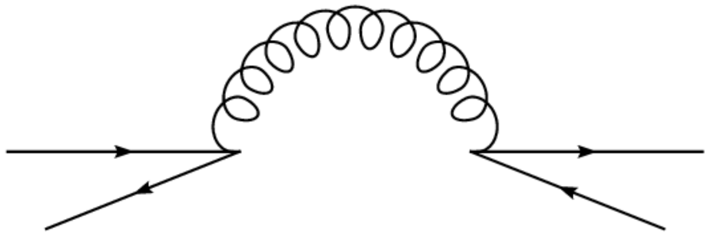
Janko, Maly, Levin, PRB**56**,R11407 (1995)

Physical Origin of m_T & Plasmino

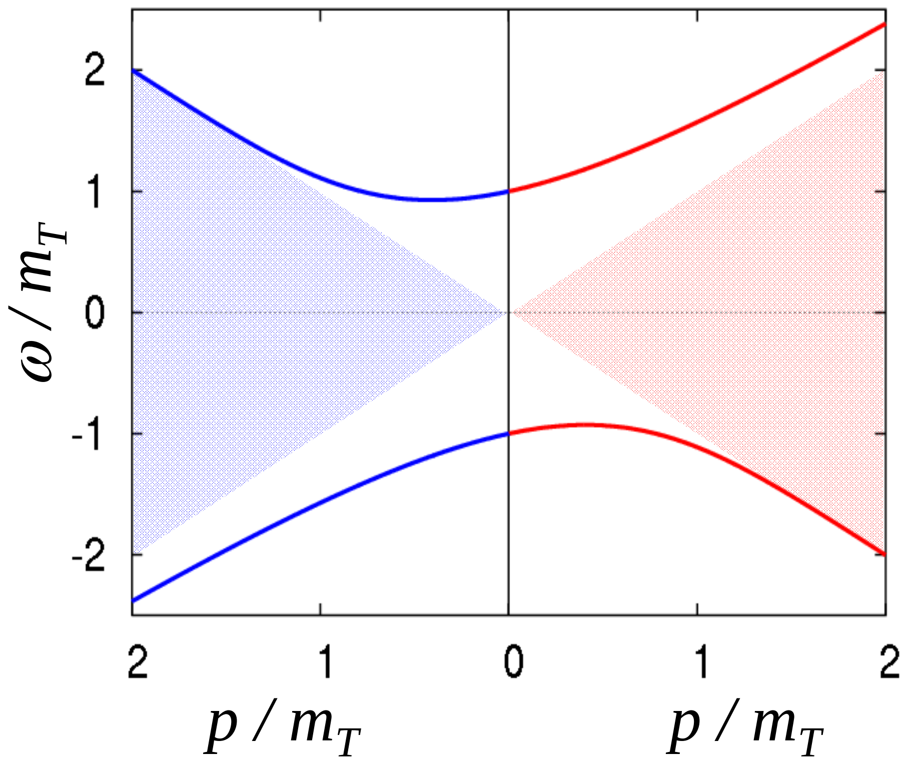
Weldon, '89
MK, *et al.*, '06

$$S_{\mathrm{HTL}}(\omega,\mathbf{p}) = \frac{\Lambda_+(\mathbf{p})\gamma^0}{\omega - p - \Sigma_+} + \frac{\Lambda_-(\mathbf{p})\gamma^0}{\omega + p - \Sigma_-}$$

= A quark is scattered by a gluon.



= A quark and a thermally-excited anti-quark annihilate and produce a gluon.



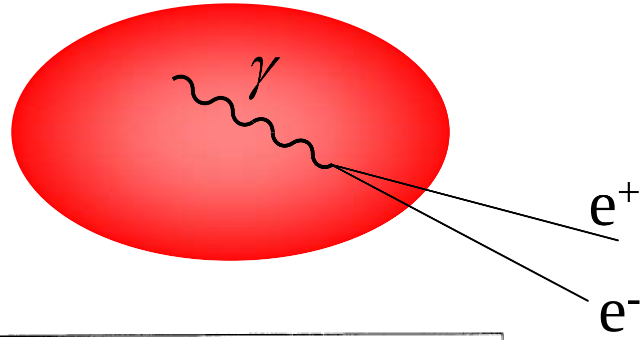
⇒ The quark turns into the "**anti-quark hole**".

Effects of Plasmino Minimum

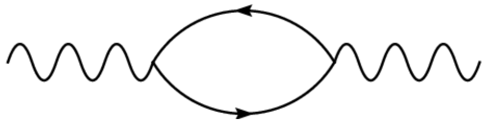
minimum @ p>0 ⇒ divergence of DoS
⇒ **van Hove singularity**

- Example: dilepton production rate

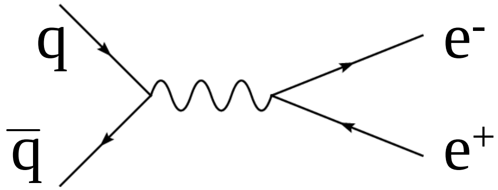
observable in heavy-ion collisions

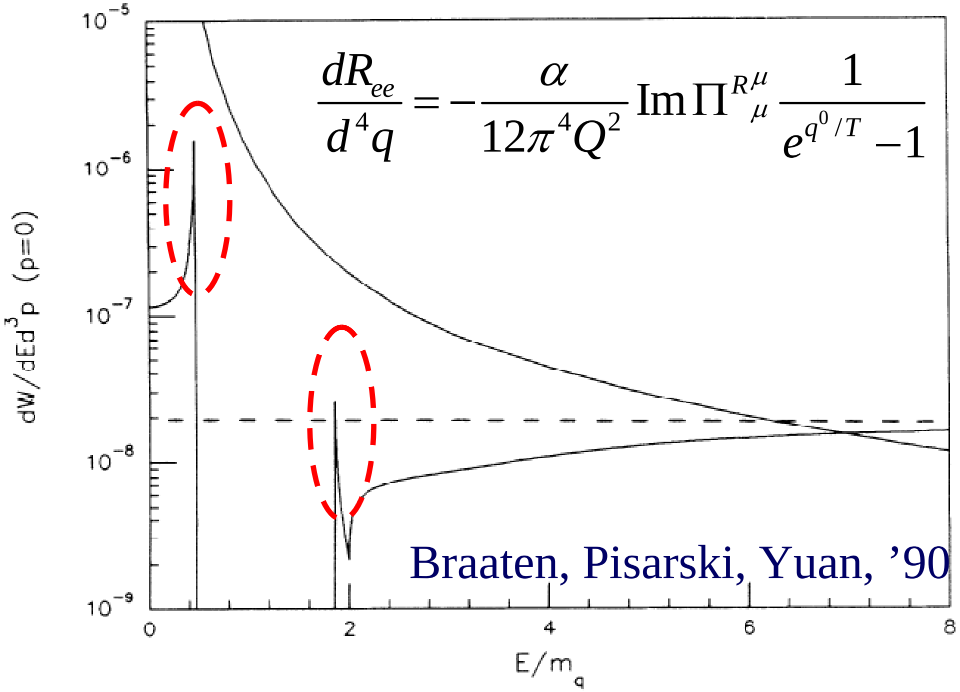


•photon self-energy:



•decay process:





Quark Spectrum as a function of m_0

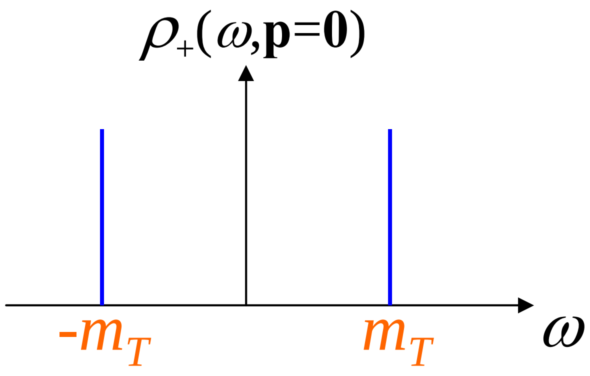
Quark propagator in hot medium at $T >> T_c$

- as a function of bare scalar mass m_0

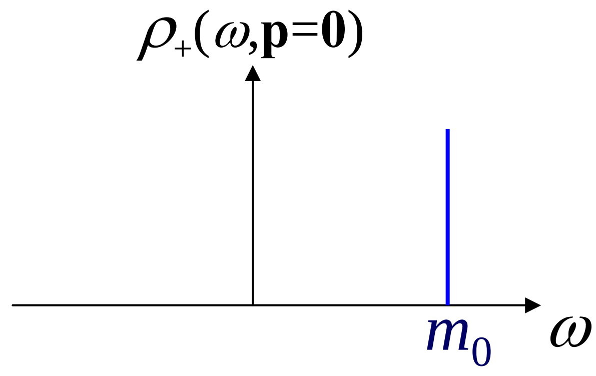
$$\rho(p,\omega) = \rho_+(p,\omega)\Lambda_+(\vec{p})\gamma^0 + \rho_-(p,\omega)\Lambda_-(\vec{p})\gamma^0$$

We know two gauge independent limits:

$m_0 << gT$



$m_0 >> gT$



- How is the interpolating behavior?
- How does the plasmino excitation emerge as $m_0 \rightarrow 0$?

Fermion Spectrum in Yukawa Models

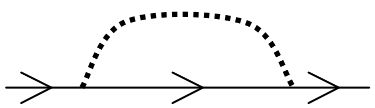
massive fermion + massive scalar boson

$$L = i\bar{\psi}(i\partial\!\!\!/ - m_f - g\sigma)\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + m_b^2\sigma^2\right)$$

Model parameter : g, m_f, m_b, (T)

(1) $m_f>0$, $m_b=0$

- massive fermions in gauge theories
- electrons at $T \sim m_e$
- heavy quarks in QGP



1-loop approx.

Baym, Blaizot, Svetitsky, '92

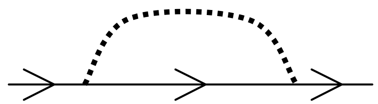
(2) $m_f=0$, $m_b>0$

- light quarks in the QGP phase coupled with massive bosons
- excitations in graphene
- leptons coupled with W and Z at $T \sim m_{W,Z}$

MK, Kunihiro, Nemoto, '06,'07
Boyanovsky, '06

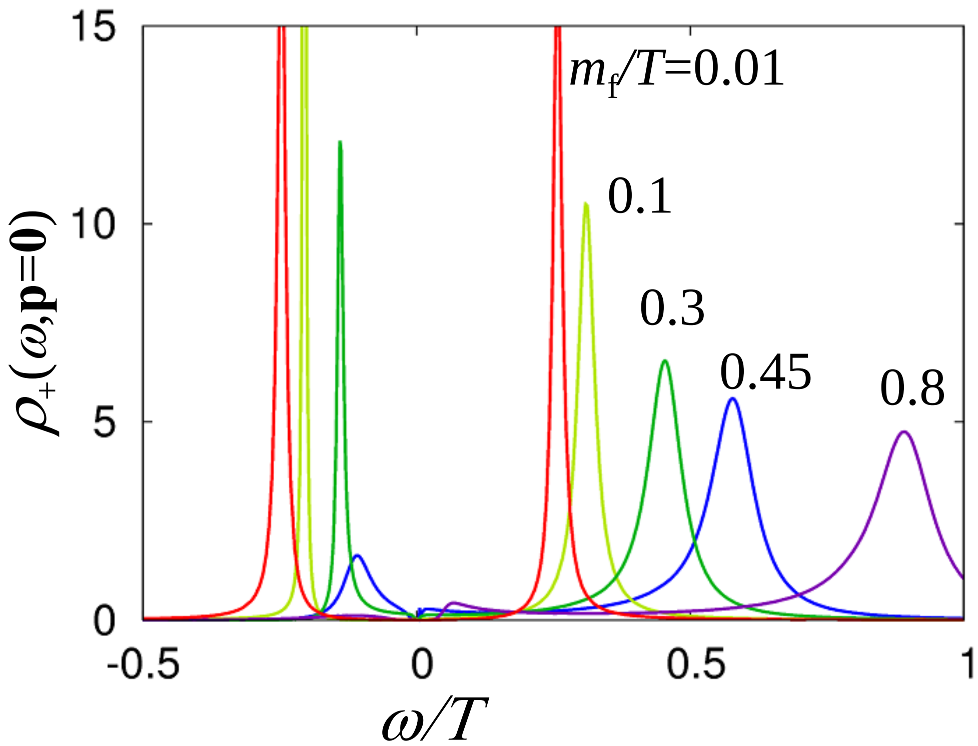
Case1 : $m_f>0$, $m_b=0$

one-loop approx.



- quark spectral function in 1-loop approx.:

Spectral Function for $g=1$, $T=1$

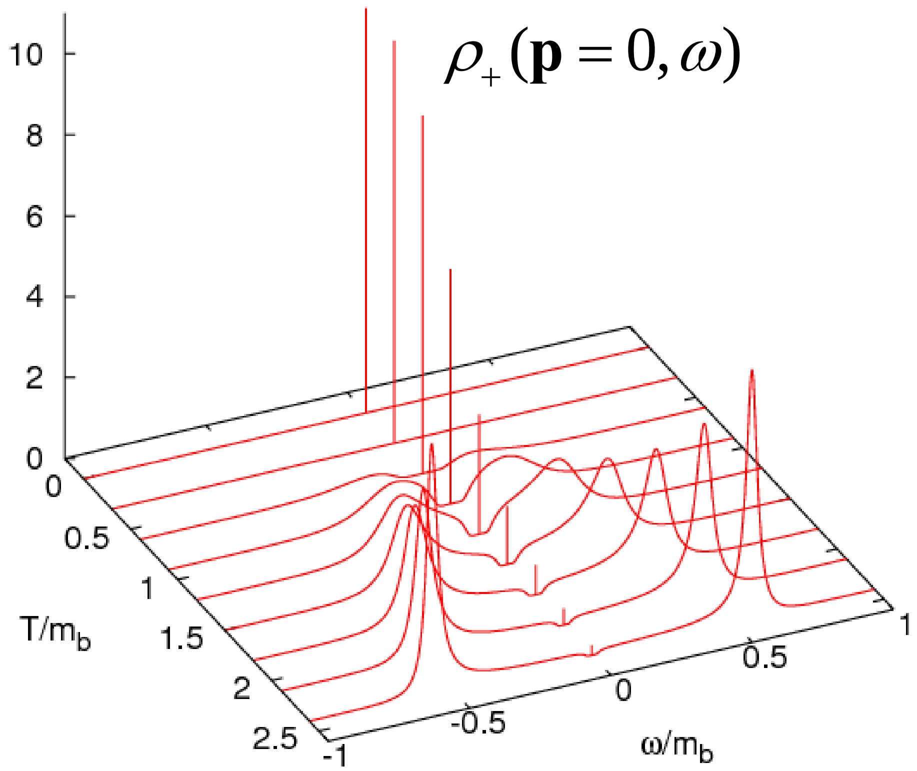


- $m_f/T \ll 1$
  thermal mass $m_T=gT/4$
- $m_f/T \gg 1$
  single peak at m_f

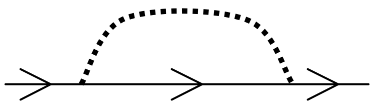
•Two limits are connected continuously.
•Plasmino peak disappears as m_f/T becomes larger.

•Note: Qualitative result hardly changes with a variation of g, and in Yukawa models with PS, V, AV interactions, and in QED.

Case2 : m_f=0, m_b>0

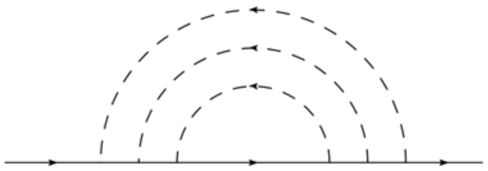


g=1
one-loop approx.

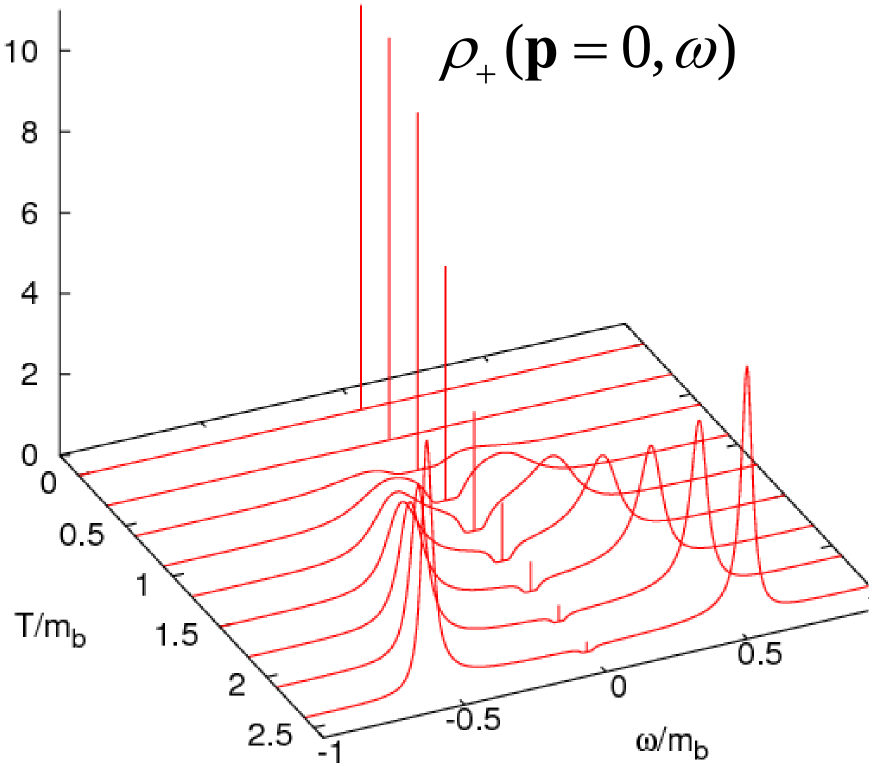


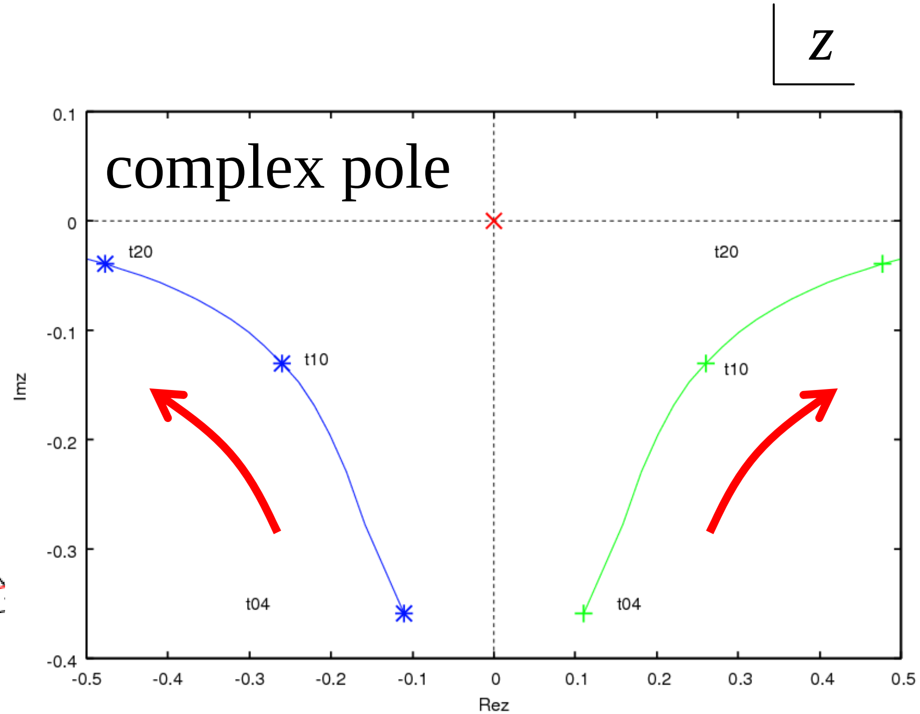
- Three peak structure is formed at T/m_b~1 as an interpolating behavior between single- and two-pole limits.
- Similar result is obtained even with Schwinger-Dyson approach.

Harada, Nemoto, '08



Case2 : *T* dependence for *p*=0





- Three peak structure is formed at $T/m_b \sim 1$ as an interpolating behavior between single- and two-pole limits.
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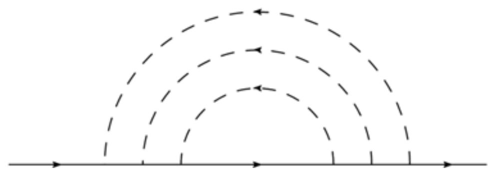
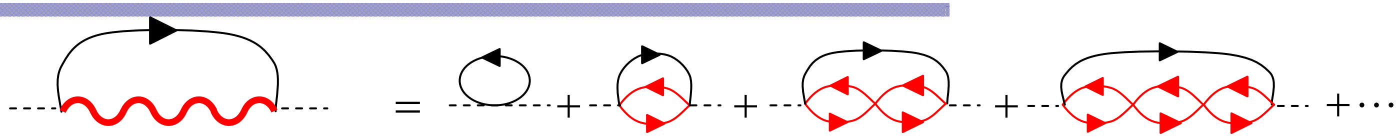
Harada, Nemoto, '08

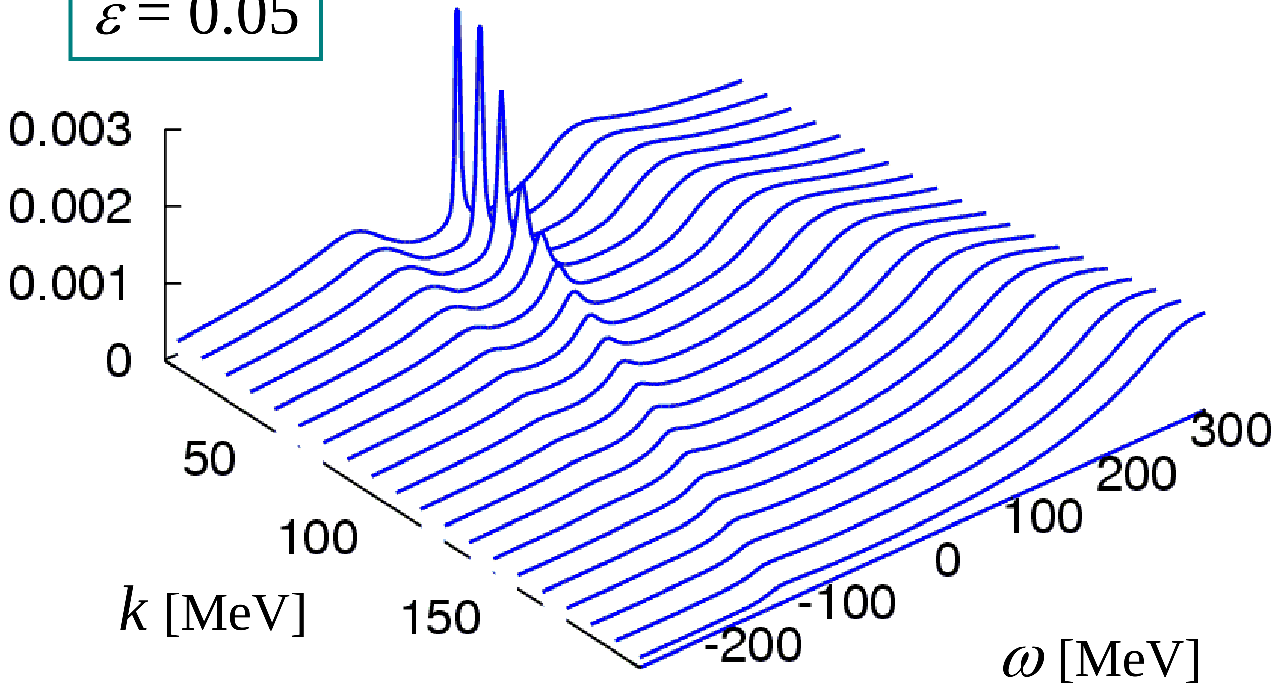


Spectral Function in NJL Model

$\Sigma(\mathbf{k},\omega) =$



$\varepsilon = 0.05$

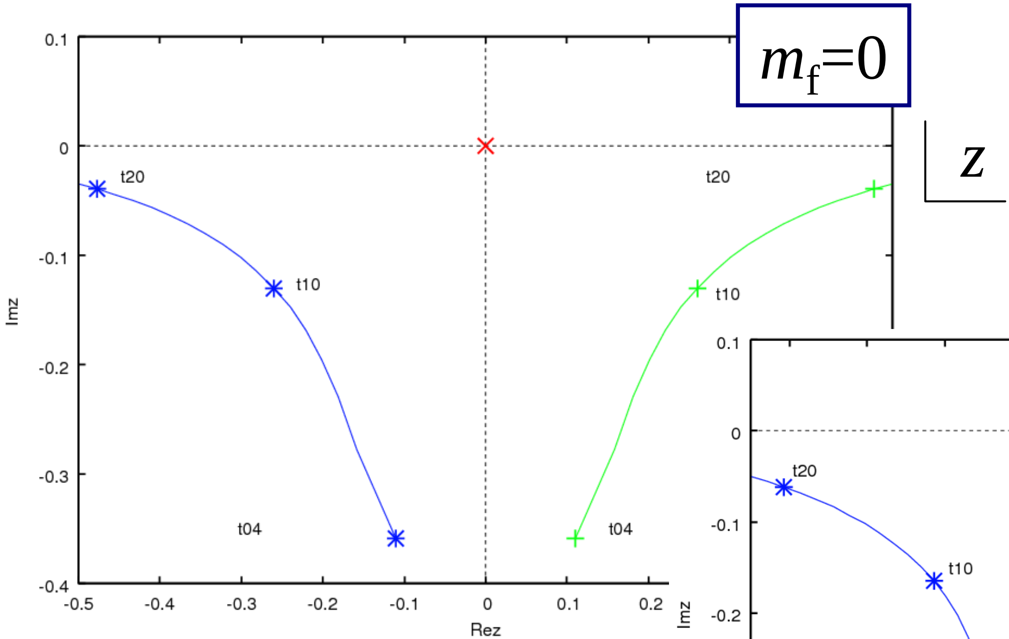


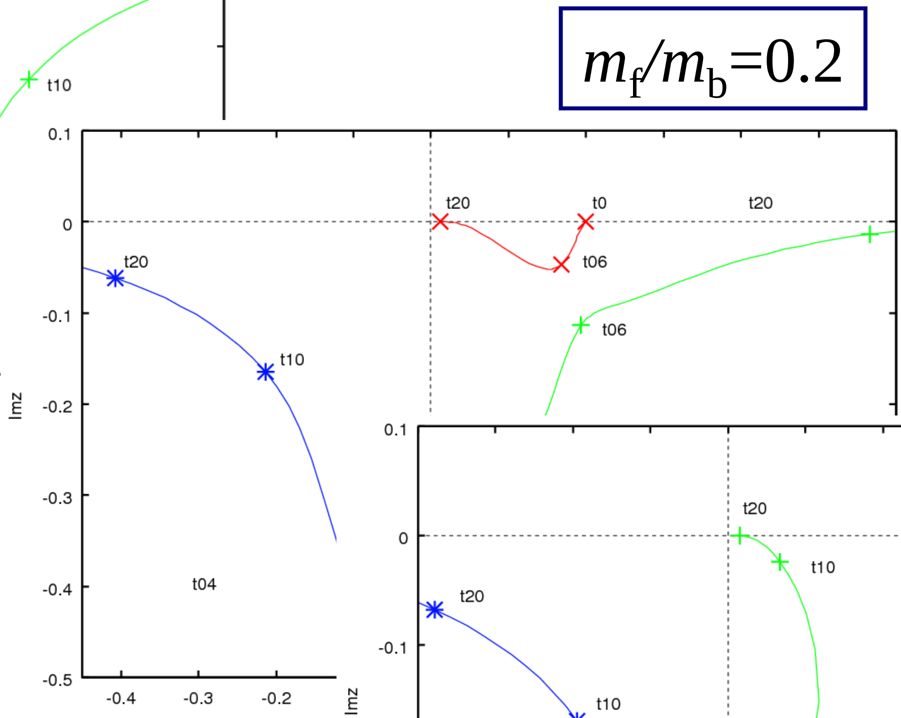
$$\varepsilon = \frac{T - T_c}{T_c}$$

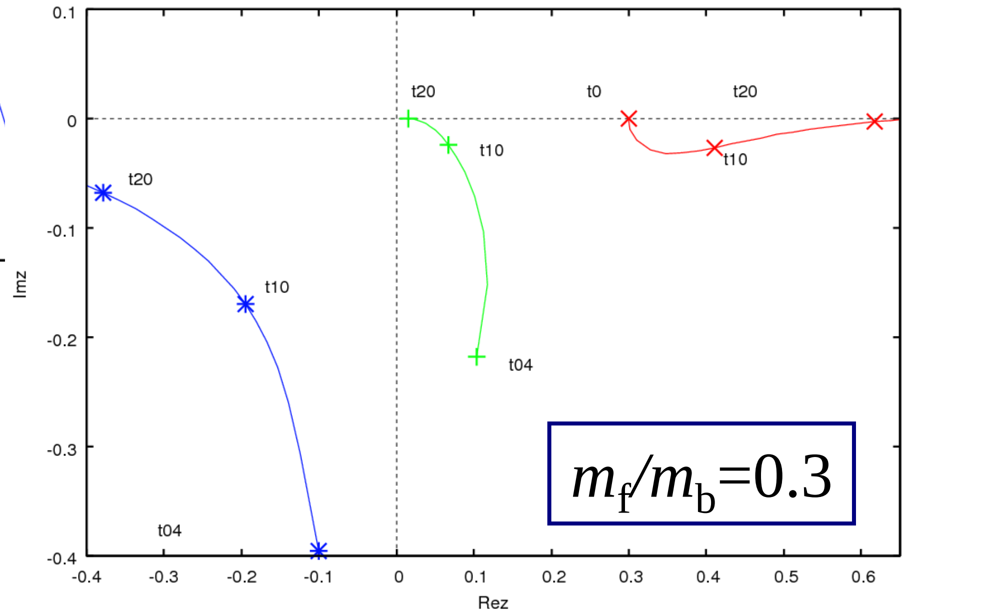
- Three-peak structure emerges.
- The peak around the origin is the sharpest.

Case3 : $m_b>0$, $m_f>0$

Mitsutani, et al. '08





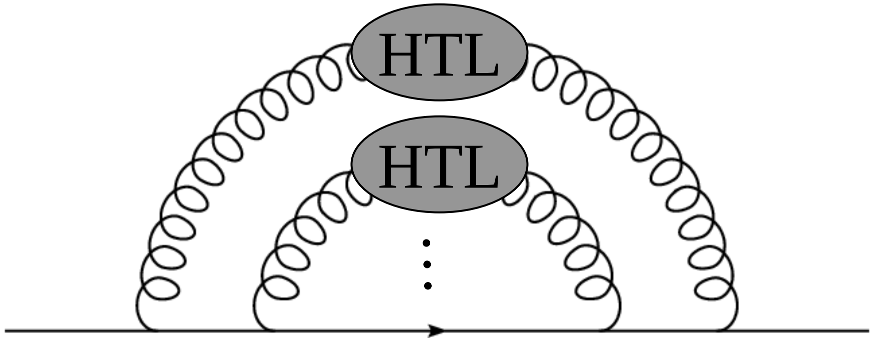


How do cases 1 & 2 connected?

"level crossing" between two poles!

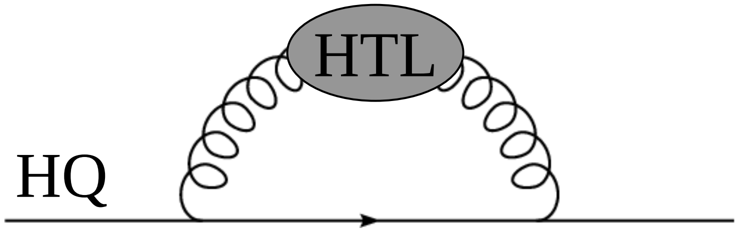
Higher Order Analysis

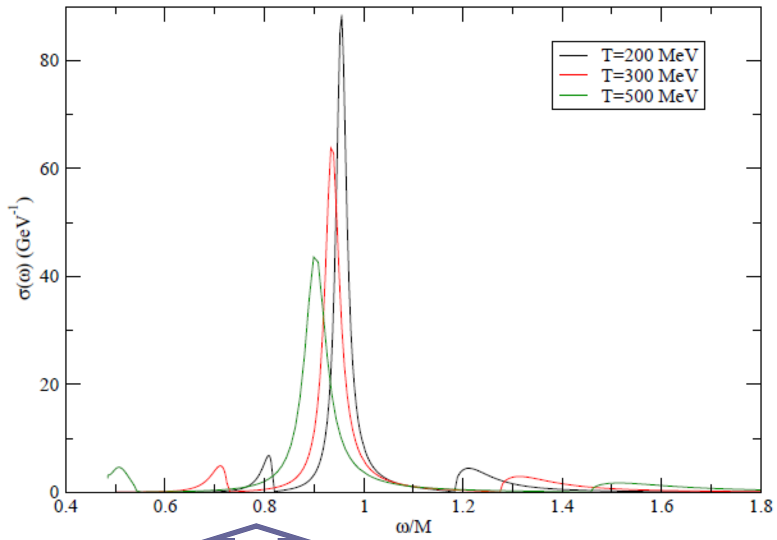
Harada, Yoshimoto, '09



Quark decay width grows as gluon screening mass becomes heavier.

Beraudo, Blaizot, *et al*. '09





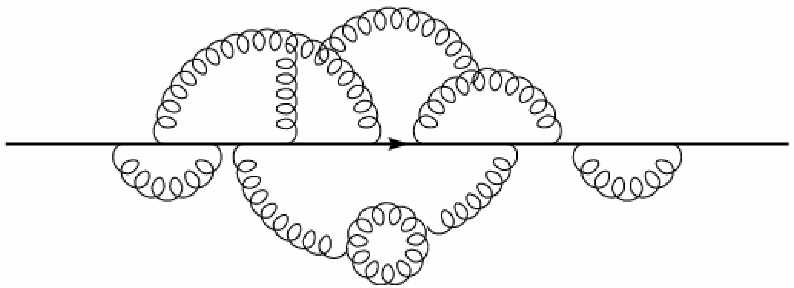
another peak
in charm spectrum

3, Lattice Study of Quarks at nonzero *T*

Simulation Setup

Let's start exploring
non-perturbative quark propagator!

- **quenched** approximation
- **Landau** gauge fixing



- analyze m, p, and T dependences of numerical results.

NEW!!

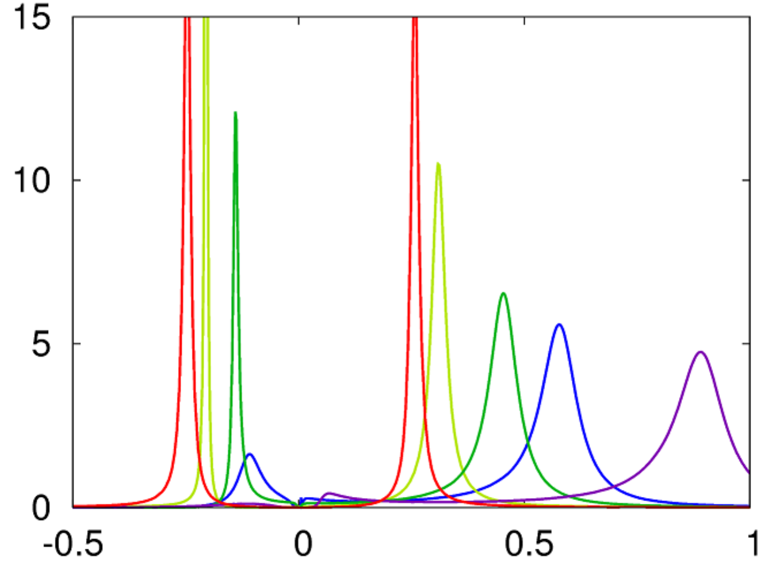
T	β	size
$3T_c$	7.45	**128^3x16**
		64^3x16
		48^3x16
	7.19	48^3x12
$1.5T_c$	6.87	64^3x16
		48^3x16
	6.64	48^3x12
$1.25T_c$	6.72	64^3x16
		48^3x16
$0.9T_c$	6.42	48^3x16
$0.55T_c$	6.13	48^3x16

Correlator and Spectral Function

$$C_+(\tau) = \int_{-\infty}^{+\infty} d\omega \frac{e^{(\beta/2-\tau)\omega}}{e^{\beta\omega/2}+e^{-\beta\omega/2}} \rho_+(\omega)$$

observable on the lattice

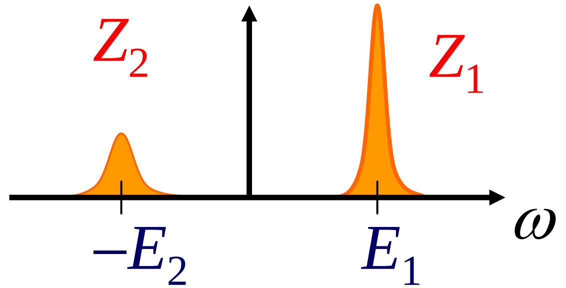
dynamical information



- 2-pole structure may be a good assumption for $\rho_+(\omega)$.

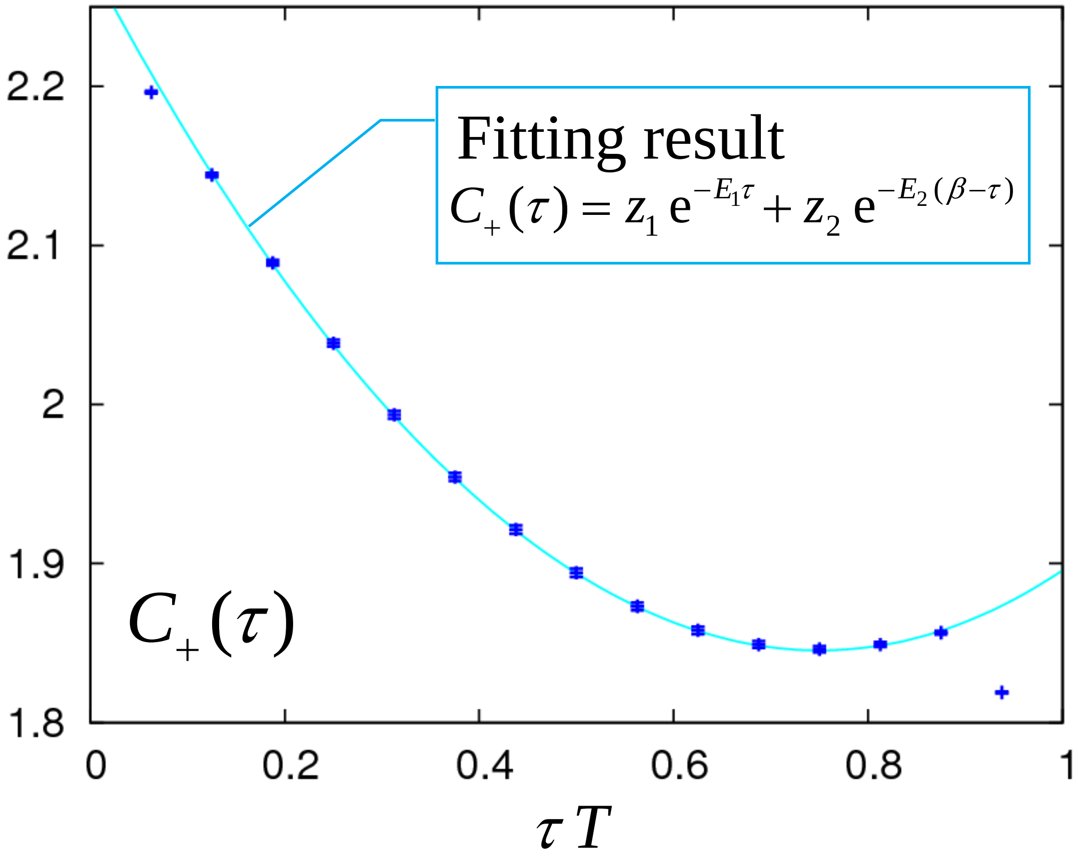
$$\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2)$$

4-parameter fit E_1, E_2, Z_1, Z_2



Correlation Function

$$C(\tau,\mathbf{0}) = C_+(\tau)\Lambda_+\gamma^0 + C_-(\tau)\Lambda_-\gamma^0$$



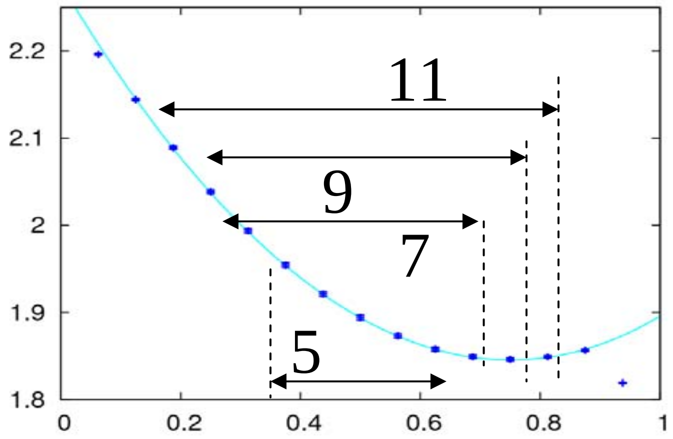
64^3x16, $\beta = 7.459$ ($3T_c$),
$\kappa = 0.1337$, 51confs.

- 2-pole ansatz works quite well. (χ^2/dof.~1 in correlated fit.)

⇒
- Quark excitations would be good quasi-particles.
- Gauge dependence of E_1 and E_2 should be small.

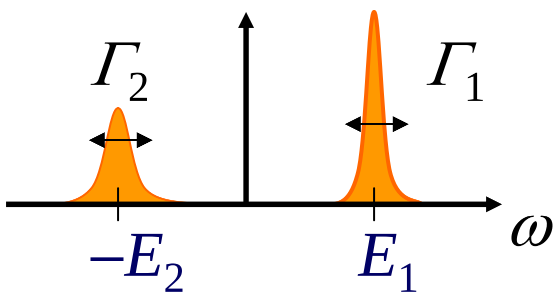
How Reliable is the 2-Pole Ansatz?

Check 1 fits with different data points N_{data}



Fitting results with different N_{data}= 5,7,9, and 11 coincide within statistical error.

Check 2 6-parameter fit with Gaussian width



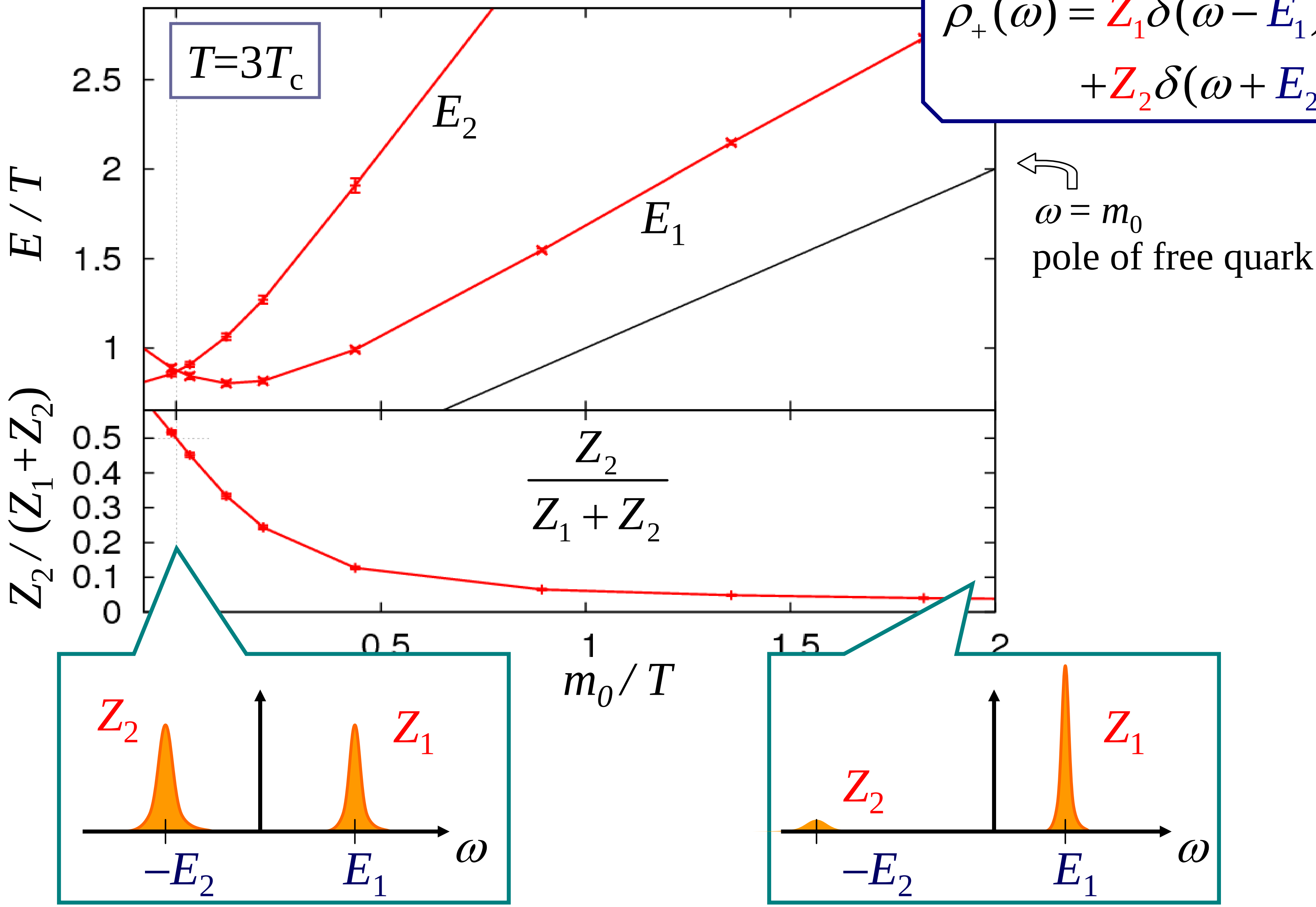
Minimum of χ^2 is always at Γ_1=Γ_2=0.

Note: The above results are unchanged even with N_σ=128.
χ^2 is large for heavy quarks; χ^2/dof >10 for charm quark.

Spectral Function

$T = 3T_c$ 64^3x16 ($\beta = 7.459$)

$$\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2)$$



Spectral Function

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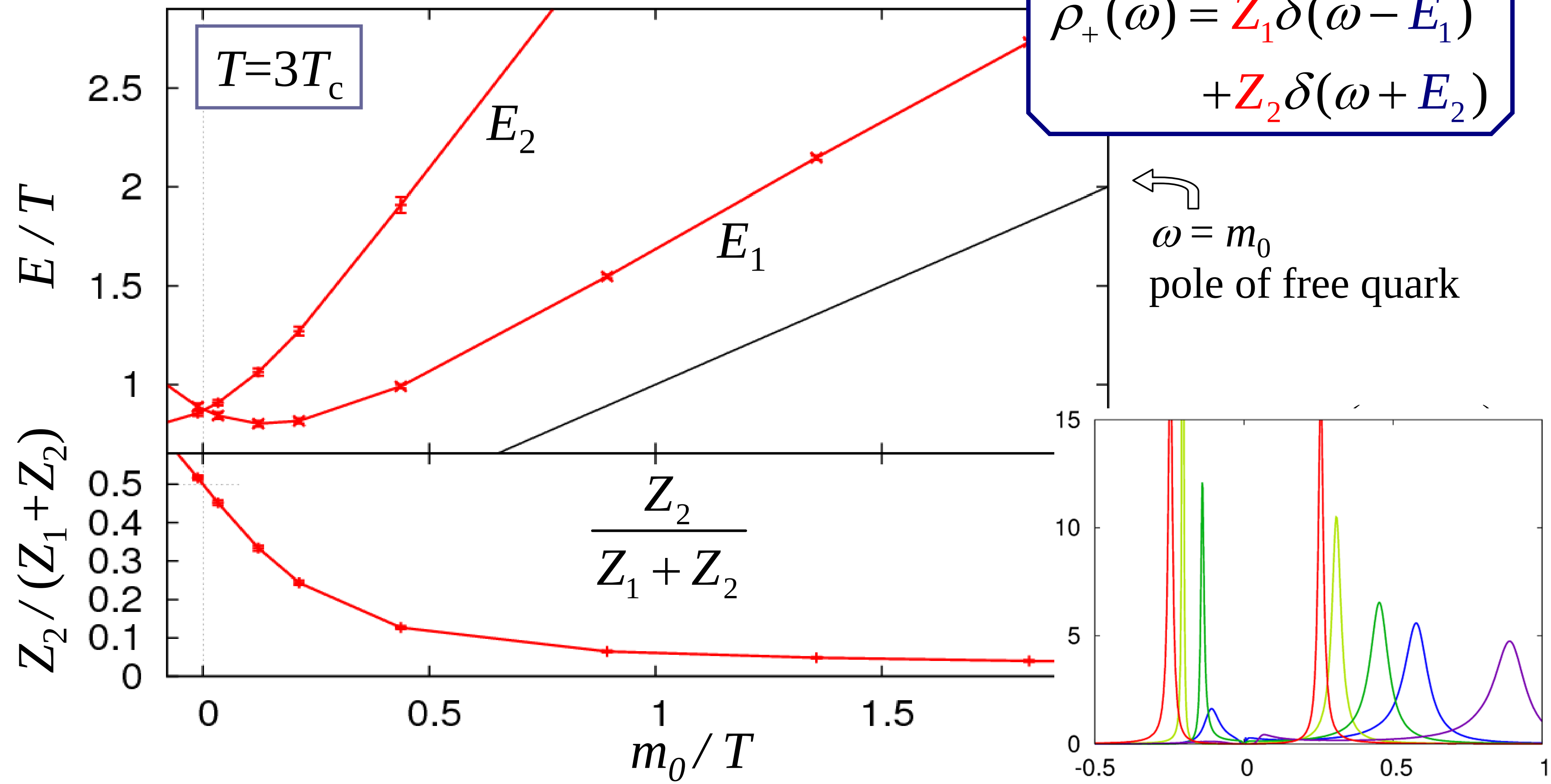
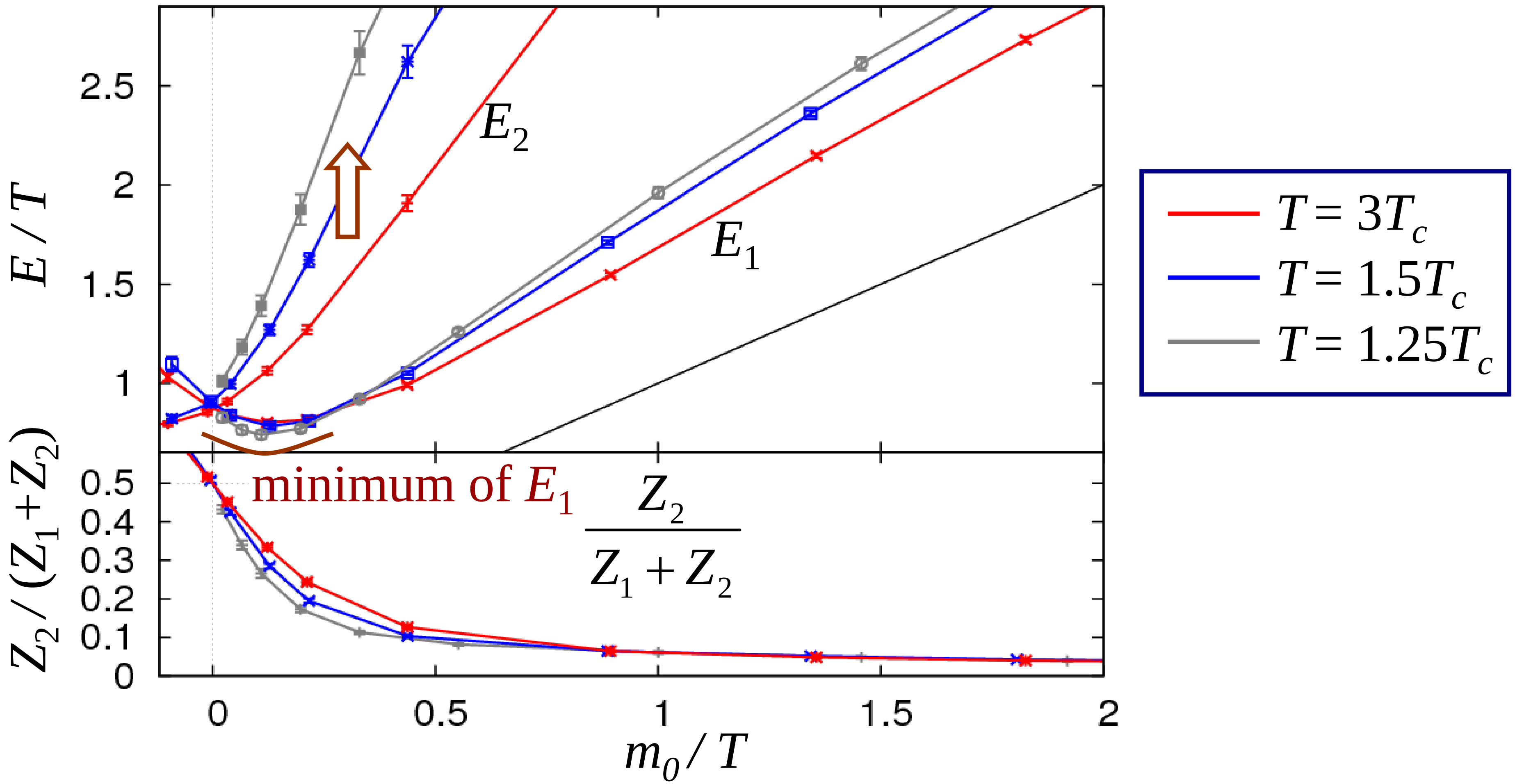


- Limiting behaviors for $m_0 \to 0, m_0 \to \infty$ are as expected.
- Quark propagator approaches the chiral symmetric one near m_0=0.
- E_2>E_1 : qualitatively different from the 1-loop result.

Temperature Dependence

64^3x16

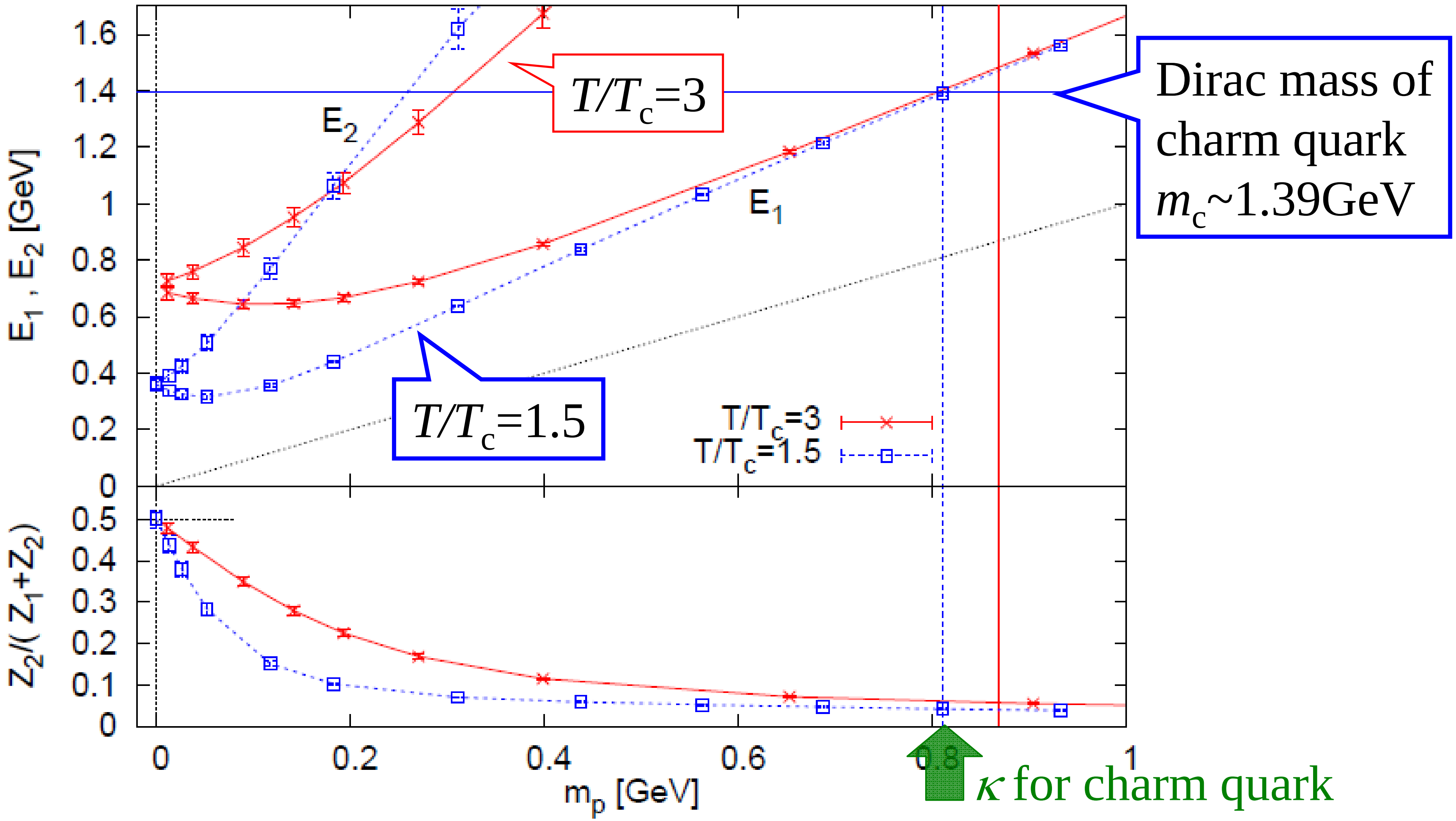


- m_T/T is insensitive to T.
- The slope of E_2 and minimum of E_1 is much clearer at lower T.

➡ 1-loop result might be realized for high T.

Charm Quark & J/ψ

κ from Datta et al. PRD**69**,094507(2004).



- Charm quark is free-quark like, rather than HTL.
- The J/ψ peak in MEM exists above $2m_c$.

Charm Quark & J/ψ

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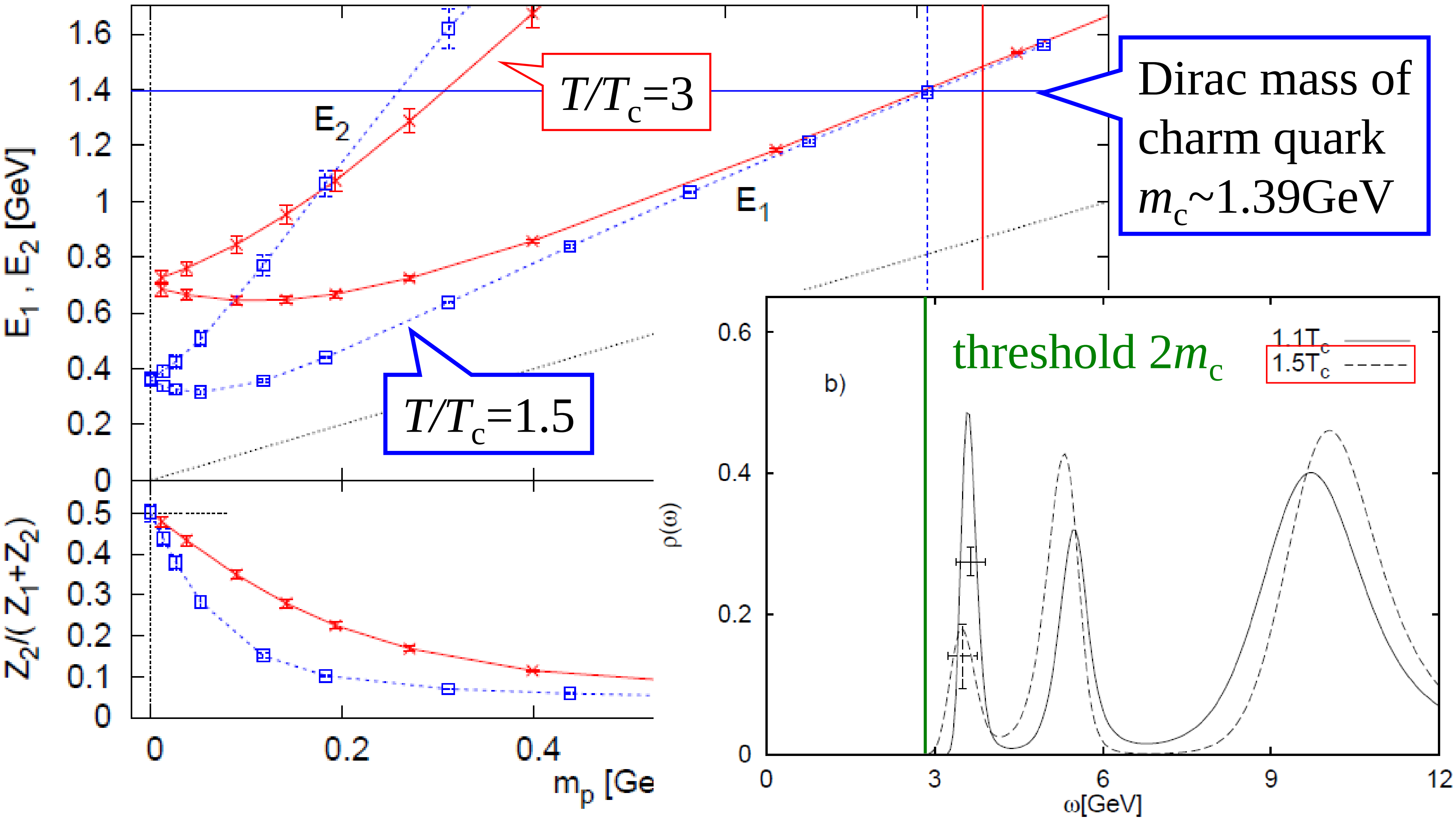

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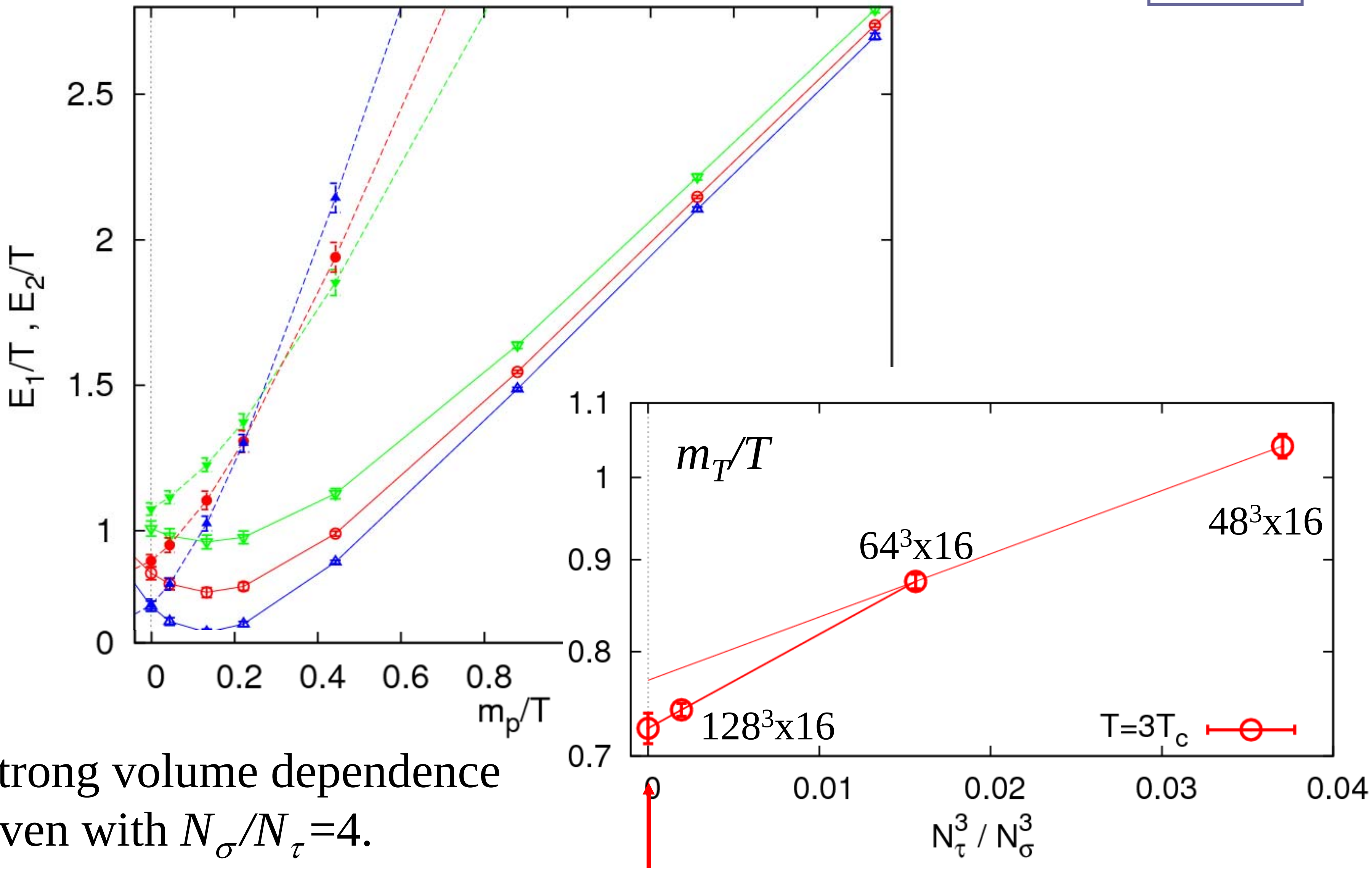
Spatial Volume Dependence

$T=3T_c$



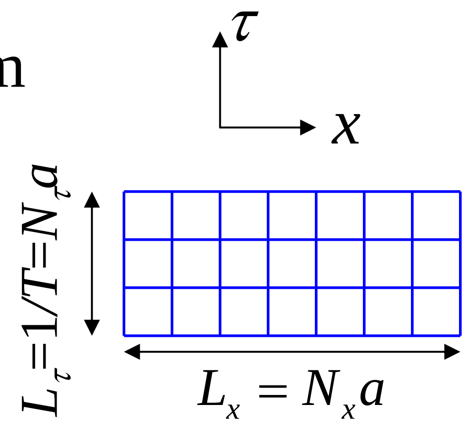
- Strong volume dependence even with $N_\sigma/N_\tau=4$.

$m_T/T=0.725(14)$

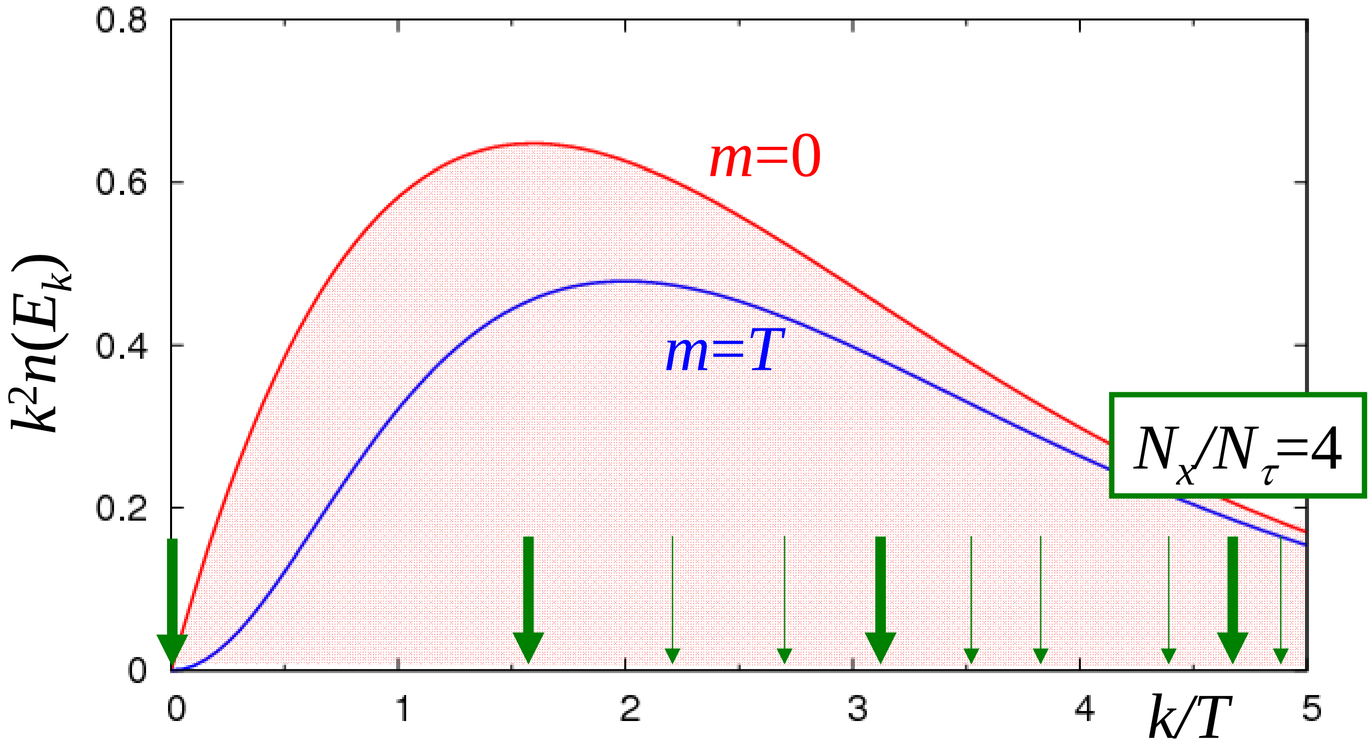
Finite Volume Effects

- Finite spatial volume → discretization of momentum

$$p_{\min} = \frac{2\pi}{L_x} = 2\pi T \frac{N_\tau}{N_x} \quad \Longleftarrow \quad L_x = \frac{N_x}{N_\tau}\frac{1}{T}$$

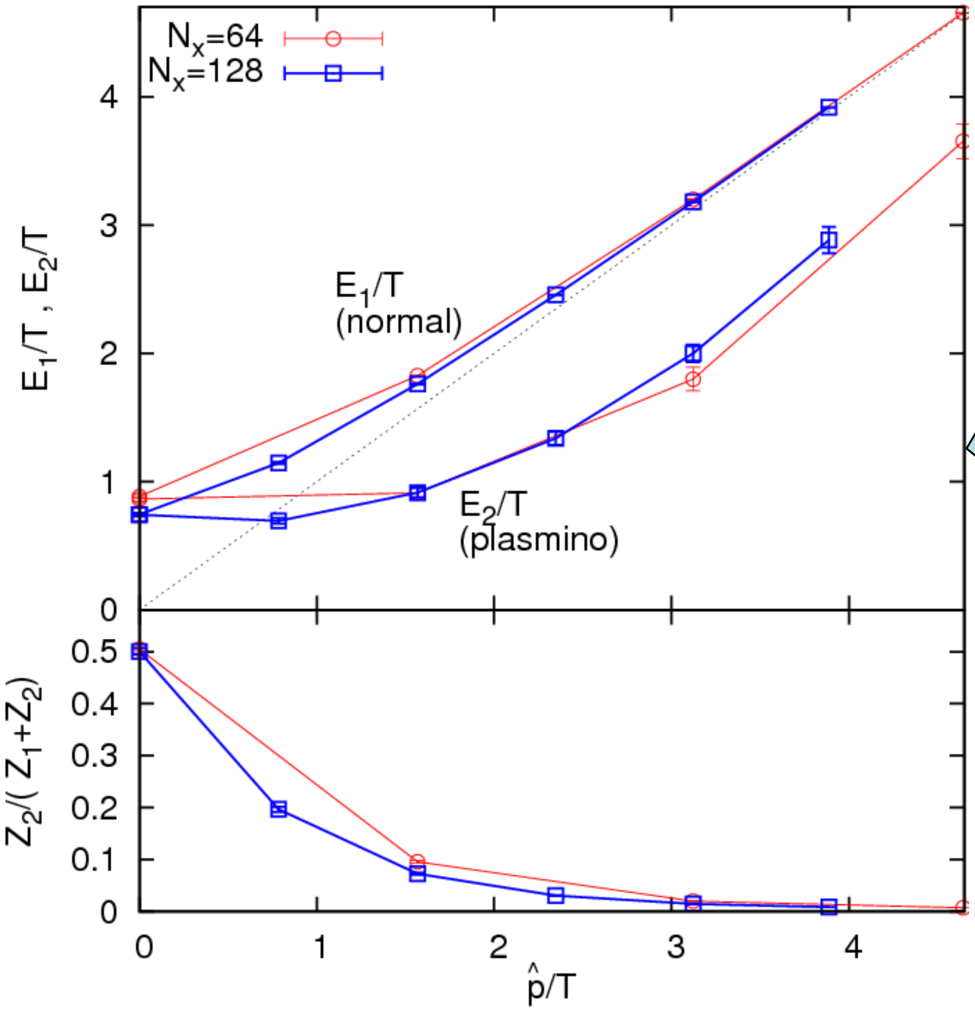


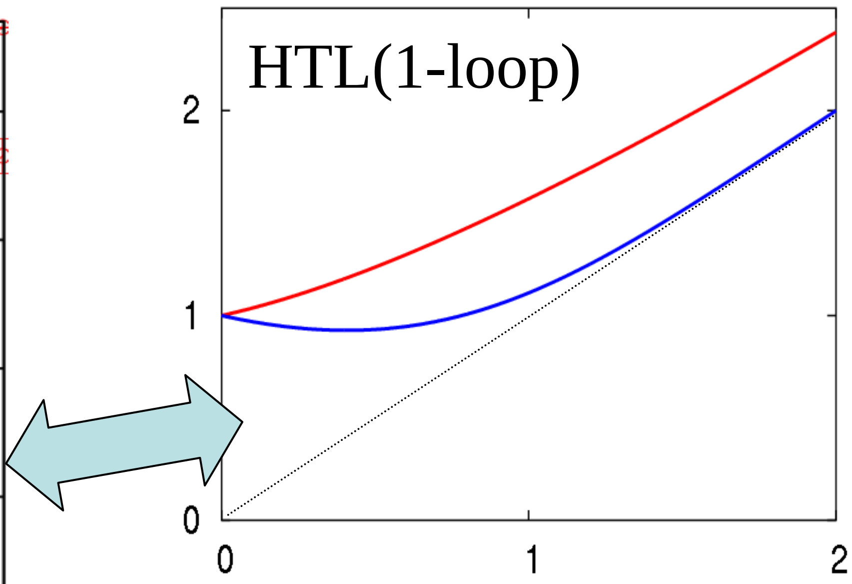
- $N_x/N_\tau = 4$ → $p_{\min} = (\pi/2)T \sim 1.57T$
- $N_x/N_\tau = 8$ → $p_{\min} = (\pi/4)T \sim 0.79T$



Quark Dispersion Relations

$T=3T_c$



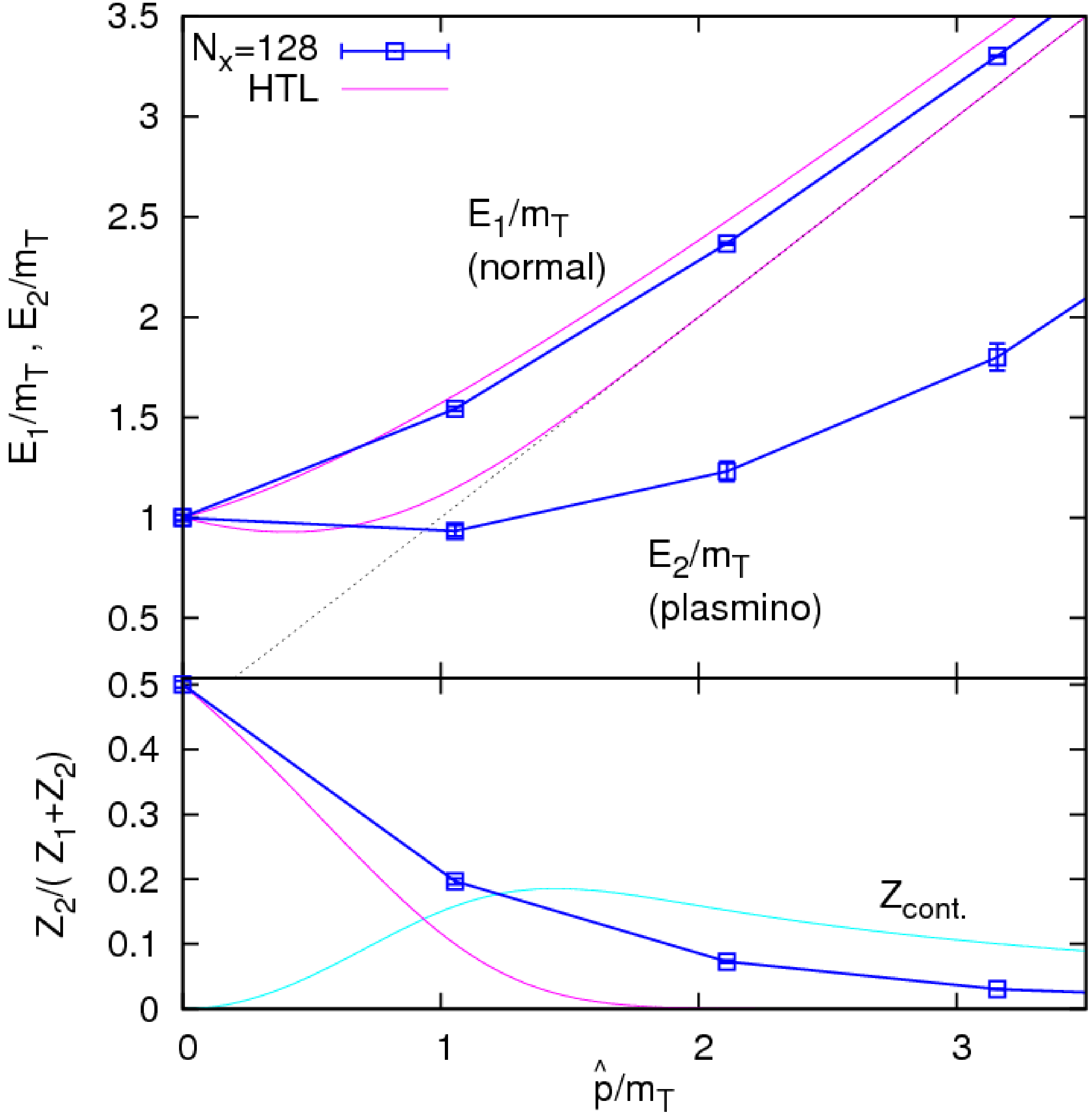


• $E_2 < E_1$; consistent with the HTL result.

• **minimum of plasmino confirmed!**

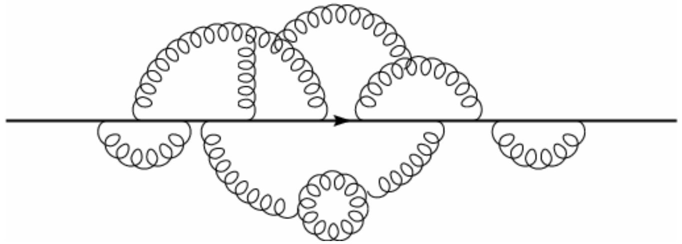
Comparison with HTL Dispersion

$T = 3T_c$

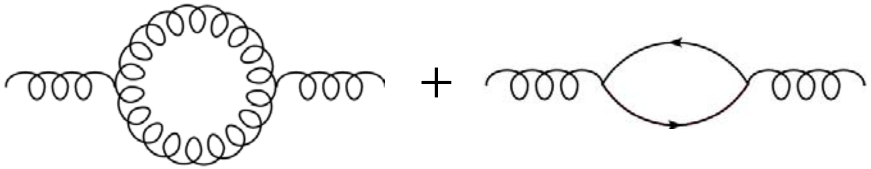


Effect of Dynamical Quarks 1

- Our calculation
  in quenched approx.



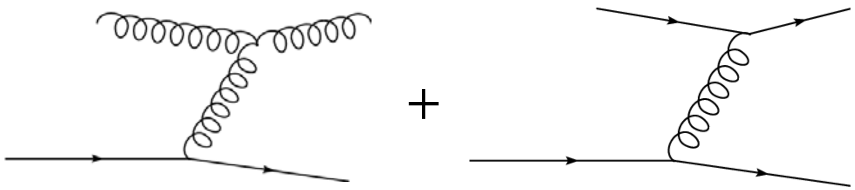
In full QCD, more gluon screening



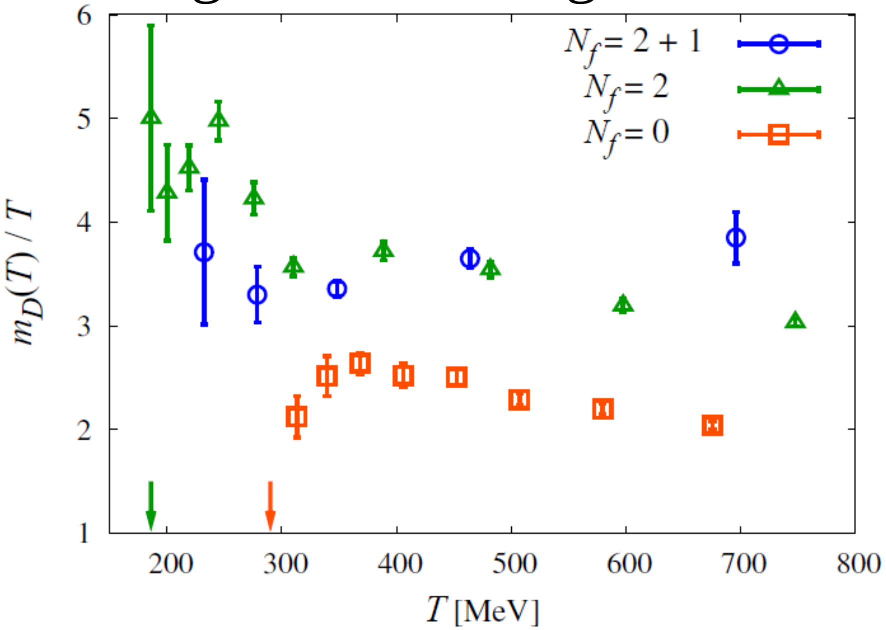
- smaller quark mass
  ⇨ quark-dominated QGP?

- but, larger decay width

Harada, Yoshimoto

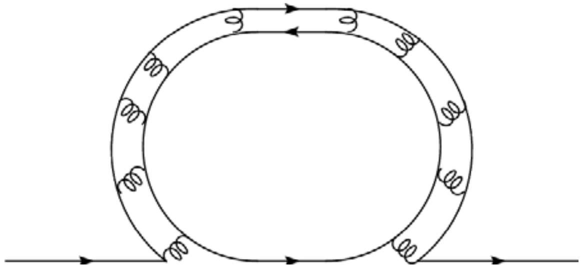


gluon screening mass



Effect of Dynamical Quarks 2

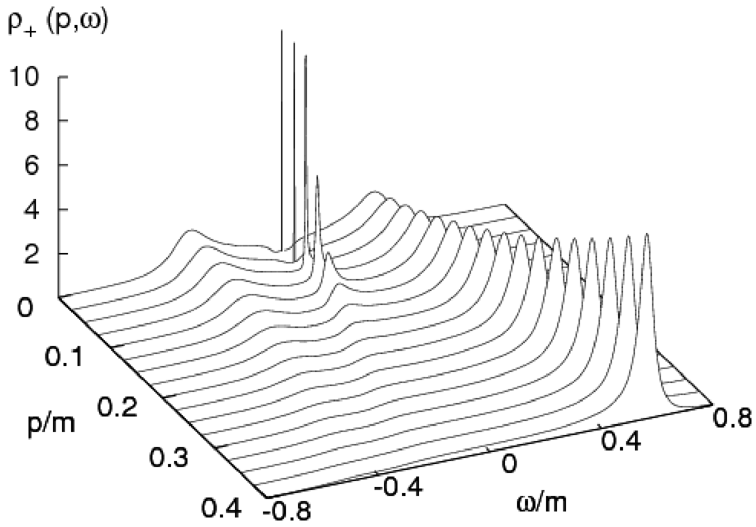
- Another diagram in full QCD:



should be important when mesonic fields form a well-developed collective mode.

coupling between
- a light quark
- a massive boson

3-peak structure?



Summary

- Fermion spectrum at relativistic temperatures has a rich and complex quasi-particle properties.

- Quarks in quenched approximation acquires thermal mass m_T~0.7T even near T_c.
- Minimum of the plasmino dispersion is confirmed.

- Quark thermal mass has a strong spatial-volume dependence. We need much finer and larger lattice to clarify detailed quasi-particles properties of quarks.

- Are “quasi-particles” useful to analyze non-perturbative region of the QGP phase?

Finite Momentum

$$S(\mathbf{p},\tau) = \frac{1}{V}\sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \psi(\mathbf{x},\tau)\bar{\psi}(\mathbf{y},0)\rangle$$

$$\rho(\mathbf{p},\omega) = \rho_0(\mathbf{p},\omega)\gamma^0 - \rho_V(\mathbf{p},\omega)\vec{p}\cdot\vec{\gamma} + \rho_S(\mathbf{p},\omega)$$

p=0

$$\rho(\mathbf{0},\omega) = \rho_0(\omega)\gamma^0 + \rho_S(\omega)$$
$$= \rho_+(\omega)\Lambda_+^0\gamma^0 + \rho_-(\omega)\Lambda_-^0\gamma^0$$

$$\Lambda_\pm^0 = \frac{1\pm\gamma^0}{2}$$

m=0

$$\rho(\mathbf{p},\omega) = \rho_0(\omega)\gamma^0 - \rho_V(\omega)\boldsymbol{\gamma}\cdot\mathbf{p}$$
$$= \rho_+(\omega)\Lambda_+^p\gamma^0 + \rho_-(\omega)\Lambda_-^p\gamma^0$$

$$\Lambda_\pm^p = \frac{1\pm\gamma^0\boldsymbol{\gamma}\cdot\mathbf{p}}{2}$$

- Spectral function in each channel is positive definite!

Below T_c

$C_+(\tau)$ for 48^3x16 lattices

T/T_c= 3, 1.5, 1.25,
0.93, 0.55.

Quark correlator below T_c

- is convex upward.
- indicates a negative norm.
- does not approach the chiral symm. one in the chiral limit.