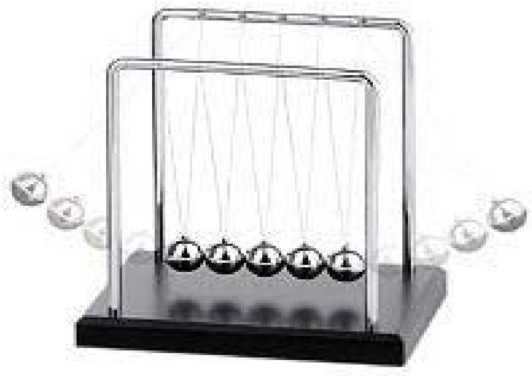
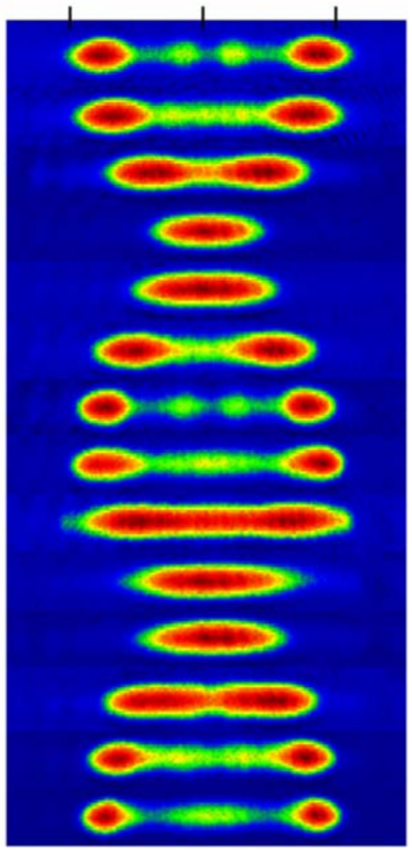
Non-Equilibrium1D Bose Gases
Integrability and Thermalization





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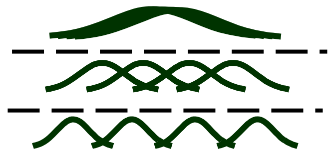
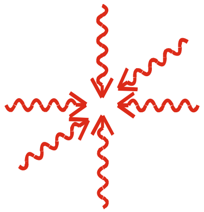
Trevor Wenger
Prof. David S. Weiss

Outline

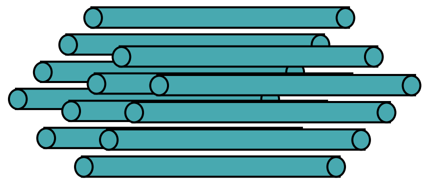


1D Bose gas theory



Equilibrium 1D Bose gas experiments

- Total energy
- 1D Cloud Size
- Local Pair Correlations



I will describe briefly in this talk.

Non-Equilibrium 1D Bose gas experiments

- the Quantum Newton's cradle

1次元ボゾン

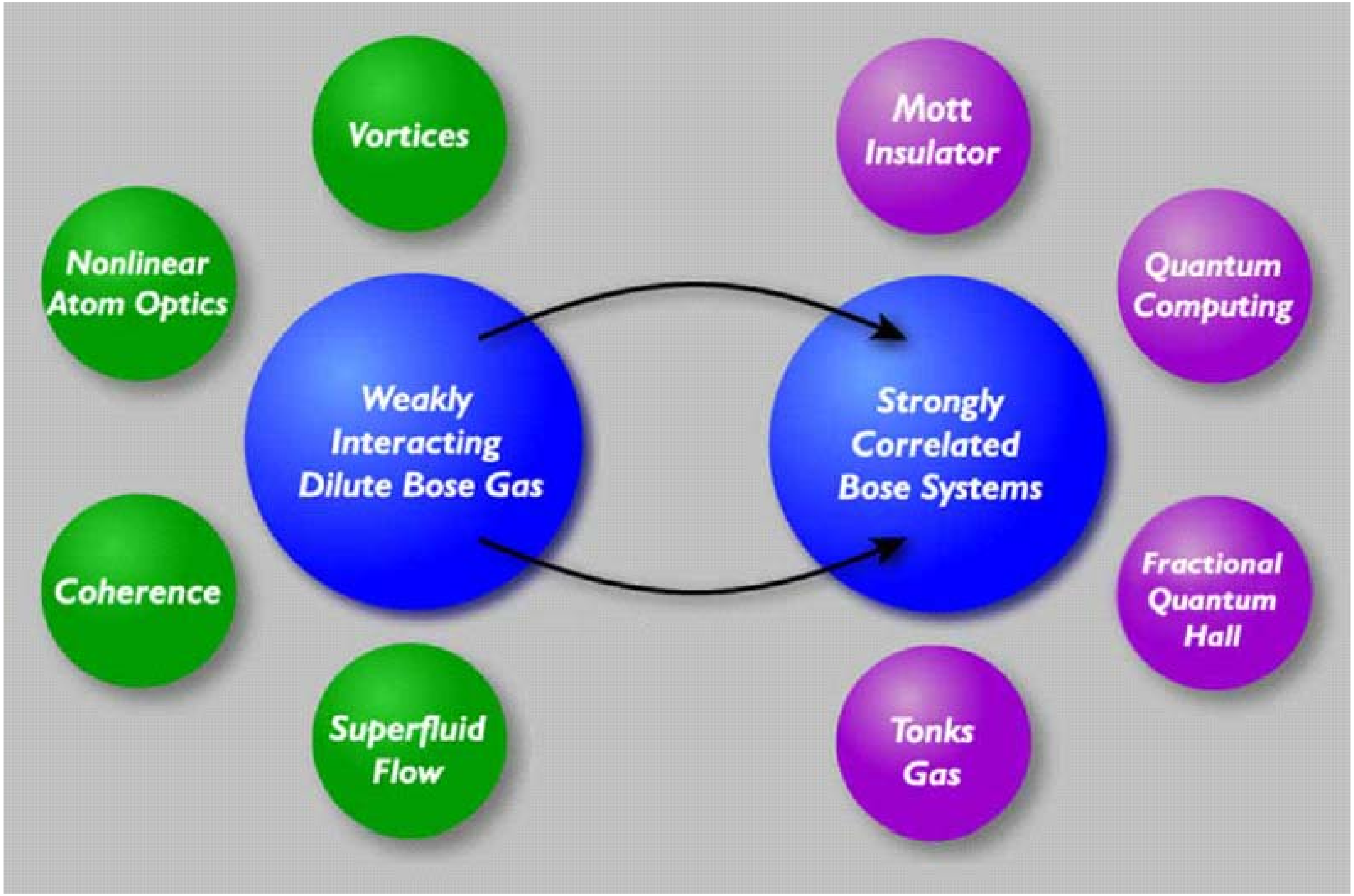
Theory

- Exactly solvable from weakly interacting
  to strongly correlated regimes
- Integrable system

Experiment

- Better understanding of strongly correlated system
  for condensed matter physics,
  for atom entanglement schemes
- Direct comparison to Theory
- Test ground for other (more complicated) correlated systems
  fundamental properties
  method to extract correlation properties
- Process from Non-equilibrium states

Post BEC の1つの流れ

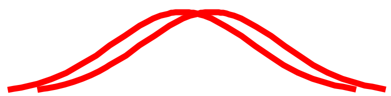


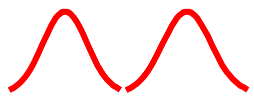
How to make strongly correlated system with a dilute gas.....

$E_{kinetic}$ ~~vs~~ $E_{interaction}$

Weakly Interacting

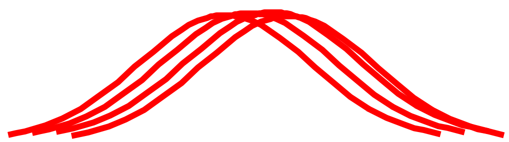
Strongly Interacting

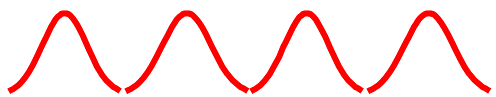




重なりによる相互作用エネルギーの上昇

局在による力学的エネルギーの上昇

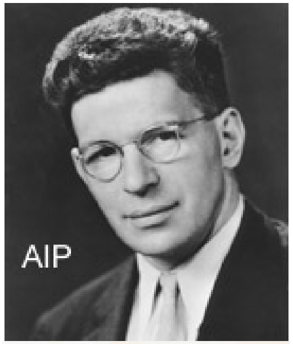




平均場近似

強相関系

1D Bose gases with infinite hard core interactions

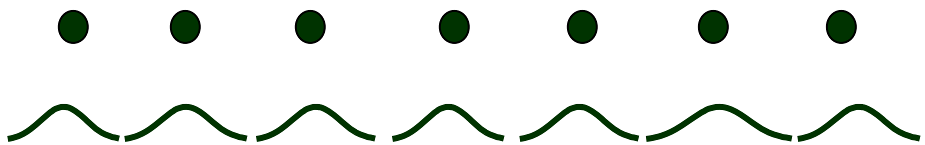


Lewi Tonks, 1936: Eq. of state of a 1D classical gas of hard spheres

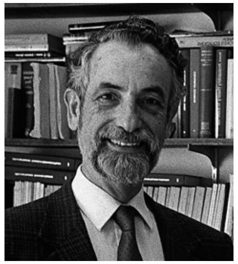
Marvin Girardeau, 1960: 1D Bose gases with infinite hard core repulsion

In 1D, if no two single particle wavefunctions overlap

$$\Rightarrow \quad \psi_{\text{bosons}} = \left|\psi_{\text{fermions}}\right|$$ “Fermionization”



1D Bose gases with variable point-like interactions



Elliot Lieb and Werner Liniger, 1963: Exact solutions for 1D Bose gases with arbitrary $\delta(z)$ interactions

$$H_{1D} = -\sum_{\substack{\text{all}\\\text{atoms}}} \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \sum_{\substack{\text{all}\\\text{pairs}}} g_{1D}\delta(z)$$

$(g_{1D} > 0)$

Solutions parameterized by $\gamma = \dfrac{m}{\hbar^2}\dfrac{g_{1D}}{n_{1D}}$

$\gamma \gg 1$
Tonks-Girardeau gas

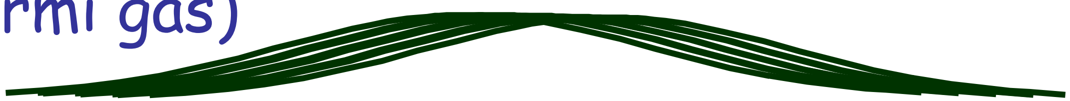
kinetic energy dominates

large g_{1D}
low density

$\gamma \ll 1$
mean field theory (Thomas-Fermi gas)

mean field energy dominates

small g_{1D}
high density

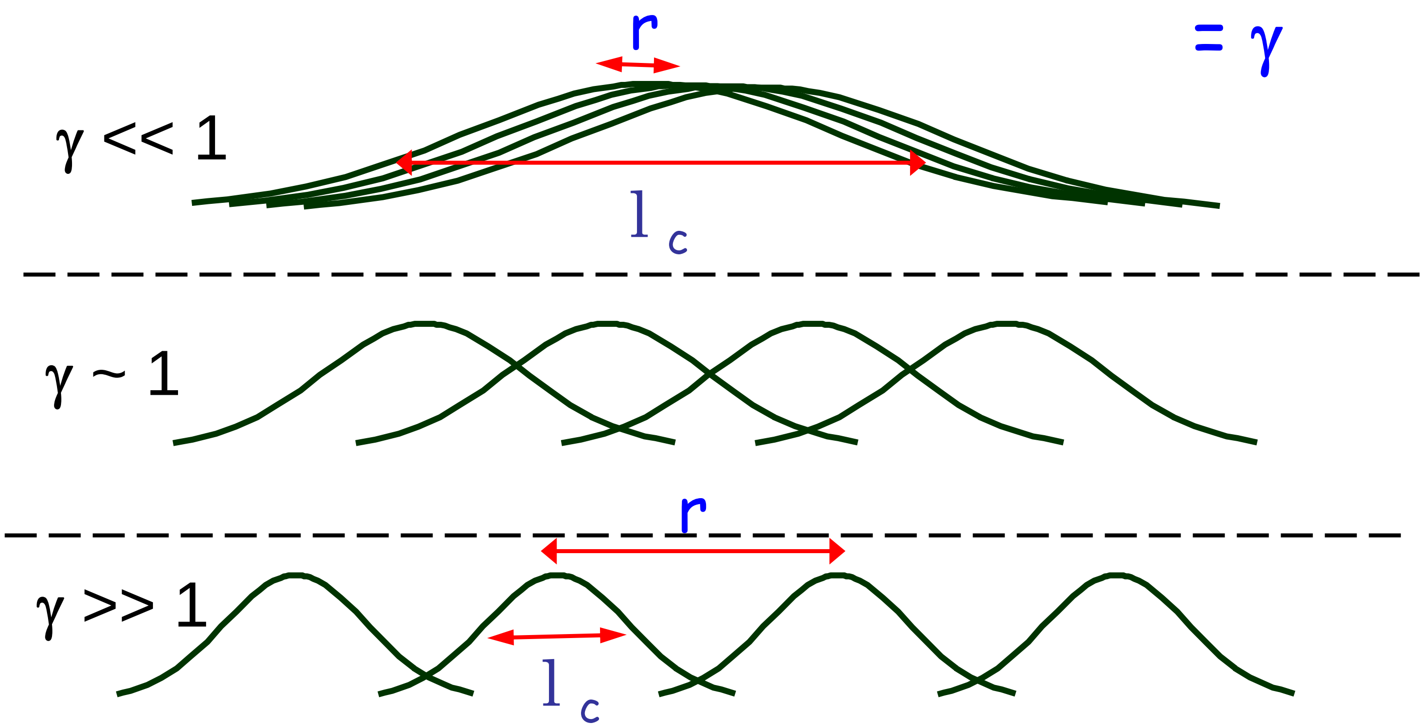


3D Bose Gas Cartoons

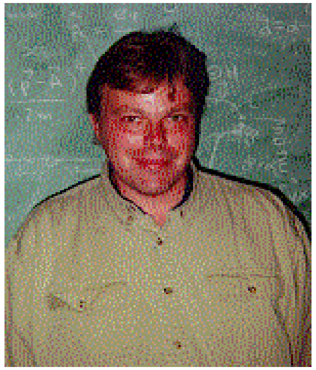
interparticle separation $r = \frac{1}{(n_{3D})^{1/3}}$

correlation length
(λ_{dB} associated with the mean field E_{int}, $\frac{1}{2}\, g_{3D}\, n_{3D}$) $l_c = \frac{\hbar}{\sqrt{\mu\, g_{1D}\, n_{1D}}}$

$$\left(\frac{r}{l_c}\right)^2 = \frac{\mu g_{3D}\, n_{3D}}{\hbar^2 (n_{3D})^{2/3}} = \frac{\mu g_{3D} \sqrt[3]{n_{3D}}}{\hbar^2} = \gamma$$



1D Bose atomic gases



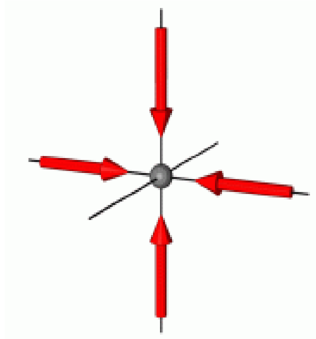
Maxim Olshanii, 1998: Adaptation to real atoms

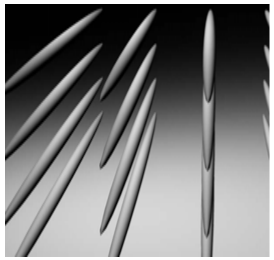
$$\gamma \approx \frac{2 \quad a_{3D}}{a_\perp^2 \quad n_{1D}}$$

1D waveguide

a_{3D} = 3D scattering length

$a_\perp$ = transverse size of wavefunction





$\gamma\uparrow$ when $a_{3D}\uparrow$, $\underline{n_{1D}\downarrow}$ or $\underline{a_\perp\downarrow}$

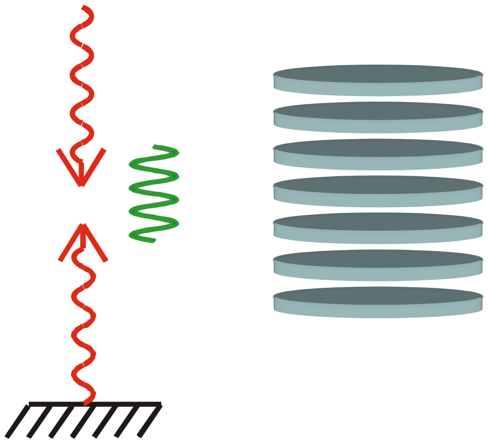
Optical Lattices

Calculable, versatile atom traps

$U_{AC} \propto$ Intensity

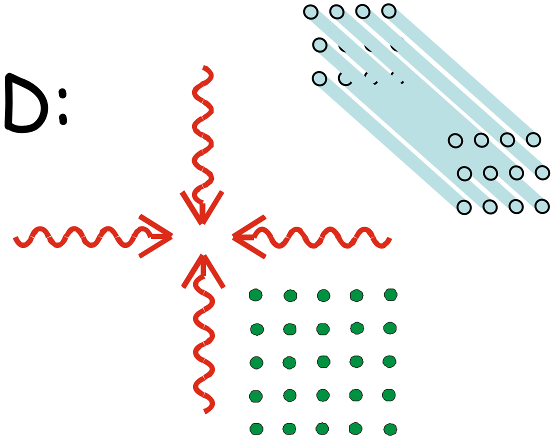
Far from resonance,
no light scattering

1D:

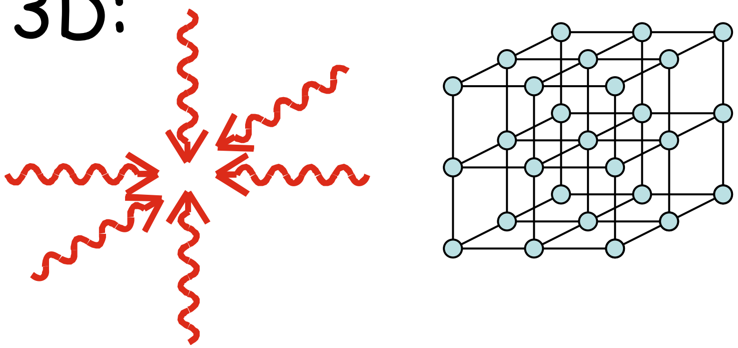


1D Bose gases

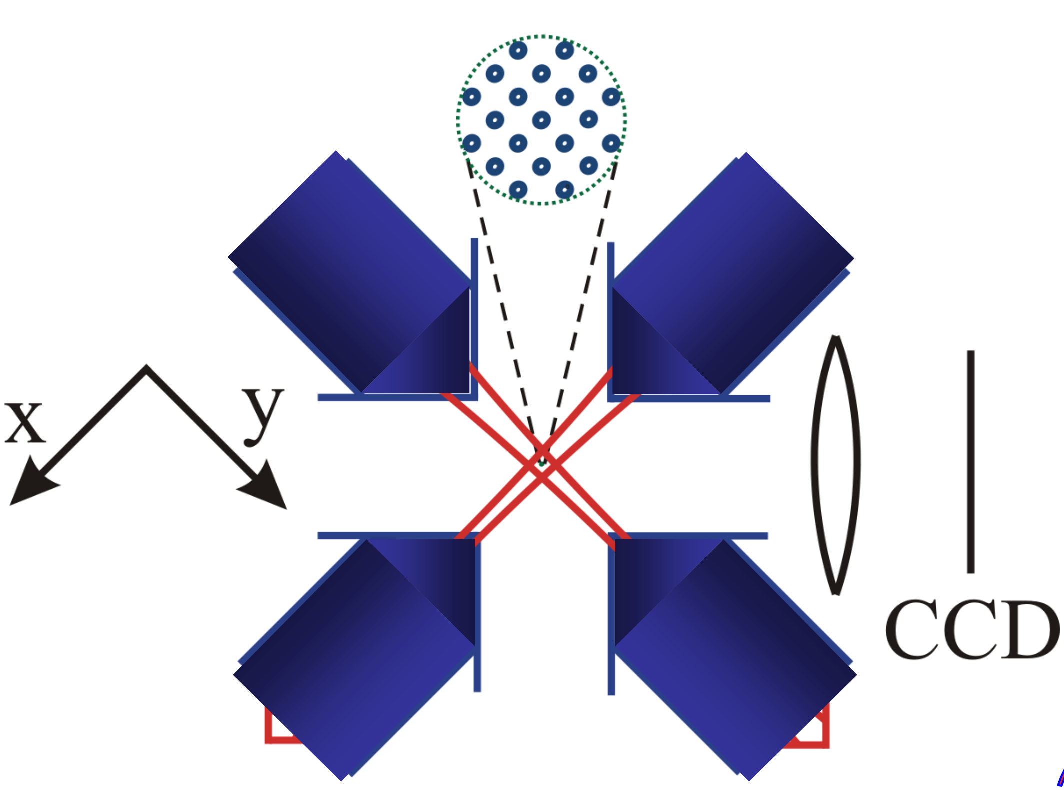
2D:

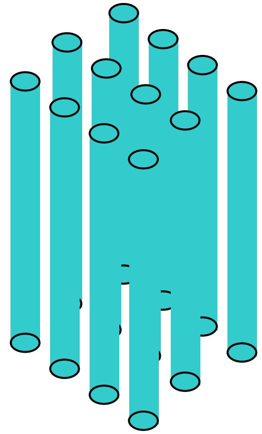


3D:



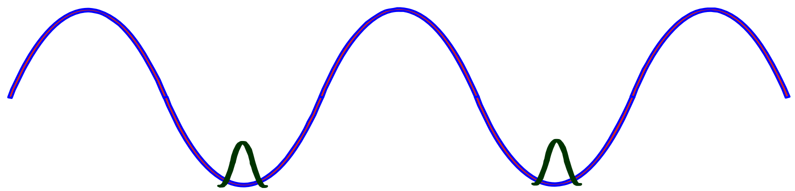
Bundles of 1D Systems





detuning
3.2THz
w_0~ 600 um
up to 85E_{rec}

Blue-detuned Lattice
minimizes
spontaneous emission



For 1D: negligible tunneling;
all energies << $\hbar\omega_\perp$

Recall: $\gamma\uparrow$ when $a_\perp\downarrow$ or $n_{1D}\downarrow$

Independently adjust
longitudinal and transverse
trapping

So $\gamma\uparrow$ when the lattice power $\uparrow$
or the dipole trap power $\downarrow$

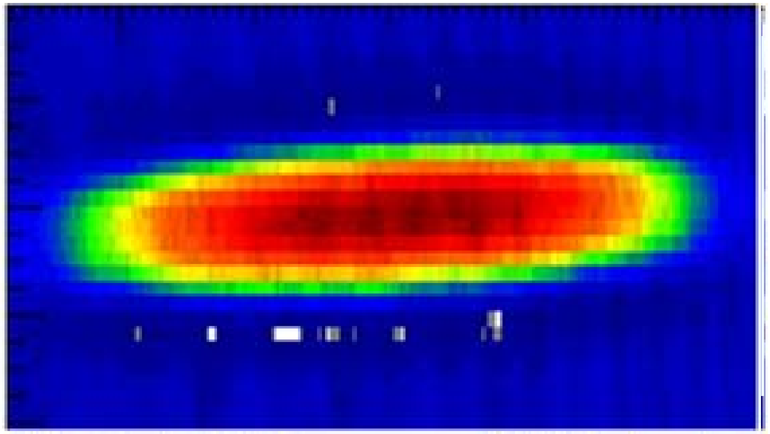
Expansion in the 1D tubes

0 ms

7 ms

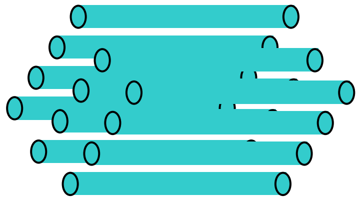
17 ms

← up



50-300
atoms/tube

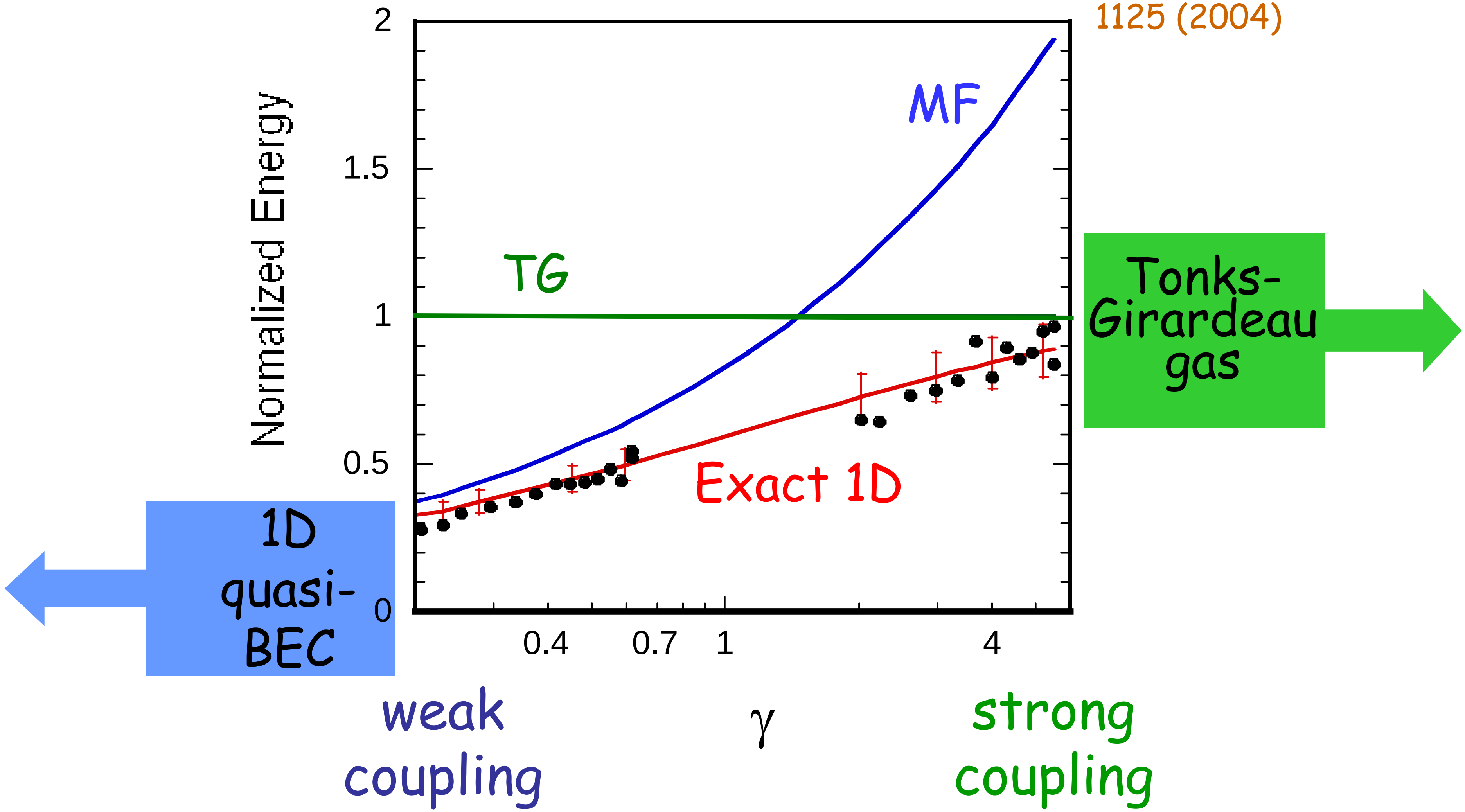
1000-8000
tubes



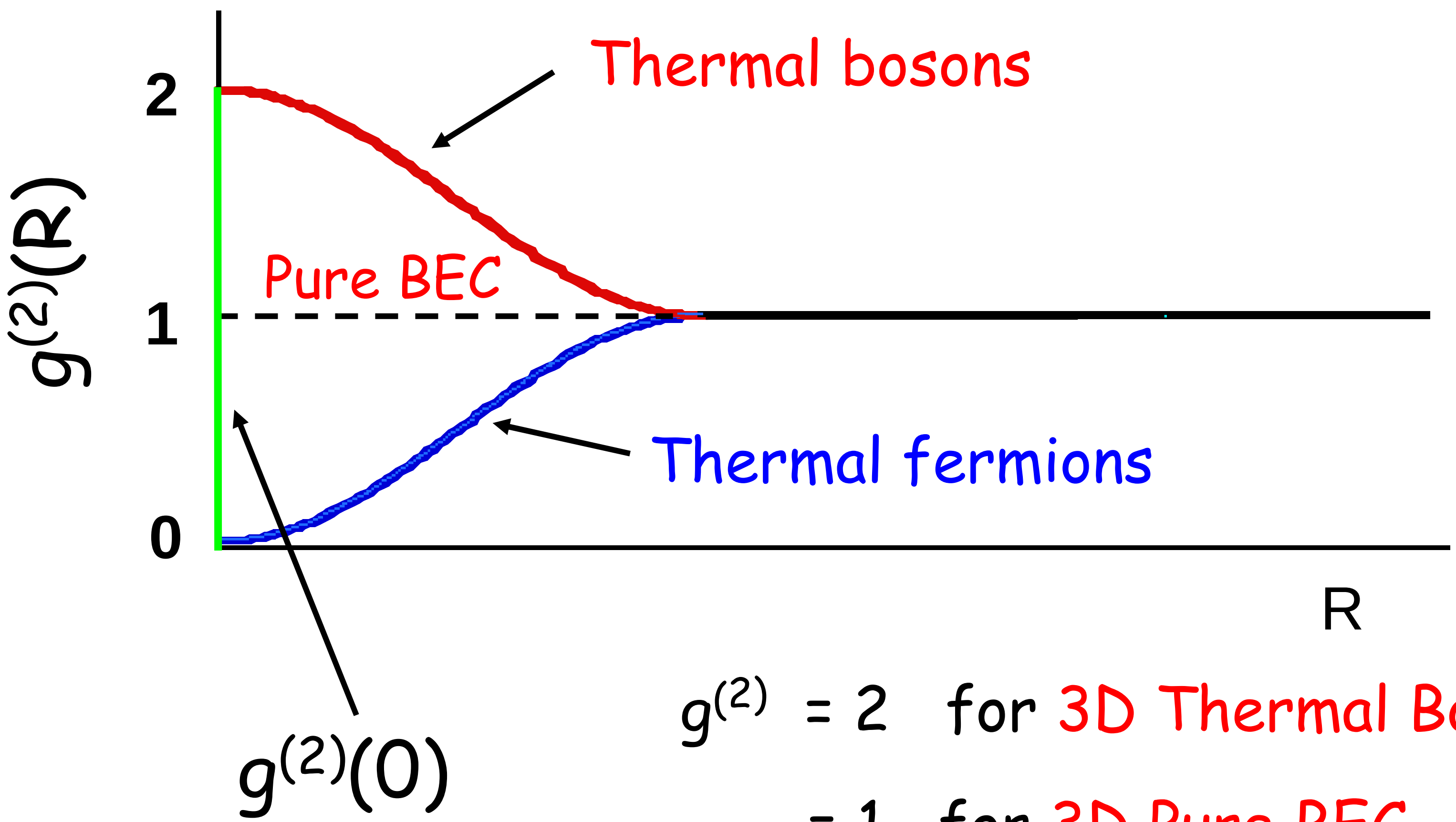
aspect ratio
150 ~ 700

Family of curves parameterized by γ

Kinoshita, Wenger, DSW, *Science* **305**, 1125 (2004)



Pair (Two-Particle) Correlations



Local Pair Correlations

$g^{(2)}$ = 2 for 3D Thermal Bosons

= 1 for 3D Pure BEC

= 0 for 3D Fermions

= ? for 1D Bosons

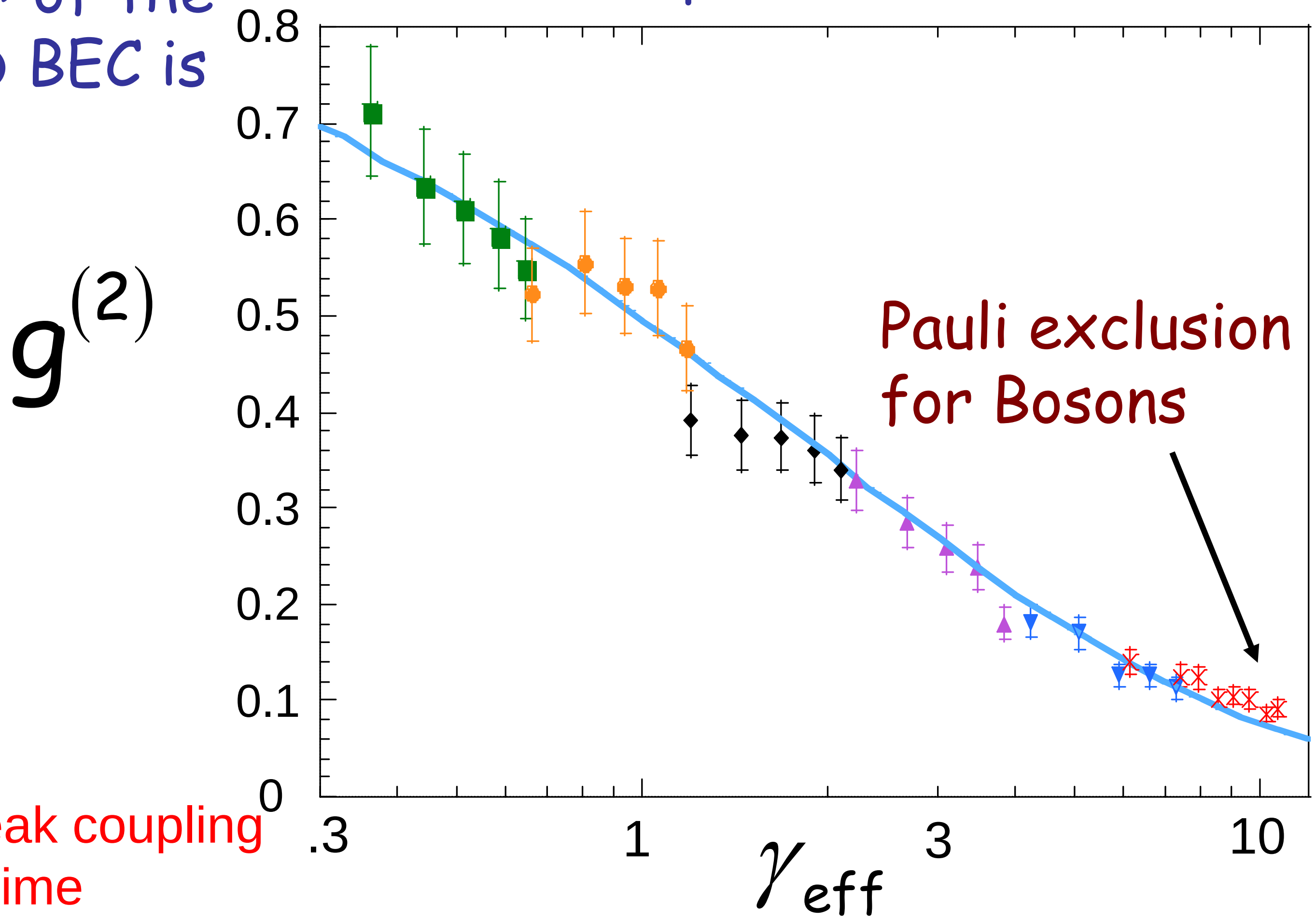
Normalized Local Pair Correlations

By photo-association

Theory: Gangardt & Shlyapnikov, PRL **90** 010401 (2003)

Expt: Kinoshita, Wenger, DSW, PRL **95** 190406 (2005)

$g^{(2)}$ of the 3D BEC is 1.



Strong coupling regime

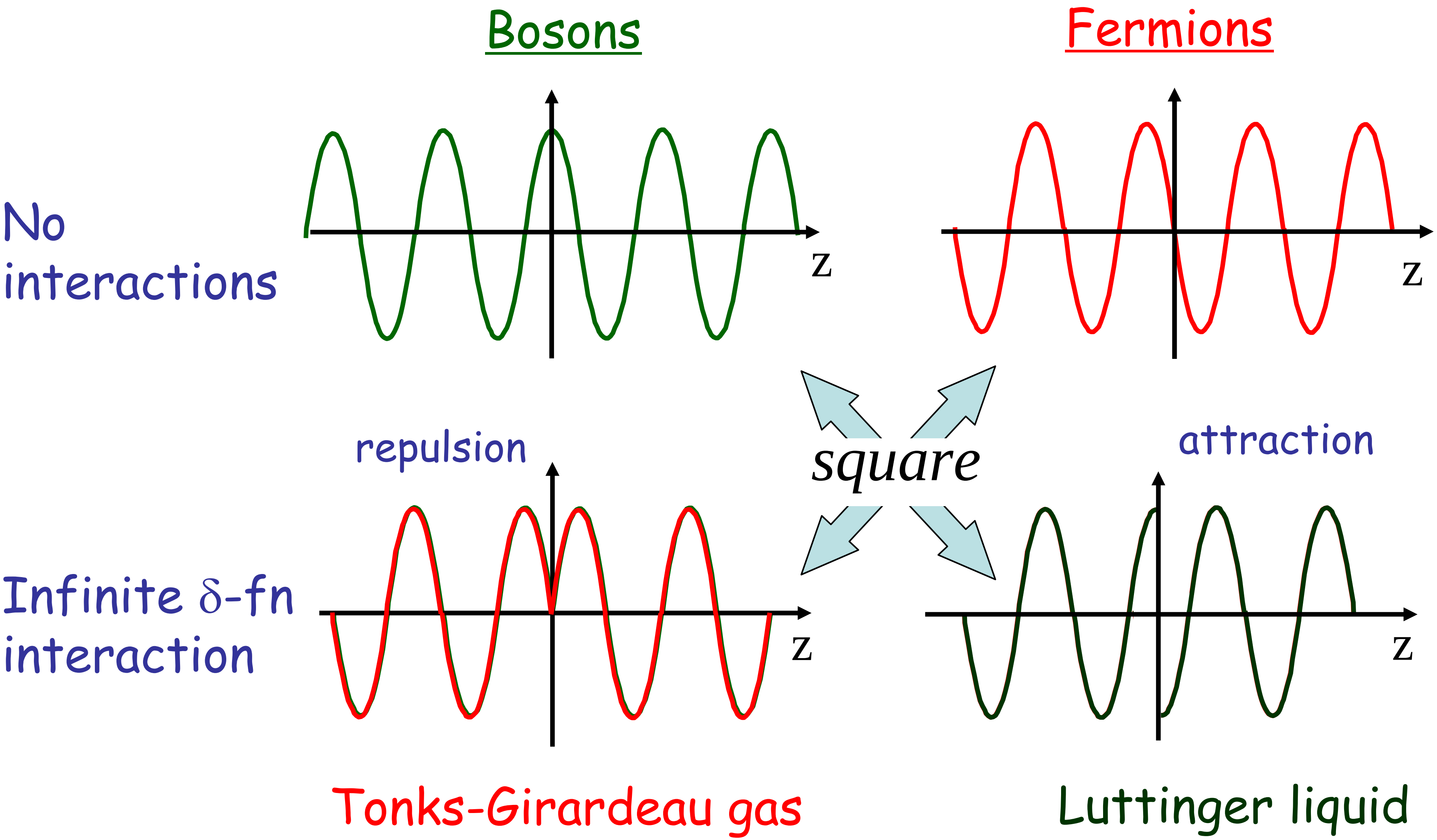
Fermionized Bosons !

$g^{(3)}$, higher order correlation also decreases

Weak coupling regime

Duality in 1D systems

Two-particle relative wavefunctions



Summary (1)

- Experiments with *equilibrium* 1D Bose gases across coupling regimes: total energy; cloud lengths, momentum distributions, local pair correlations

Experiments agree with the exact 1D Bose gas theory, from Thomas-Fermi to Tonks-Girardeau. 1D systems are a test bed for modeling condensed matter using cold atoms.

Other tests of 1D Bose gas theory : NIST(Gaithersburg), Zurich, Mainz

What happens when a 1D Bose gas is put into a *Non-Equilibrium* state ?

Does it thermalize ?

Collisions in 1D

For identical particles, reflection looks just like transmission !

➔ Two-body collisions between distinct bosons cannot change their momentum distribution.

But, the momentum distribution of a freely expanding 1D Bose gas does change, in both the TF and TG limits.

TF gas

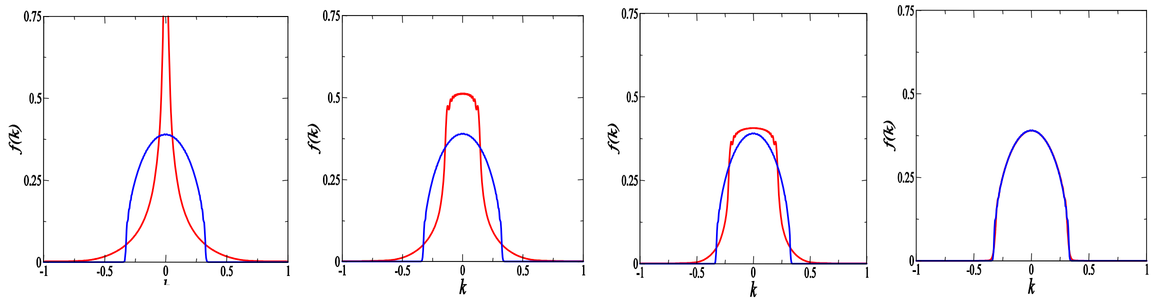
Mean-field energy

⬇

Kinetic energy

TG limit

time →



TG gas

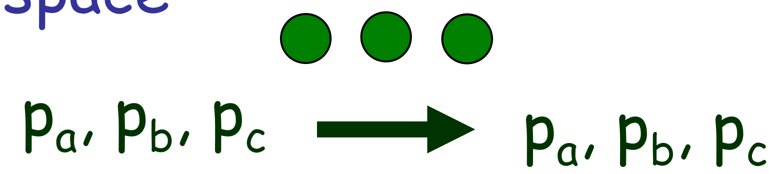
Fermions

Rigol and Muramatsu, PRL **94**, (2005)
Minguzzi and Gangardt, PRL **94**, (2005)

Does a Real 1D Gas Thermalize?

1D Bose gases with δ-fn interactions are integrable systems ➔ they do not:

- ergodically sample phase space
- ≈ become chaotic
- ≈ thermalize



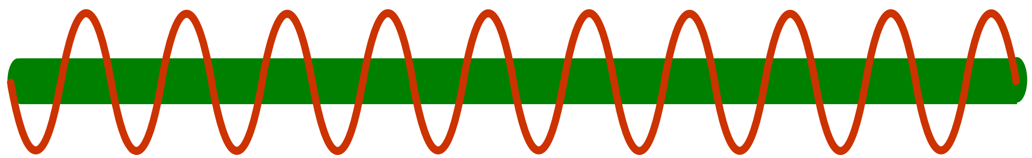
Thermalization in a real 1D Bose gas has been a somewhat open question.

Do imperfectly δ-fn interactions lift integrability enough to allow the atoms to thermalize?

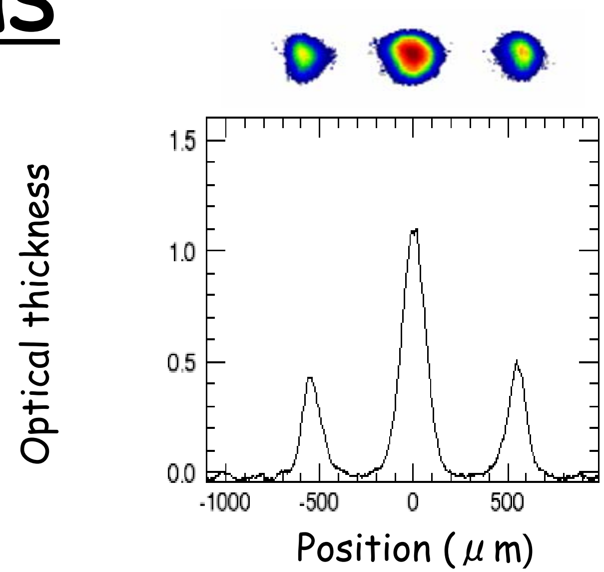
Do longitudinal potentials matter?

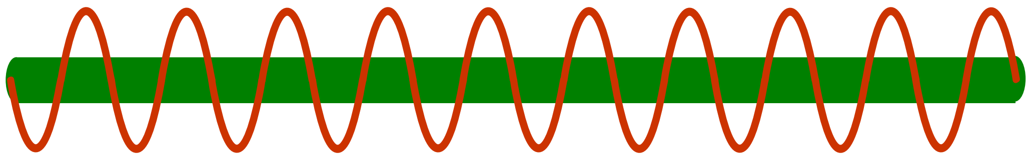
Procedure: take the 1D gas out of equilibrium and see how it evolves.

Creating Non-Equlibrium Distributions

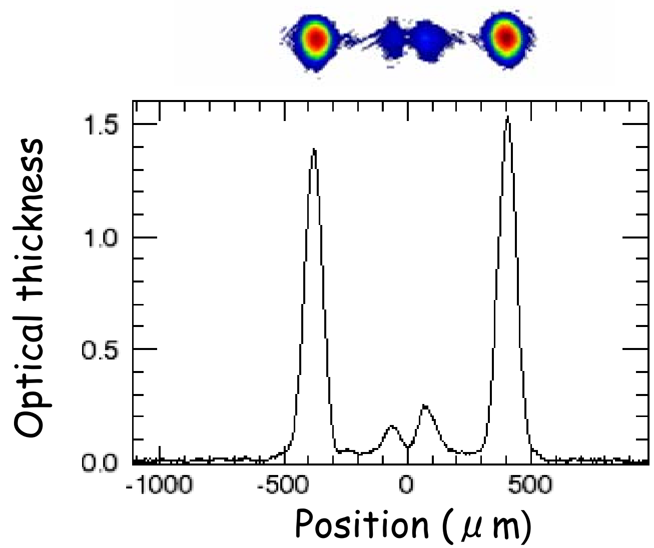


1 standing
wave pulse



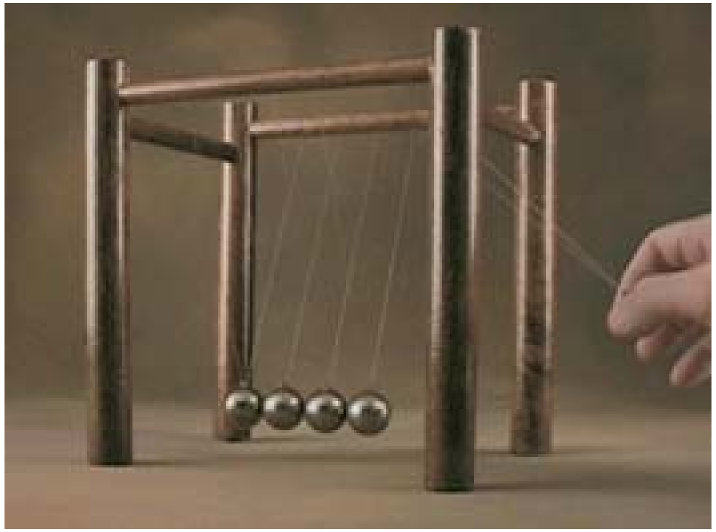


2 standing
wave pulses



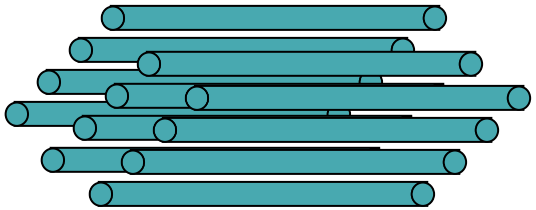
Wang, et al., PRL **94**, 090405 (2005)

Harmonic Trap Motion



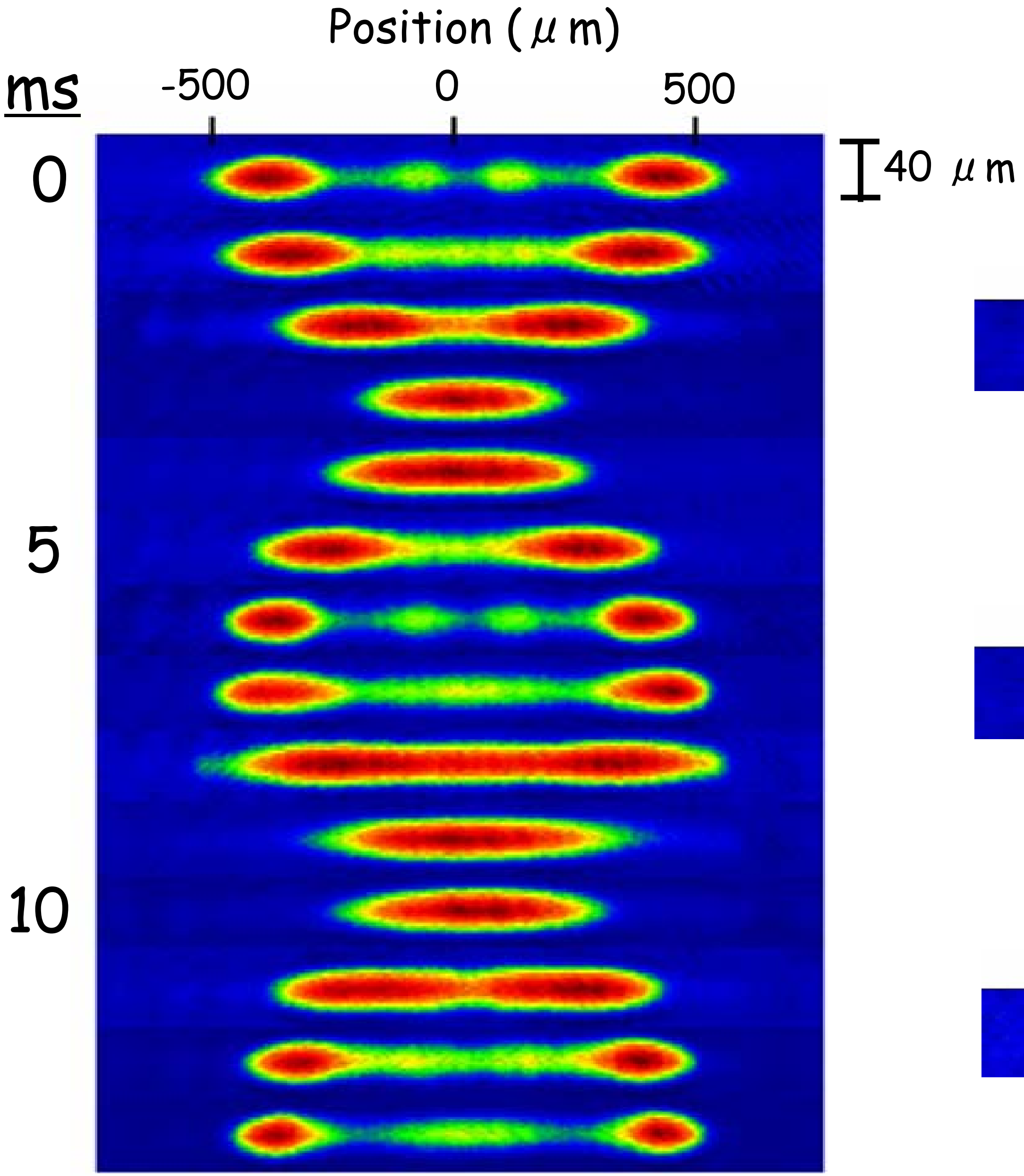
A classical Newton's cradle

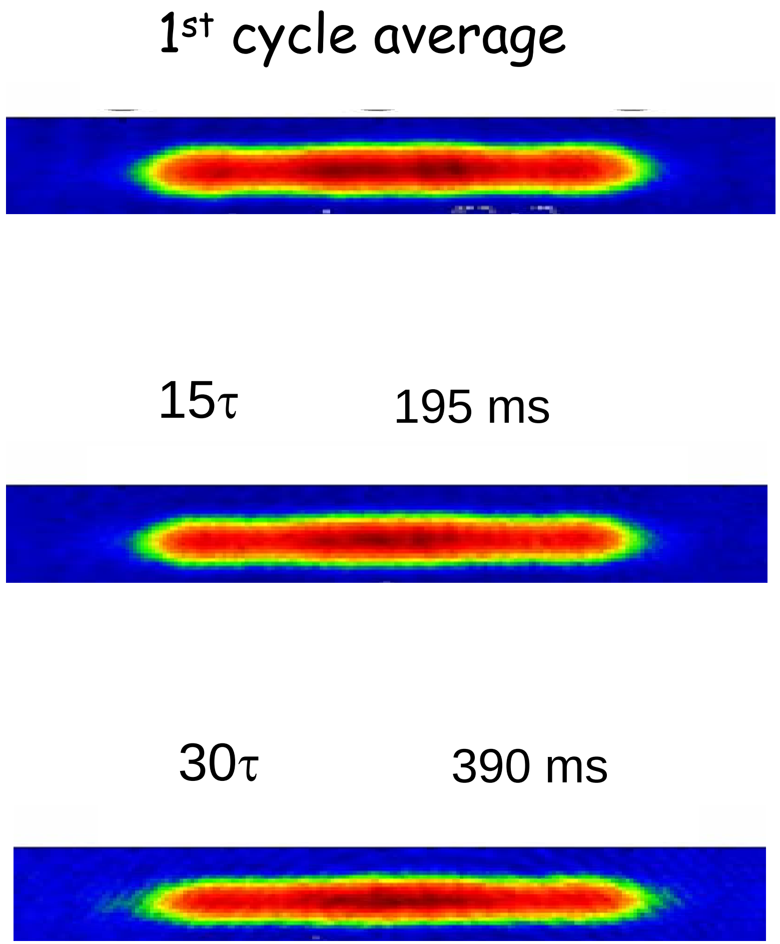
We make thousands of parallel **quantum Newton's cradles**, each with 50-300 oscillating atoms.



1D Evolution in a Harmonic Trap

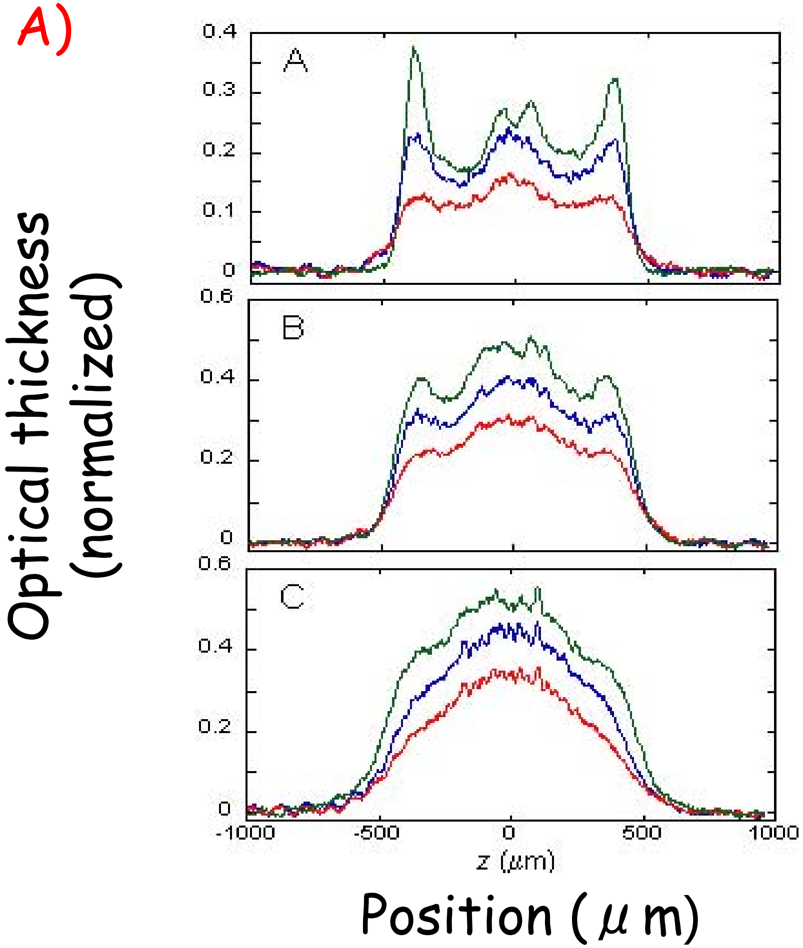
Kinoshita, Wenger, Weiss
Nature **440**, 900 (2006)





Dephased Momentum Distributions

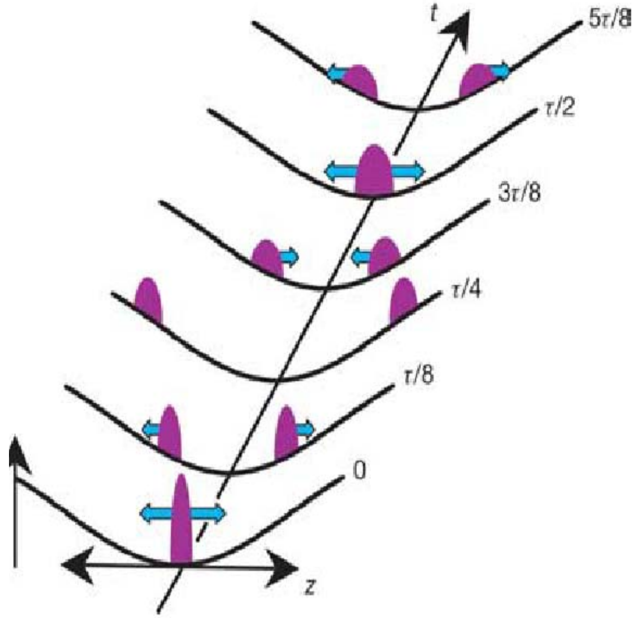
1st cycle average
15τ distribution
40τ distribution
(30τ in A)



γ=18

γ=3.2

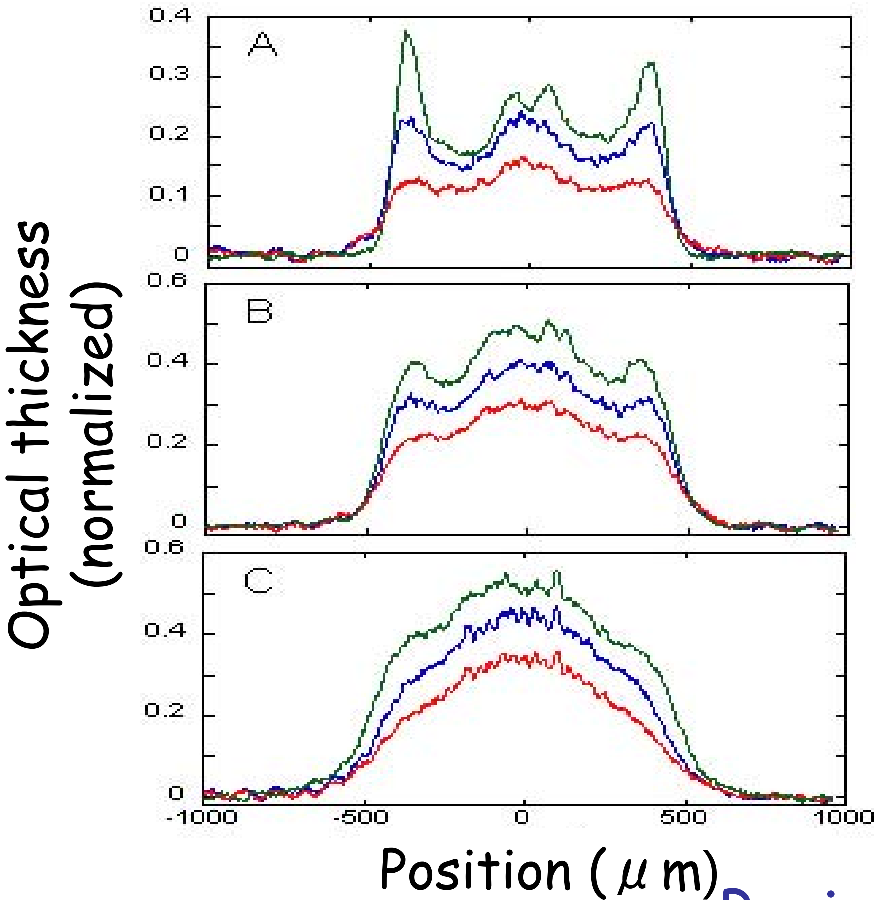
γ= 1.4



Project the evolution

Dephased Momentum Distributions

1st cycle average
15τ distribution
40τ distribution
(30τ in *A*)

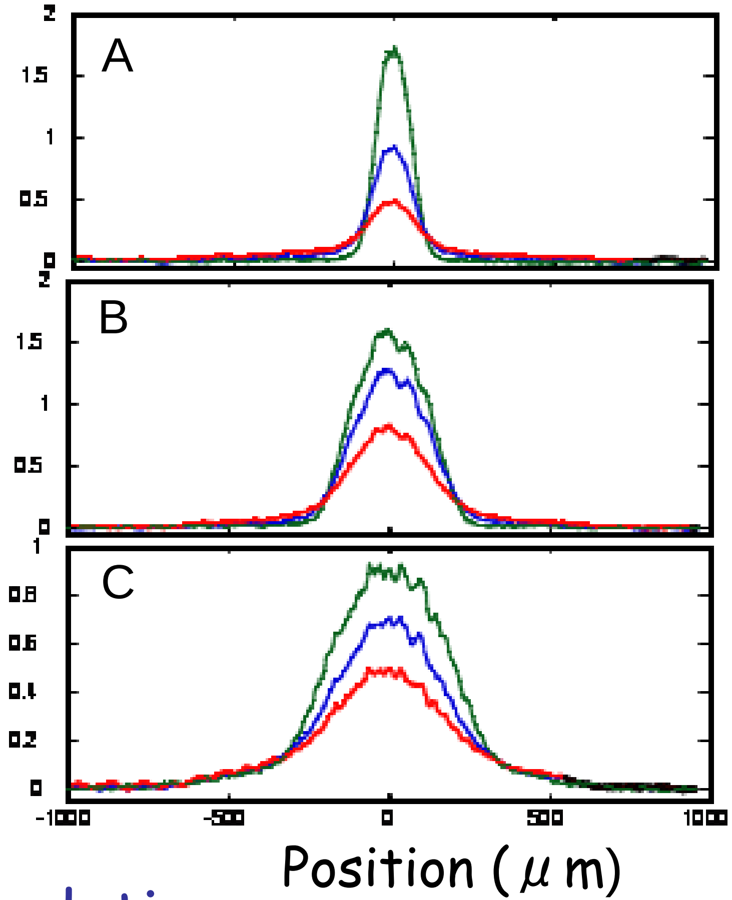


γ=18

γ=3.2

γ= 1.4

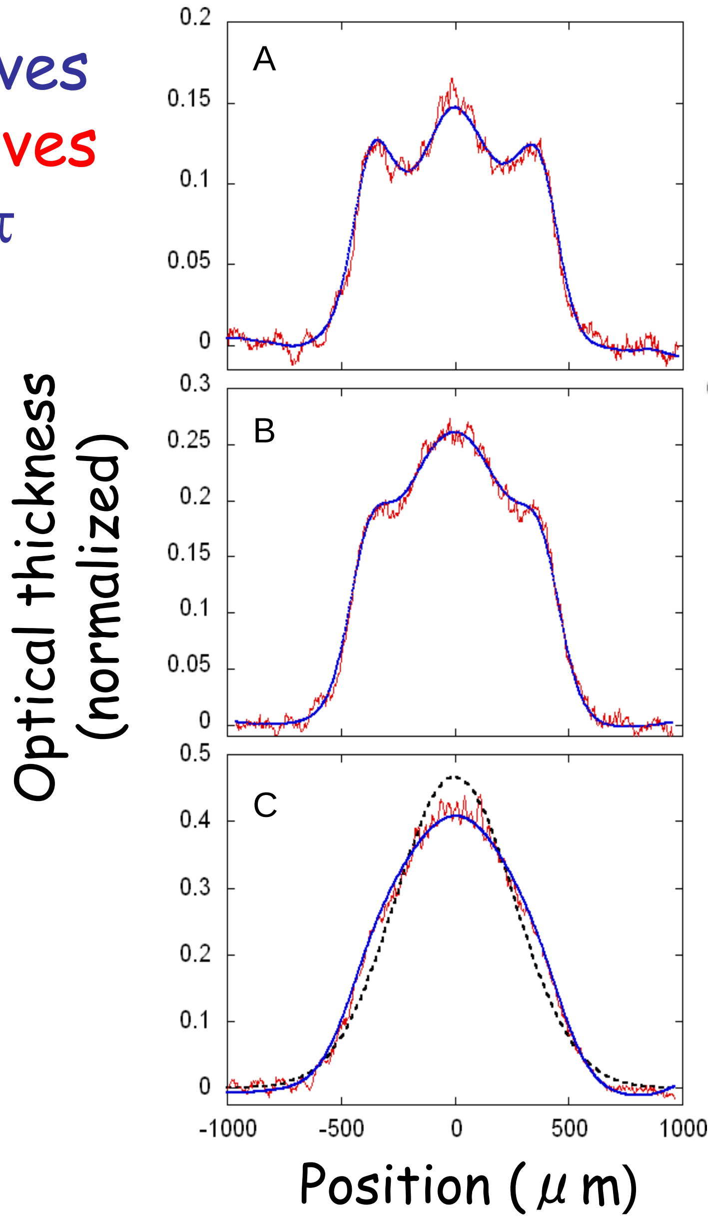
evolution without grating pulses



Project the evolution

Negligible Thermalization

Projected curves and actual curves at 30 τ or 40 τ



γ=18

$\underline{\tau_{th}}$

>390τ

γ=3.2

>1910τ

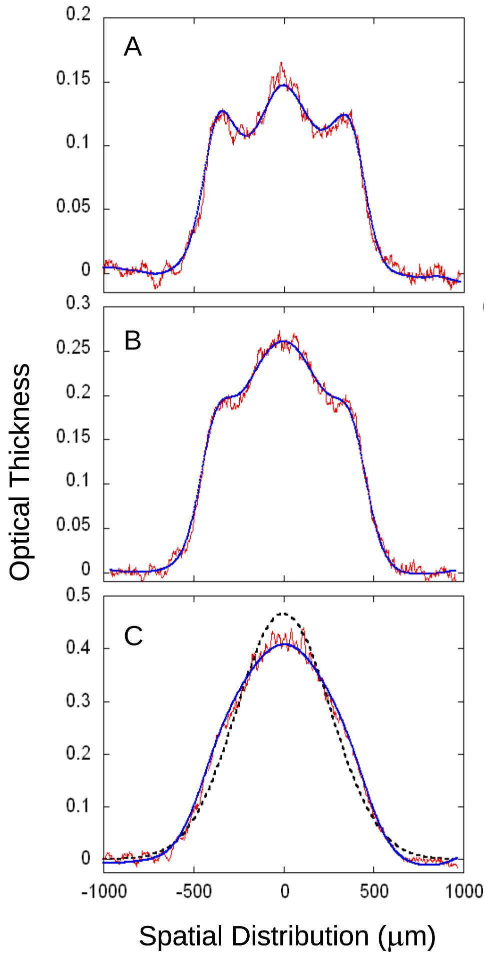
γ= 1.4

>200τ

After dephasing, the 1D gases reach a steady state that is not thermal equilibrium

Each atom continues to oscillate with its original amplitude

Lack of Thermalization



初期に与えられた、平衡から大きく
離れた運動量分布を再分布させる
機構が存在しない。

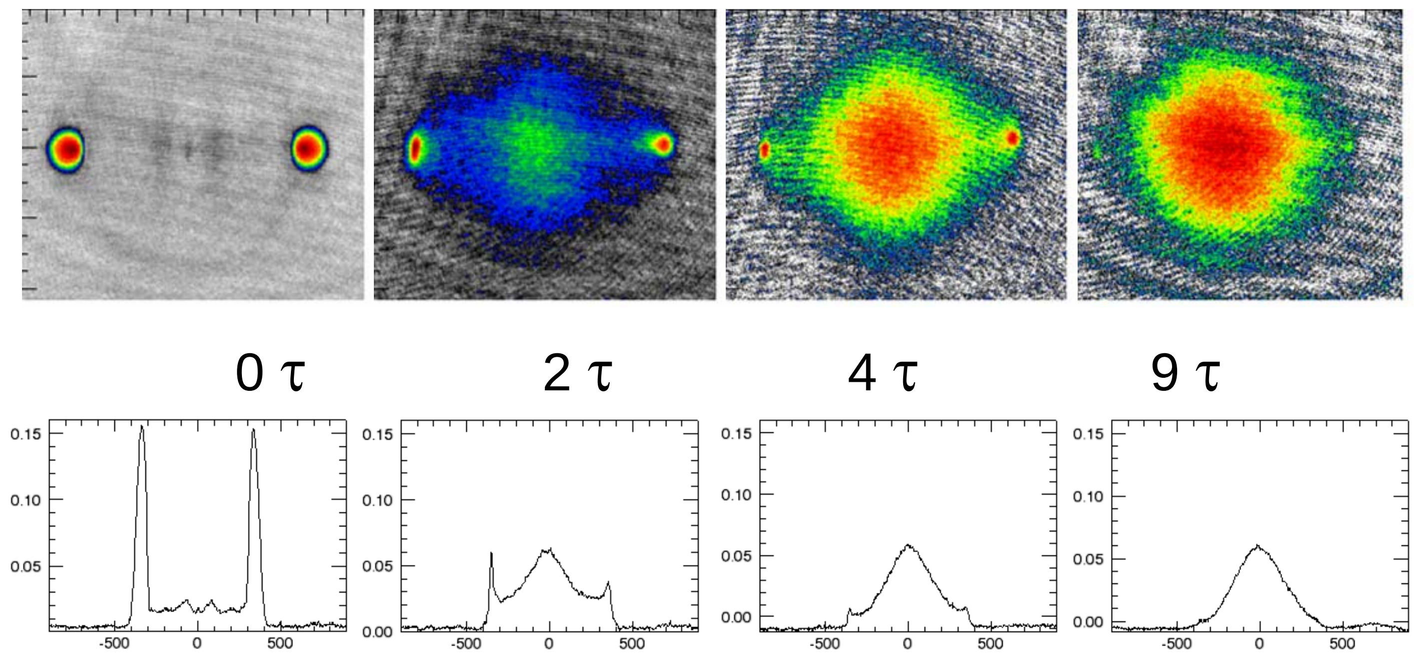
軸方向の弱いトラップポテンシャルは可積分性を崩す
ものの、熱平衡を引き起こすほどには十分でない。

This many-body 1D system
is nearly integrable.

A New Type of Experiment : Direct Control of Non-Integrability

What happens in 3D?

Thermalization occurs in ~3 collisions.



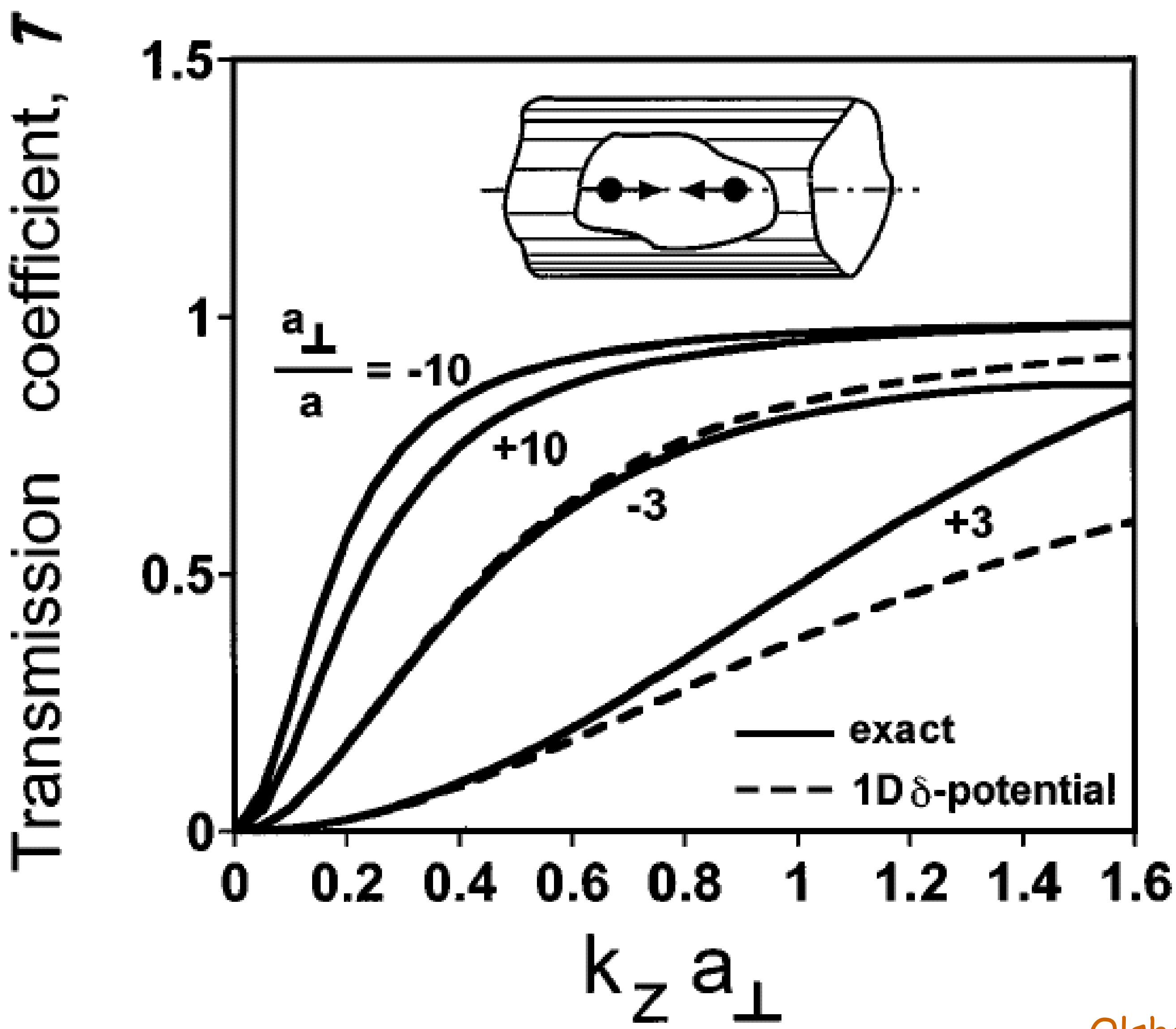
These collisions occur well above the Landau critical velocity for the 3D BEC.

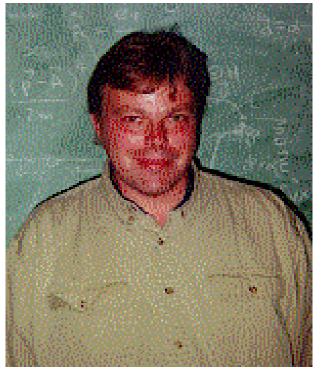
How many collisions have occurred ?

N_{tube} collisions per cycle.

$R \approx (2ka_{1D})^{-2} = \frac{1}{22}$ for $2\hbar k$ collisions

Olshanii, *PRL* **81**, 938 (1998)





Olshanii, *PRL* **81**, 938 (1998)

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N_{tube} collisions per cycle.

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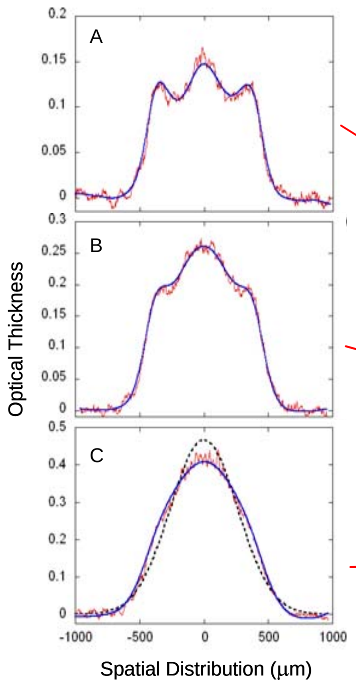
Olshanii, *PRL* **81**, 938 (1998)

We set lower limits to the number of reflections required for thermalization



>710

>9600 >>3

>2300

Lack of thermalization

This many-body 1D system is nearly integrable.

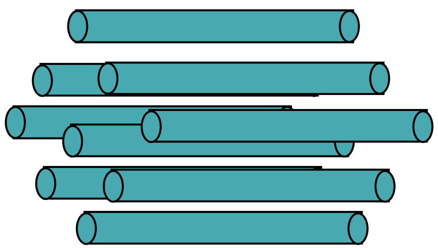
Is there a non-integrability threshold for thermalization?

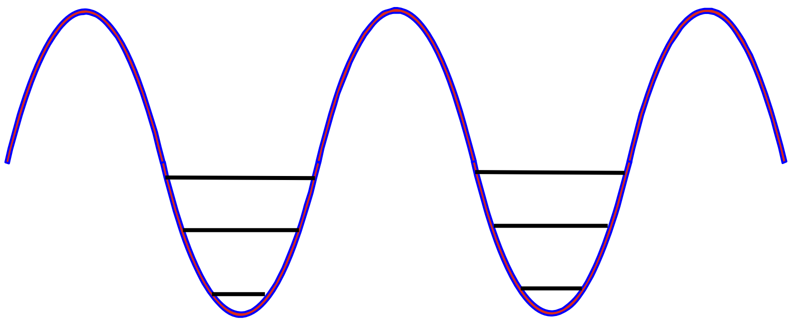
The classical KAM theorem shows that if a non-integrable system is sufficiently close to integrable, it will not ergodically sample phase space.

Is there a quantum mechanical analog?

Procedure:

controllably lift integrability and measure thermalization.

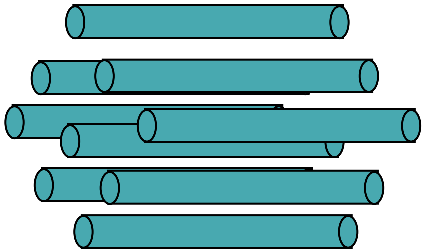


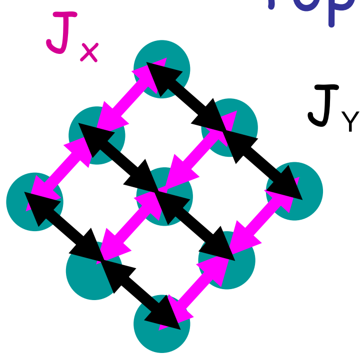


Ways to lift integrability

Allow tunneling among tubes (1D → 2D and 3D behavior); Finite range 1D interactions; Add axial potentials

Making 1D gases thermalize

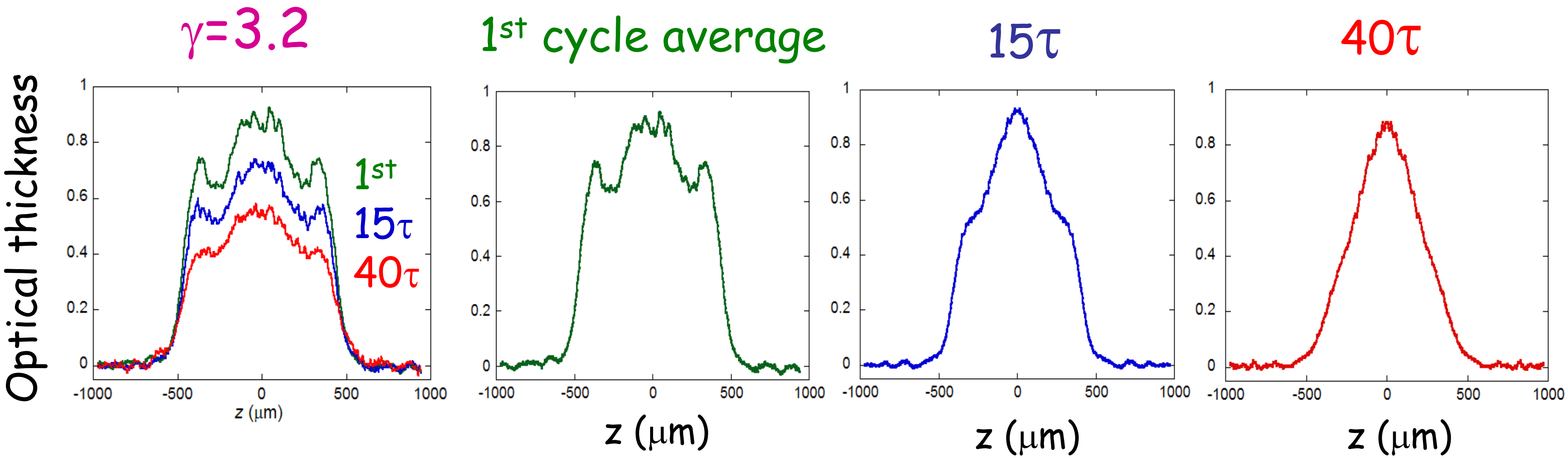




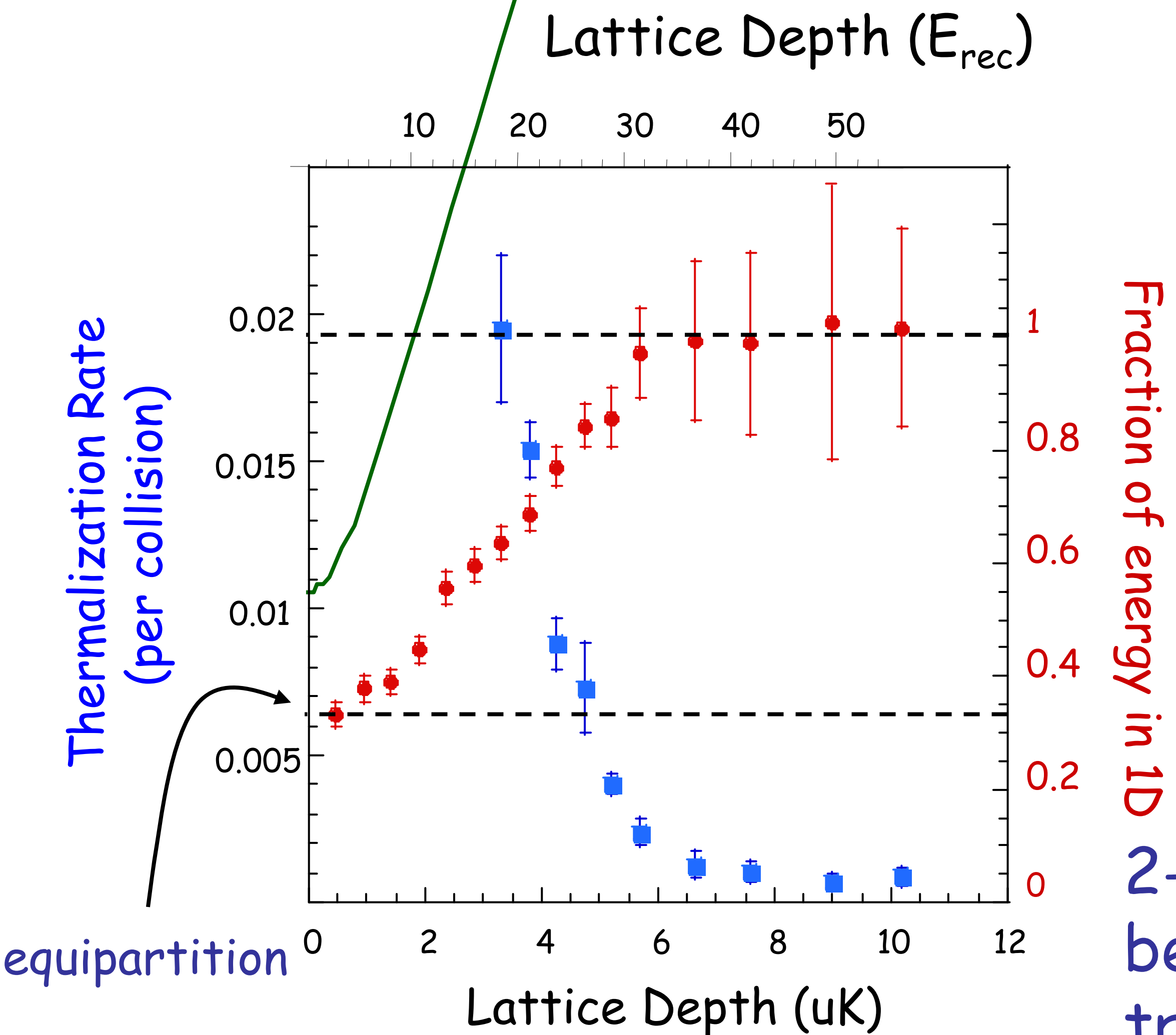
Allow tunneling among tubes ⇒ 1D ➔ 2D and 3D behavior

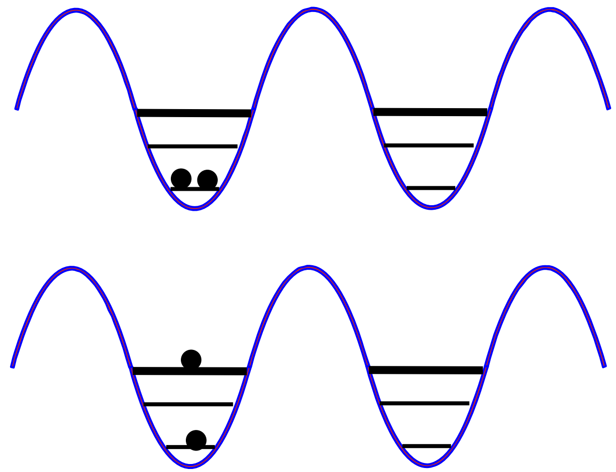
$U_x=U_Y=60\ E_{rec}$

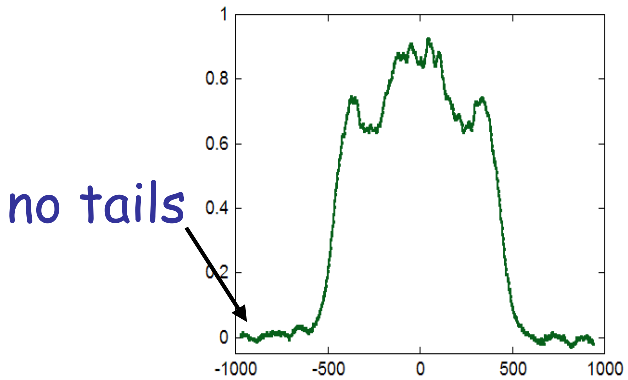
e. g. $U_x = U_Y = 21\ E_{rec}$



Thermalization in a 2D array of tubes

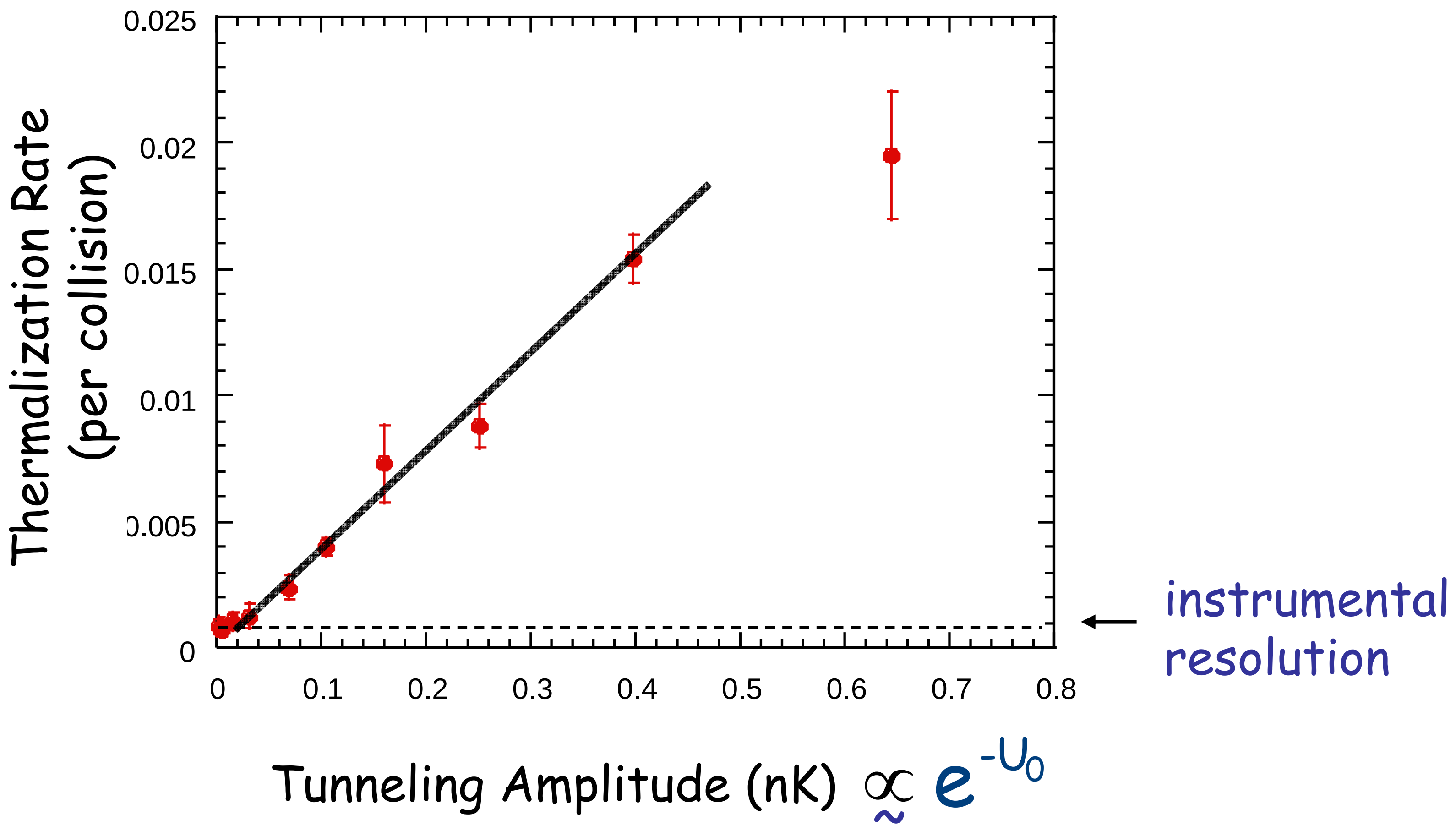






2-body collisions are well below threshold for transverse excitation.

Is there a threshold?



The experiment says "maybe".

Summary (2)

- Non-equilibrium 1D Bose gases:
  the quantum Newton's cradle

  Independent 1D Bose gases do not thermalize !

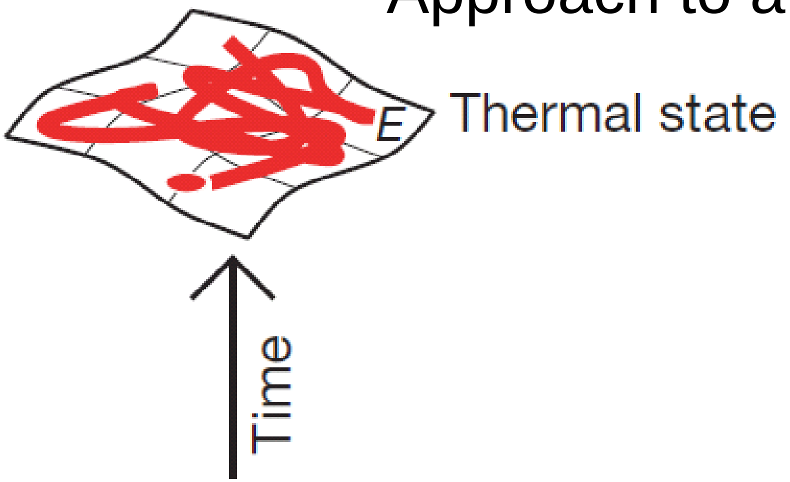
Weakly coupled 1D Bose gases do thermalize !
What is their final state ?

New tools to control non-integrability.

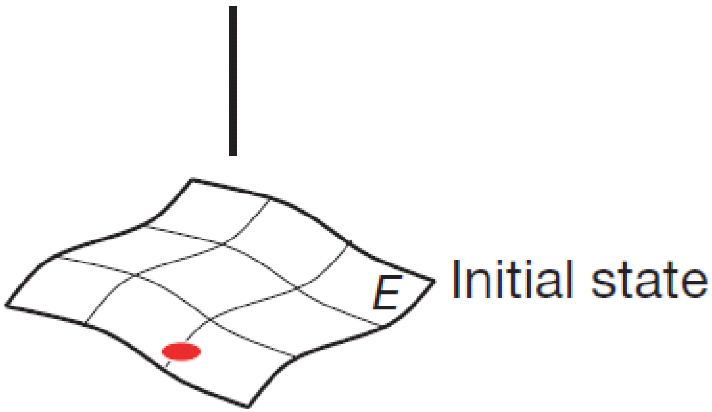
Stories After our Experiments……

Do Integral Systems Relax ?

Approach to a Thermal Equilibrium



It will ergodically sample the entire phase space (E = const.)



Integrals of Motions (conserved quantities) other than

the energy strongly restrict the sampling regions.

Integrable systems never reach a thermal equilibrium
(too many constrains)

However, they may relax to a steady state
(not a thermal equilibrium, but something else)

Maximizing Entropy $S = k_B \mathrm{Tr}[\rho \ln(1/\rho)]$

*Rigol, Dunjko, Yurovsky and Olshanii, PRL, **98**, 050405 (2007)*

Grand Canonical Distribution

→

Thermal equilibrium

$$\hat{\rho} = Z^{-1} \exp\left[-\left(\hat{H} - \mu \hat{N}_b\right)/k_B T\right]$$

$$Z = \mathrm{Tr}\left\{\exp\left[-\left(\hat{H} - \mu \hat{N}_b\right)/k_B T\right]\right\}$$

$$E = \mathrm{Tr}\left\{\hat{H}\hat{\rho}\right\}, \quad N_b = \mathrm{Tr}\left\{\hat{N}_b \hat{\rho}\right\}$$

For Integrable system

Maximize entropy S, subject to the constrains imposed by a full set of conserved quantities.

Generalized Gibbs ensemble with many Lagrange multipliers.

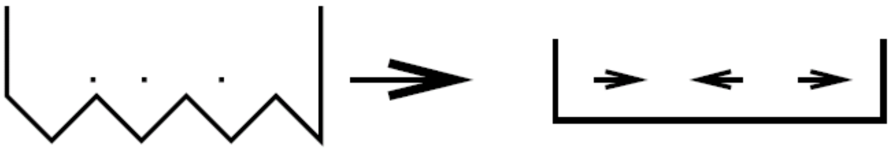
Constrained equilibrium

$$\hat{\rho}_c = Z_c^{-1} \exp\left[-\sum_m \lambda_m \hat{I}_m\right]$$

$$Z_c = \mathrm{Tr}\left\{\exp\left[-\sum_m \lambda_m \hat{I}_m\right]\right\}$$

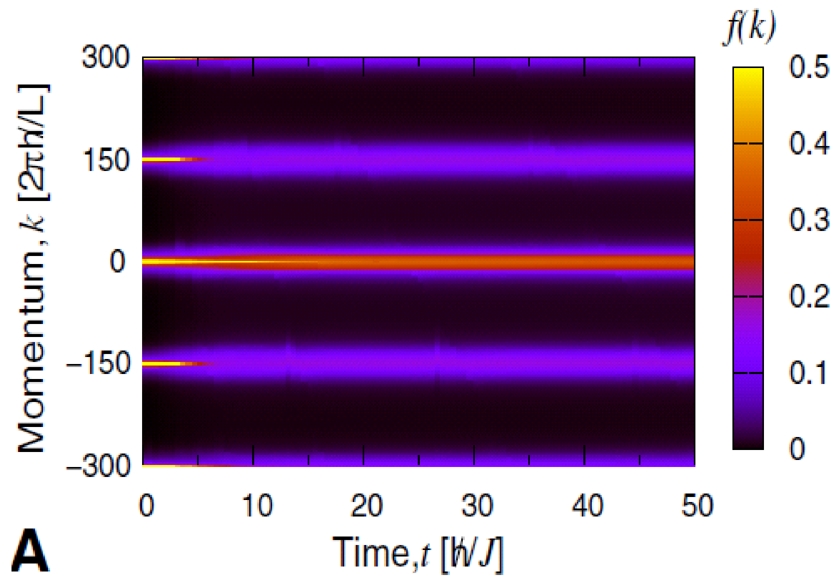
$$\langle \hat{I}_m \rangle (t=0) = \mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_c\right\}$$

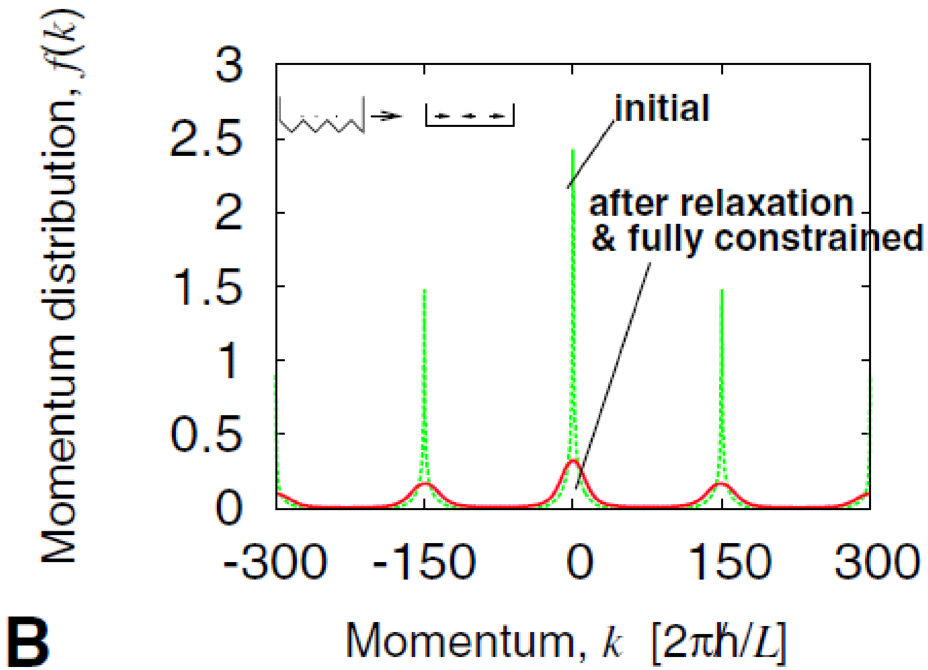
In 1D system,



Discrete Momentum Sets are created by Periodic Potentials.

Remove Potentials (Integrable system) Follow Time Evolution





Relax to a steady state, but not a thermal equilibrium.

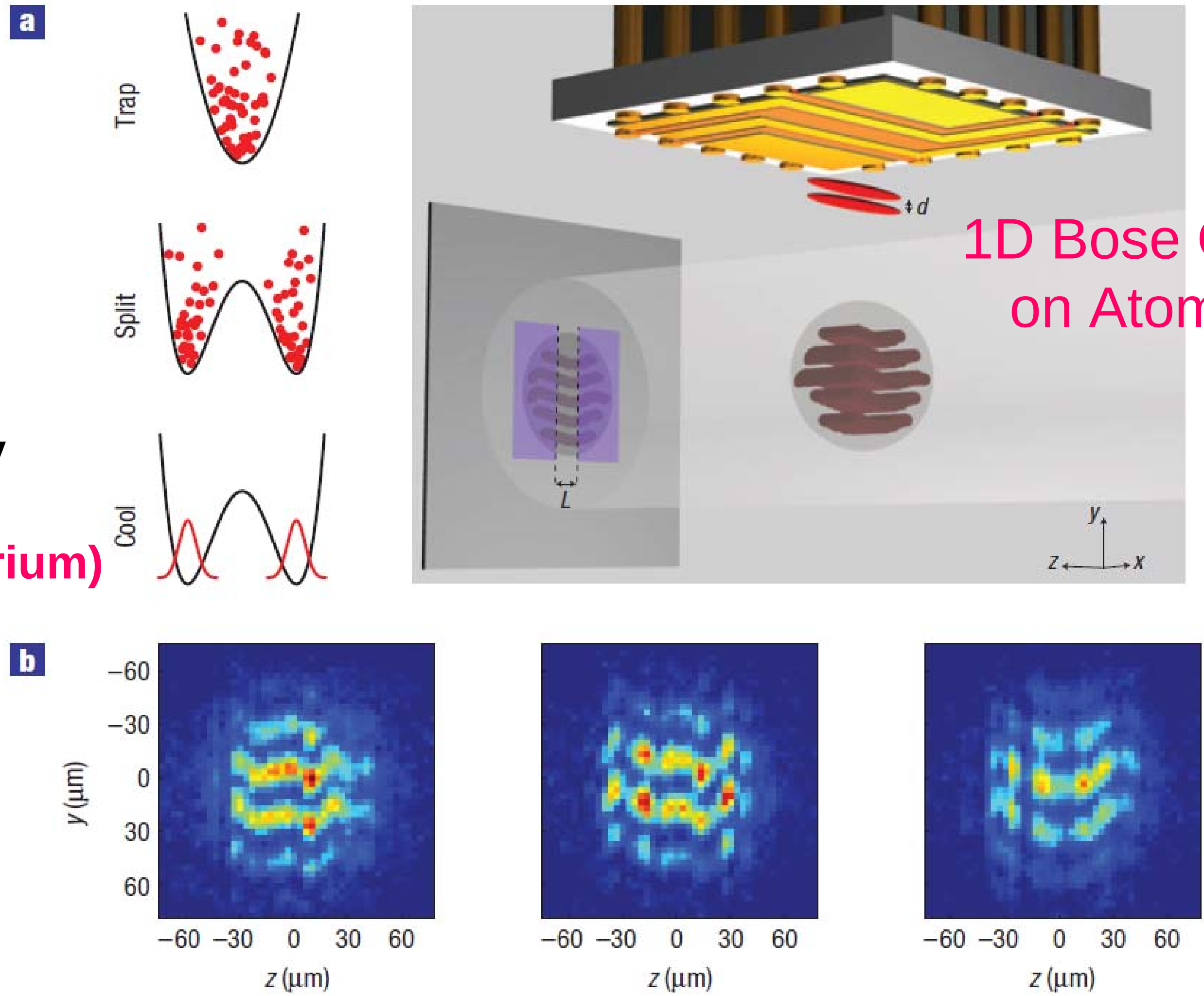
“Memory” of initial states is left.

*Rigol, Dunjko, Yurovsky and Olshanii, PRL, **98**, 050405 (2007)*

Non-Equilibrium Coherence Dynamics in One-Dimensional Bose Gases

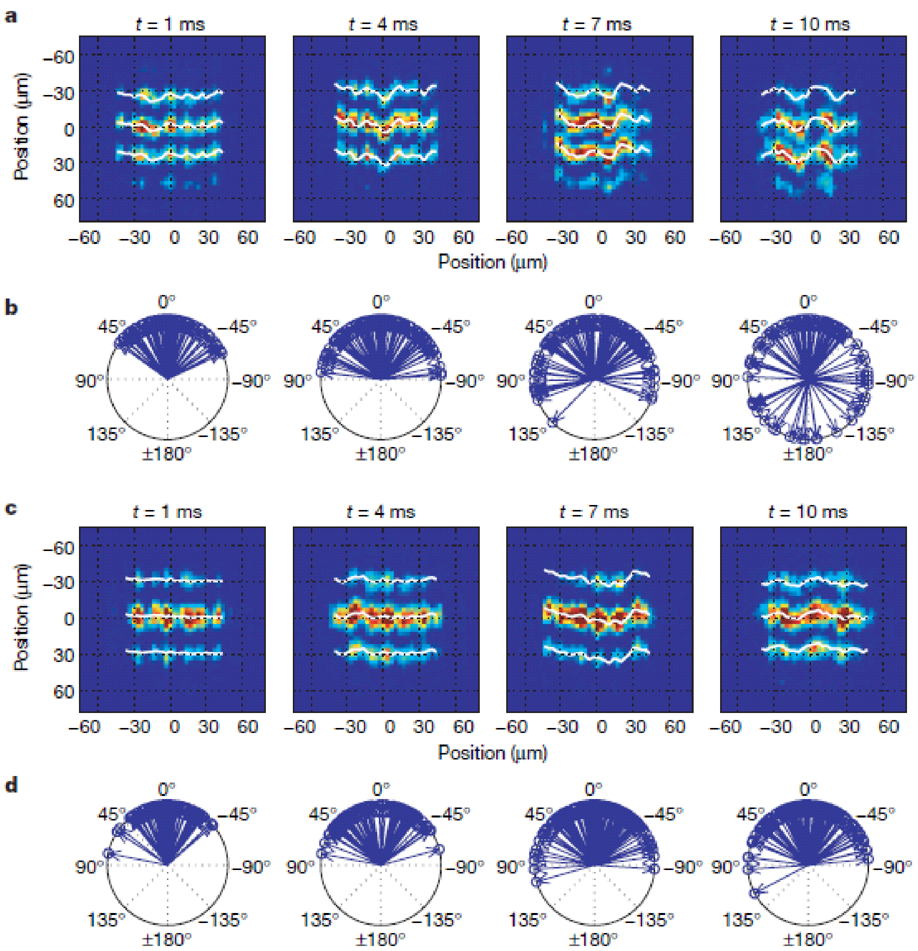
Coherently Splitting

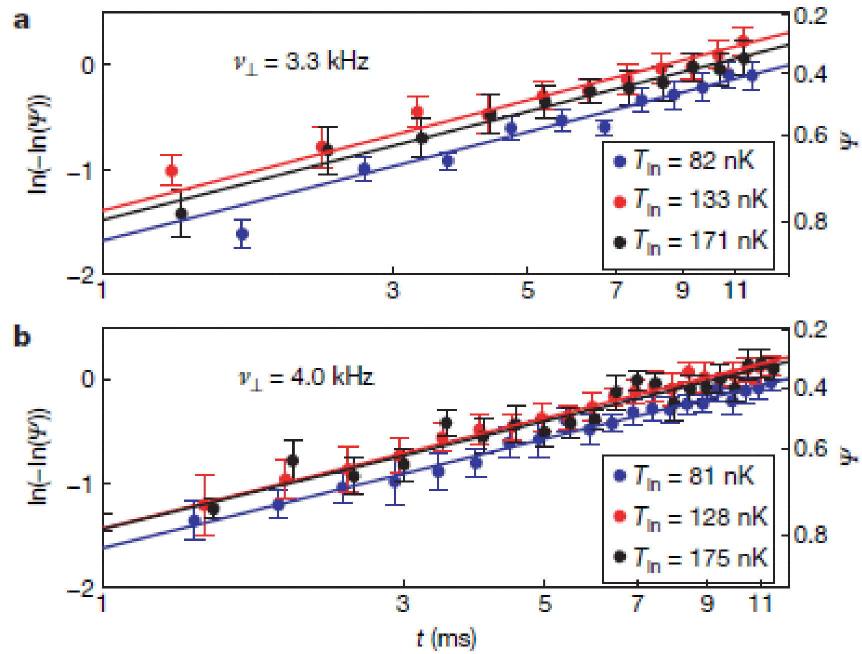
Just after splitting, very small uncertainty in relative phase
(= Highly Non-Equilibrium)



1D Bose Gas on Atom Chip

*Hofferberth, Lesanovsky, Ficher, Schumm, and Schmiedmayer, Nature **449**, 324(2007)*





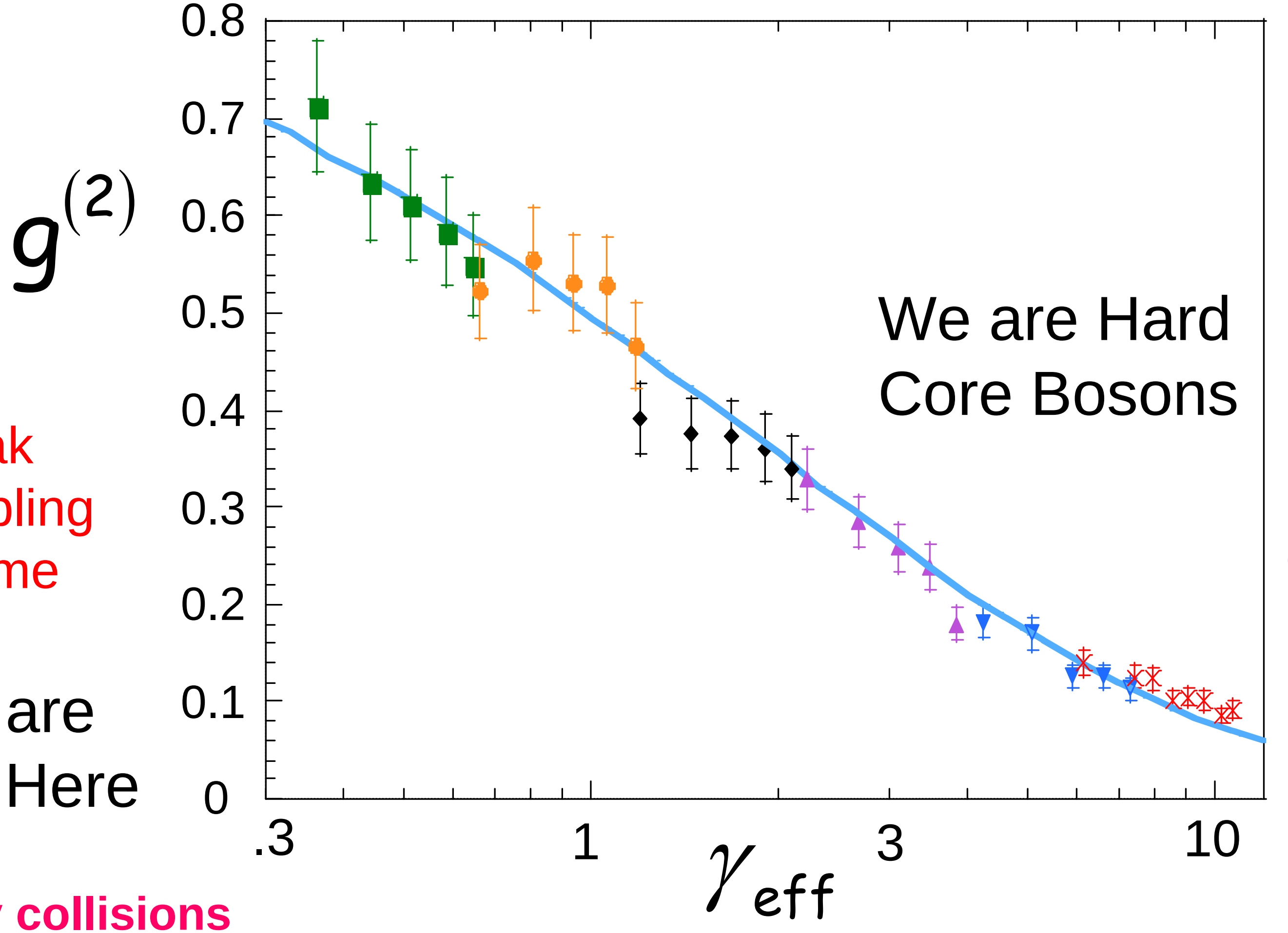
Universal sub-exponential decay of coherence

$$\Psi(t) \propto e^{-(t/t_0)^{\alpha}}$$

Dynamics is ergodic

Randomization of Local Relative Phase for Completely separated two 1D systems

Normalized Local Pair Correlations



Strong coupling regime

$g^{(3)}$, higher order correlation also decreases (3-body collision negligible)

Weak coupling regime

They are Here

3-body collisions

"Breakdown of Integrability due to 3-body collisions"
Mazets, Schumm and Schmiedmayer, PRL, 100, 210403 (2008)

Summary(3)

Understanding of Non-Equilibrium Dynamics is very important
for Condensed Matter Physics and Statistical Physics

Integrable System + Perturbation to control dynamics

1D Bosons
1D Fermions (p-wave of Hard Core particles)
$1/r^2$ interacting Gas (Calogero and Sutherland)
Fermions on a Lattice (Fermi and Habbard)

Non-Integrable system, but some constrains
what a kind of constrains, magnitude
how to lift integrability

quenched by suddenly changing parameters

Cold Atom Experiments provide nice stages
to study non-equilibrium dynamics.