Pair creations of massless fermions in electric flux tube

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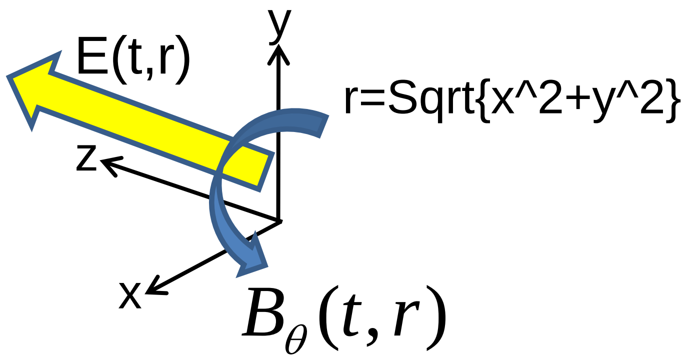
一様な強磁場Ｂ　かつ、
チューブ状の電場中での荷電粒子の対生成

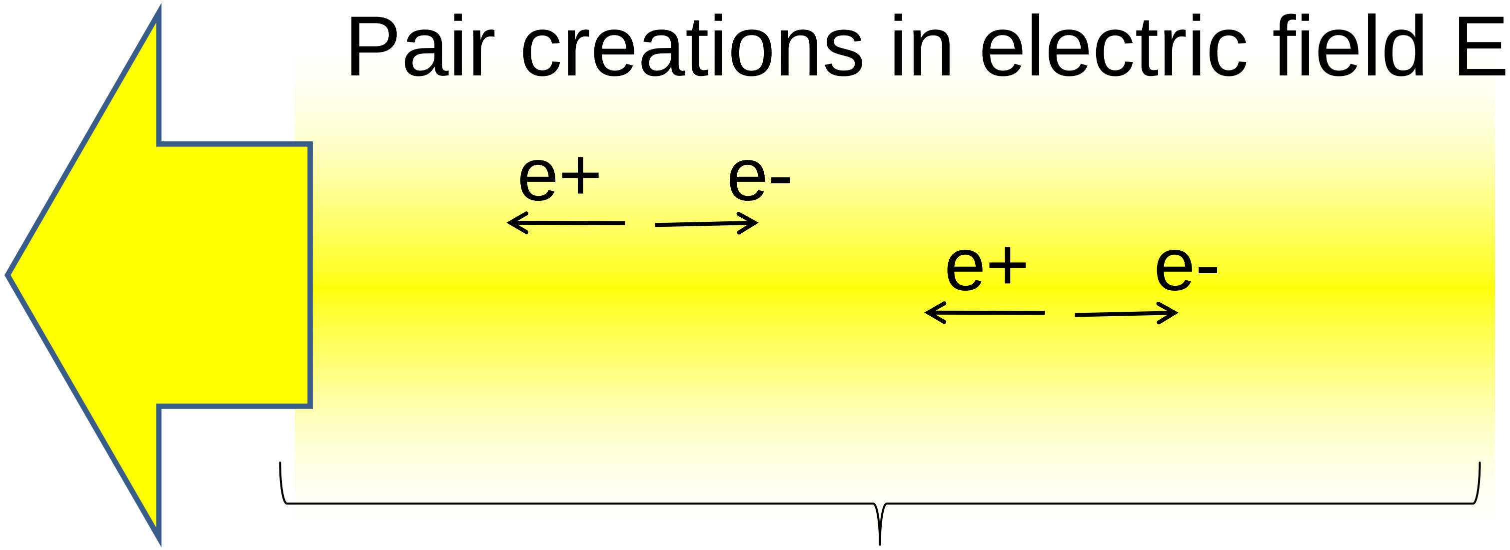
解析解
- 粒子の数密度n(t,r)
- 電場 E(t,r)
- 電流に伴う磁場$B_\theta(t,r)$



極限　B(homogeneous)　>>　E(t,r)
non-interacting massless charged fermions

Schwinger mechanism

Pair creations in electric field E



length =L

for sufficiently strong electric field

$$eEL > m_e$$

A number of references on this subject

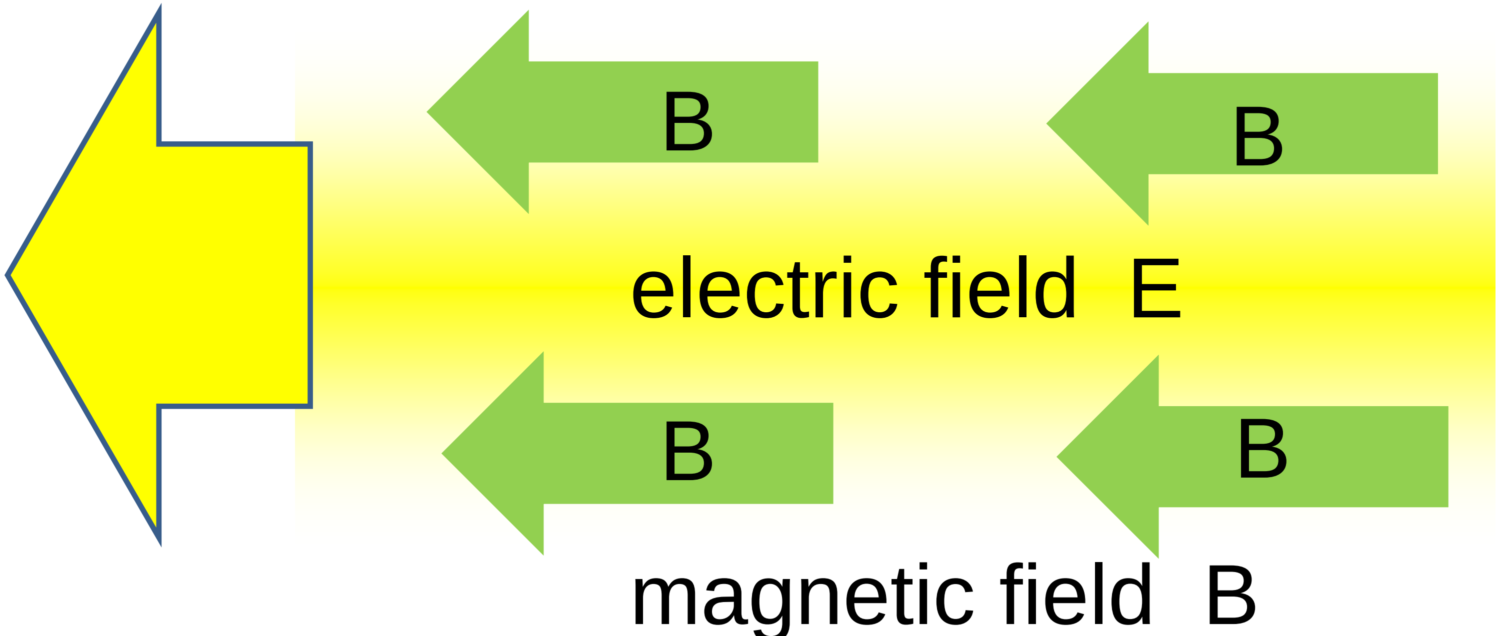
- Proper time method
  (Schwinger, 1952)

- Heisenberg –Euler effective action
  (Heisenberg and Euler, 1936.
  Dunne, hep-th/0406216)

- Canonical formalism
  (Nikishov, 1970. Tanji, hep-ph/0810.4429)

Explicit form of wave functions under electric field

Chiral anomaly of massless fermions



$$\overbrace{-\partial_i j_i^5}^{\text{vanishes}} + \partial_t (n_R - n_L) = \frac{e^2}{4\pi^2}\vec{E}\bullet\vec{B}$$

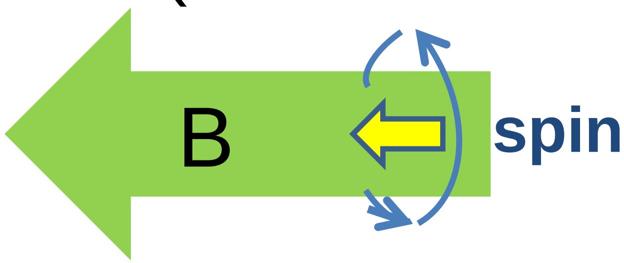
chirality= helicity for massless fermions

States of positrons under homogeneous magnetic field B=(0,0,B)

(spin parallel) (spin anti-parallel)

$$E_N = \sqrt{p_z^2 + 2NeB} \quad and \quad E_N = \sqrt{p_z^2 + 2(N+1)eB}$$

$N = 0, 1, 2, \cdots$ (Landau level)



(momentum p_z)

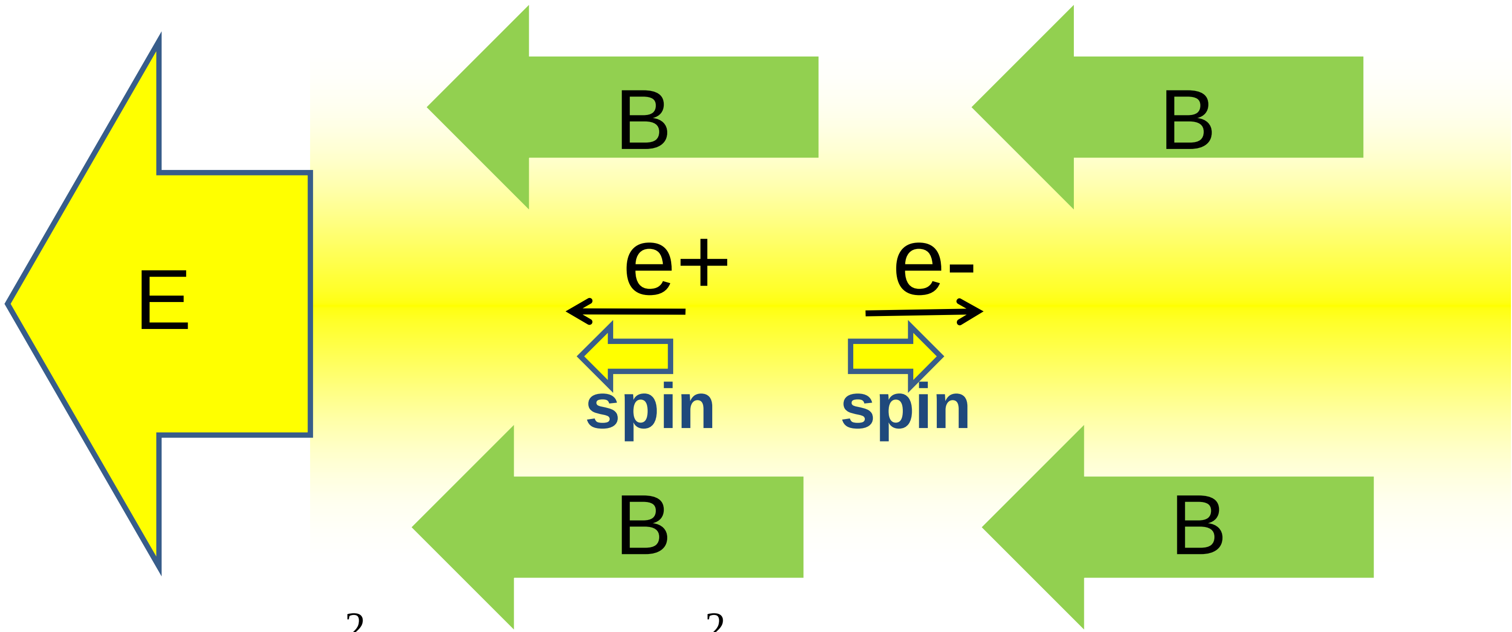
"massless" states $E_{N=0} = |p_z|$ for spin paralle

(relevant for strong B >> E)

X "massive" states $E_N^2 \propto eB$

(irrelevant for strong B >> E)

Both of electron and positron are right handed



$$\partial_t n_R(t,r) = \frac{e^2}{4\pi^2}\vec{E}\bullet\vec{B} = \frac{e^2}{4\pi^2}E(t,r)B \quad with \quad n_R(t,r) = 2n(t,r)$$

number density of electrons; $n(t,r)$

The anomaly equation describes the production rate of electrons and positrons in the limit B >>E

Lowest Landau level, N=0

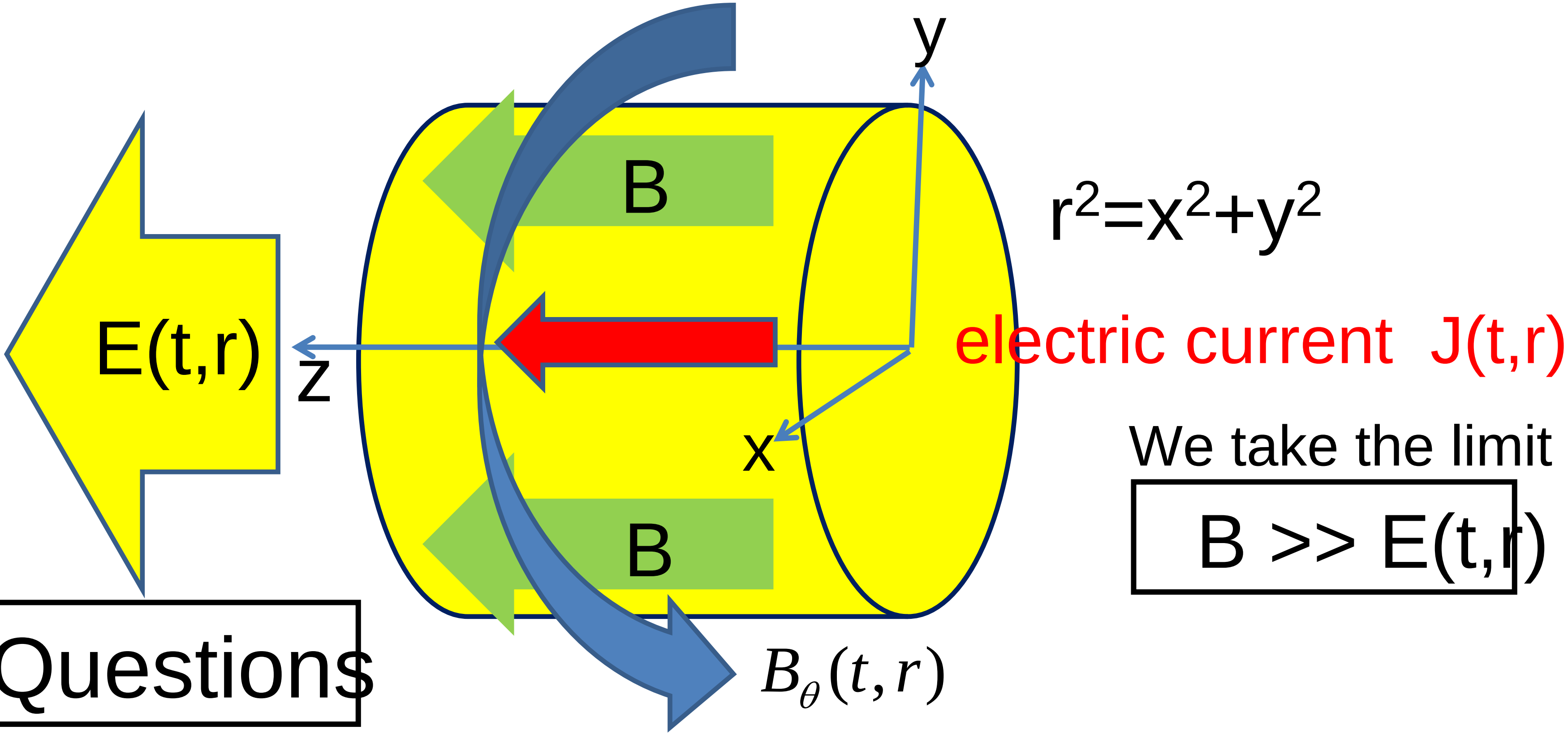
$$\partial_i j_i^5 = \underbrace{\partial_z j_z^5}_{0} + \overbrace{\partial_y j_y^5 + \partial_x j_x^5}^{0} = 0, \qquad j_i^5 = \left\langle \hat{\Psi} \gamma_5 \gamma_i \Psi \right\rangle$$

uniform in z direction

chiral anomaly

$$\underbrace{-\partial_i j_i^5}_{0} + \partial_t (n_R - n_L) = \frac{e^2}{4\pi^2} \vec{E} \bullet \vec{B}$$

Axial symmetric electric flux tube E(t,r) switched on t=(
under the background magnetic field B>>E



We take the limit

B >> E(t,r)

Questions

The temporal and spatial behaviors of
the number density n(t,r)? of electrons and positrons,
azimuthal magnetic field $B_\theta(t,r)$? by their current J(t,r)
and the electric flux tube E(t,r)?

chiral anomaly $2\partial_t n(t,r) = \frac{e^2}{4\pi^2} E(t,r) B$

Maxwell eqs

$$\begin{cases} \partial_t B_\theta(t,r) = \partial_r E(t,r), \quad \partial_t E(t,r) = \frac{\partial_r (r B_\theta(t,r))}{r} - J(t,r) \\ J(t,r) = 2en(t,r) \end{cases}$$

energy conservation

$$\partial_t \int d^3x \left(\frac{1}{2}(E^2(t,r) + B_\theta^2(t,r)) + \varepsilon(t,r) \right) = 0$$

energy of free electrons and positrons

$$\varepsilon(t,r) = n(t,r) p_F(t,r); \quad p_F(t,r) = \int_0^t dt' e E(t',r)$$

fermi momentum of positrons

Equation for E(t,r)

$$\begin{cases} \partial_t^2 E(t,r) = (\partial_r^2 + \frac{1}{r}\partial_r - \frac{e^3 B}{4\pi^2}) E(t,r) \\ E(t=0,r) = E_0 \exp(-r^2/R^2) \quad and \quad \partial_t E(t=0,r) = 0 \end{cases}$$

(initial conditions)

analytic solutions

electric field
$$E(t,r)=\frac{E_0R^2}{2}\int_0^\infty kdk\cos(t\sqrt{k^2+m^2})J_0(kr)\exp(-\frac{k^2R^2}{4})$$

number density
$$n(t,r)=\frac{e^2BE_0R^2}{16\pi^2}\left|\int_0^\infty kdk\frac{\sin(t\sqrt{k^2+m^2})}{\sqrt{k^2+m^2}}J_0(kr)\exp(-\frac{k^2R^2}{4})\right|$$

azimuthal magnetic field
$$B_\theta(t,r)=-\frac{E_0R^2}{2}\int_0^\infty k^2dk\frac{\sin(t\sqrt{k^2+m^2})}{\sqrt{k^2+m^2}}J_1(kr)\exp(-\frac{k^2R^2}{4})$$

$$(\ m^2\equiv\frac{e^3B}{4\pi^2},\qquad J_{0,1}(kr);\ Bessel\ \ functions\ )$$

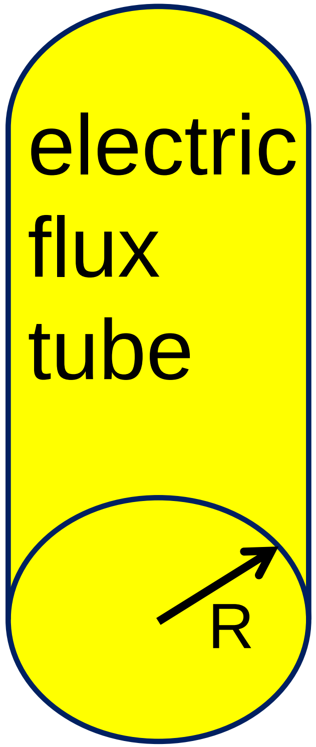
$$n(t,r)\propto\sqrt{B}\quad and\quad B_\theta(t,r)\propto 1/\sqrt{B}\quad as\ B\to\infty$$

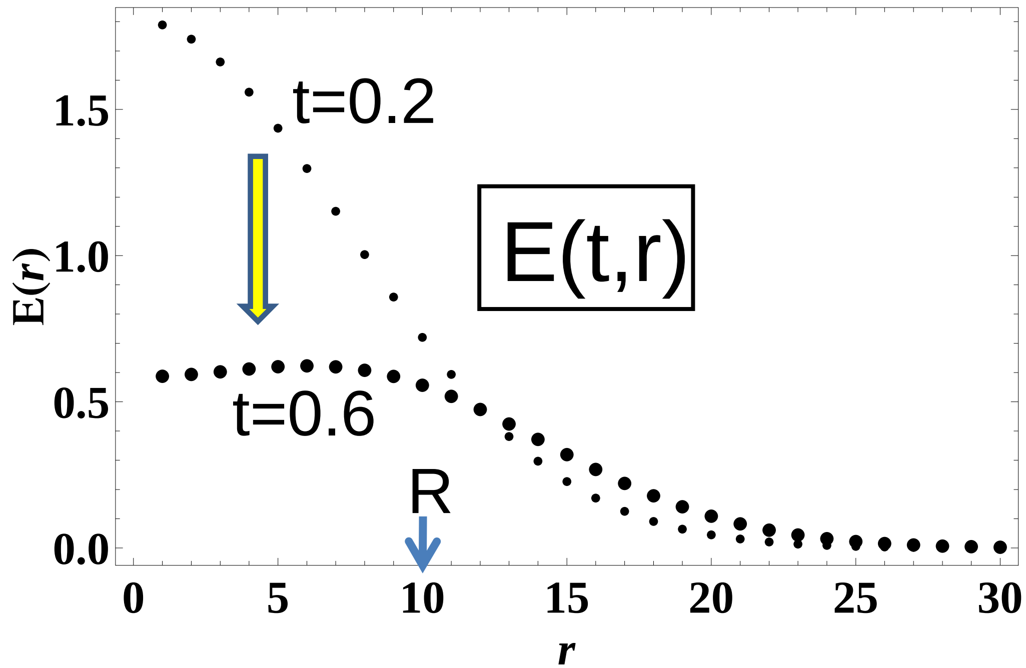
Consistency:

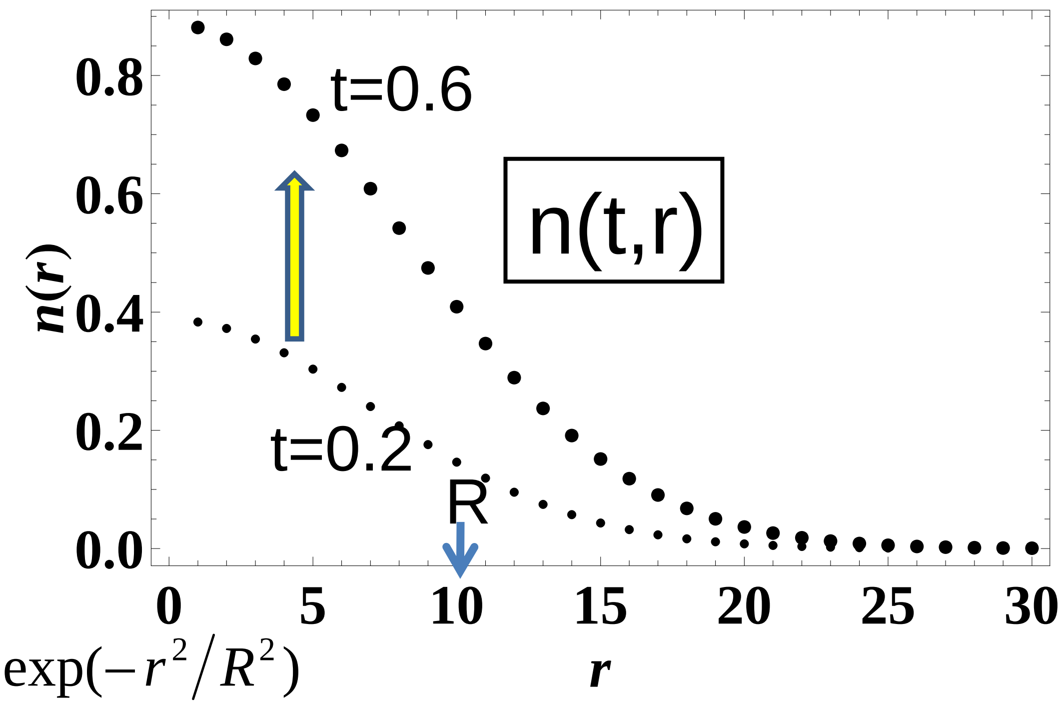
Screening of B by the magnetic moments of the particles is negligible

and

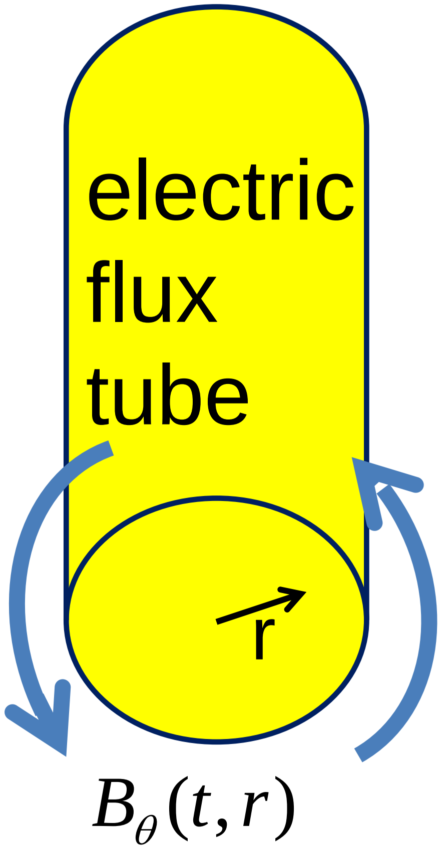
B >> $B_\theta(t,r)$ (negligible)

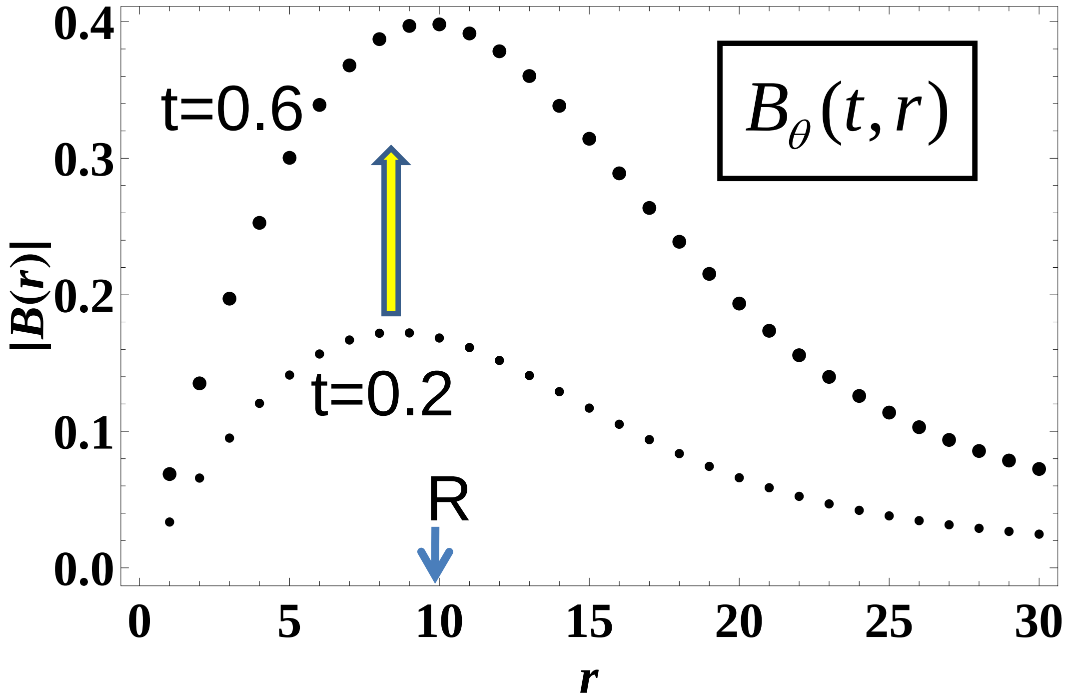


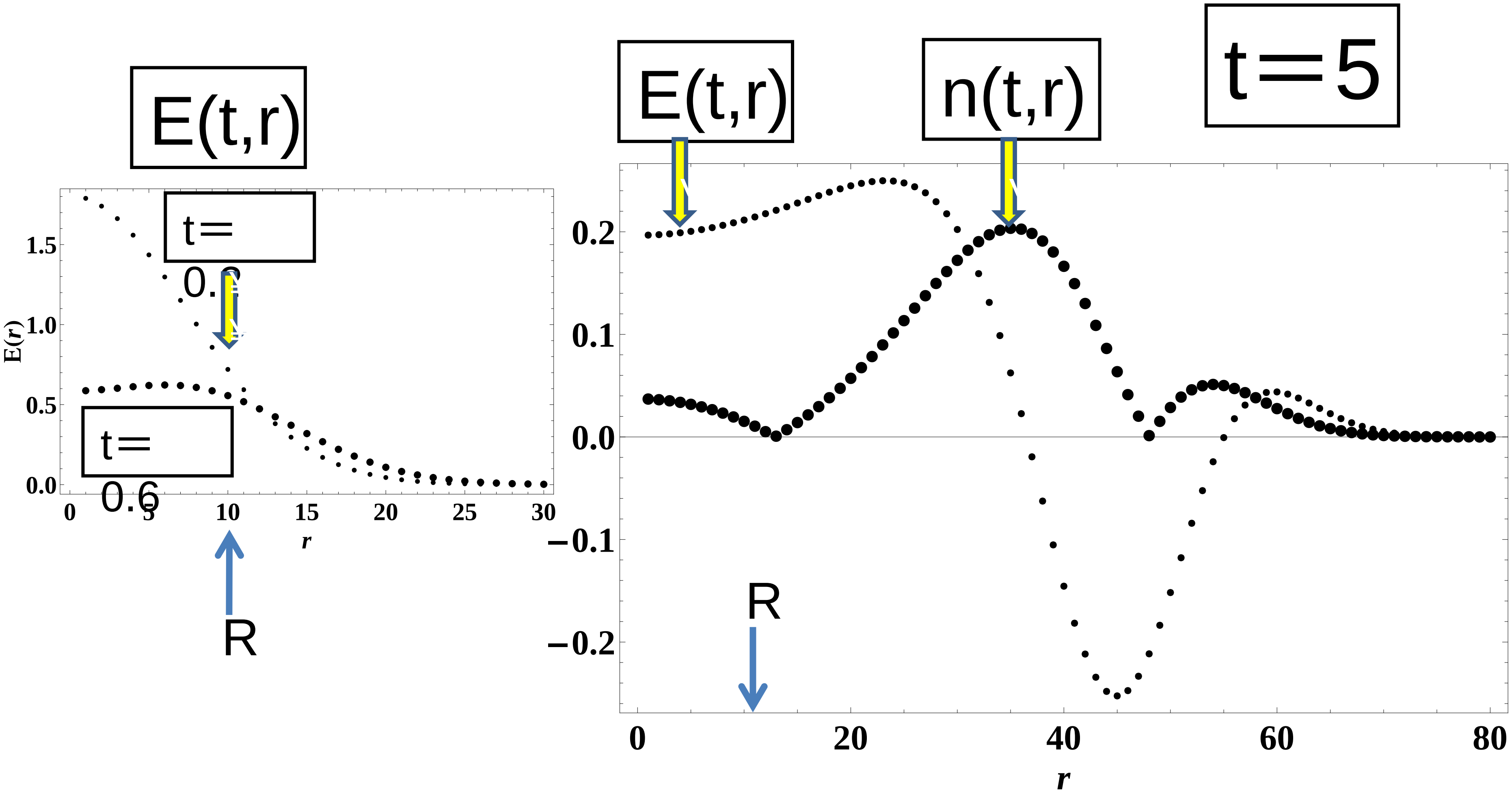




$$E(t=0,r)=E_0\exp(-r^2/R^2)$$


electric
flux
tube
r
$B_\theta(t,r)$


0.4
0.3
0.2
0.1
0.0
|B(r)|
t=0.6
t=0.2
$B_\theta(t,r)$
R
0
5
10
15
20
25
30
r



E(t,r)
t=
0.2
t=
0.6
1.5
1.0
0.5
0.0
E(r)
0
5
10
15
20
25
30
r
R
E(t,r)
n(t,r)
t=5
0.2
0.1
0.0
−0.1
−0.2
R
0
20
40
60
80
r

Homogeneous electric field ; R→∞

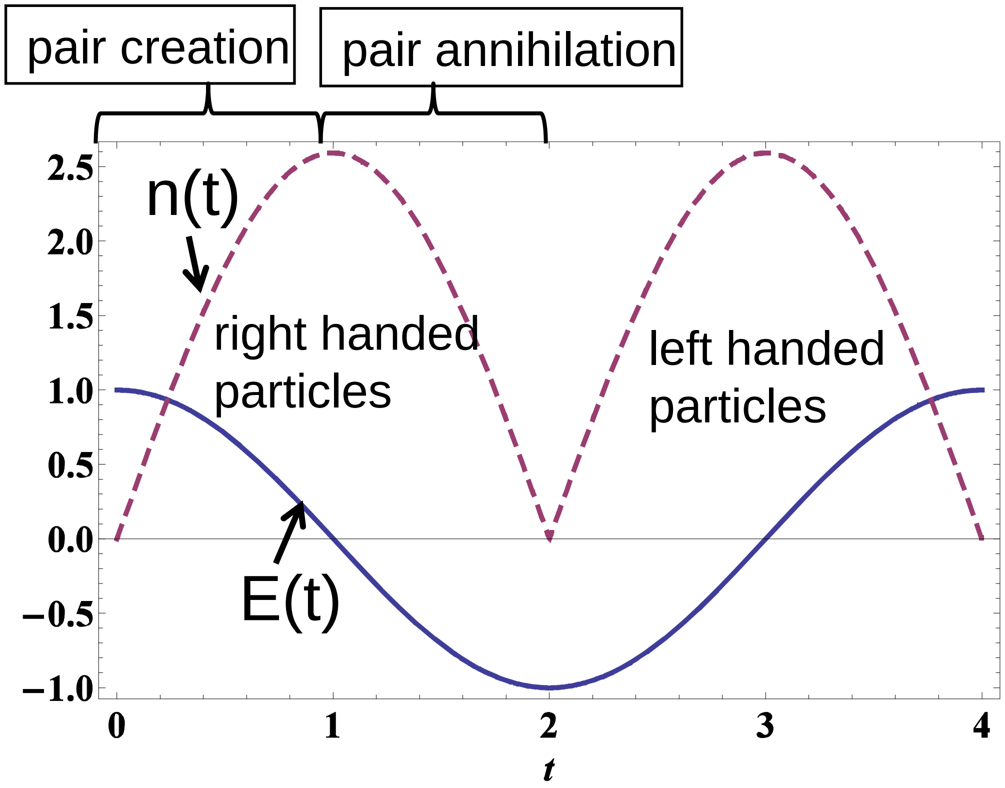
$$E(t,r) = E_0 \cos\left(t\sqrt{\alpha eB/\pi}\right)$$

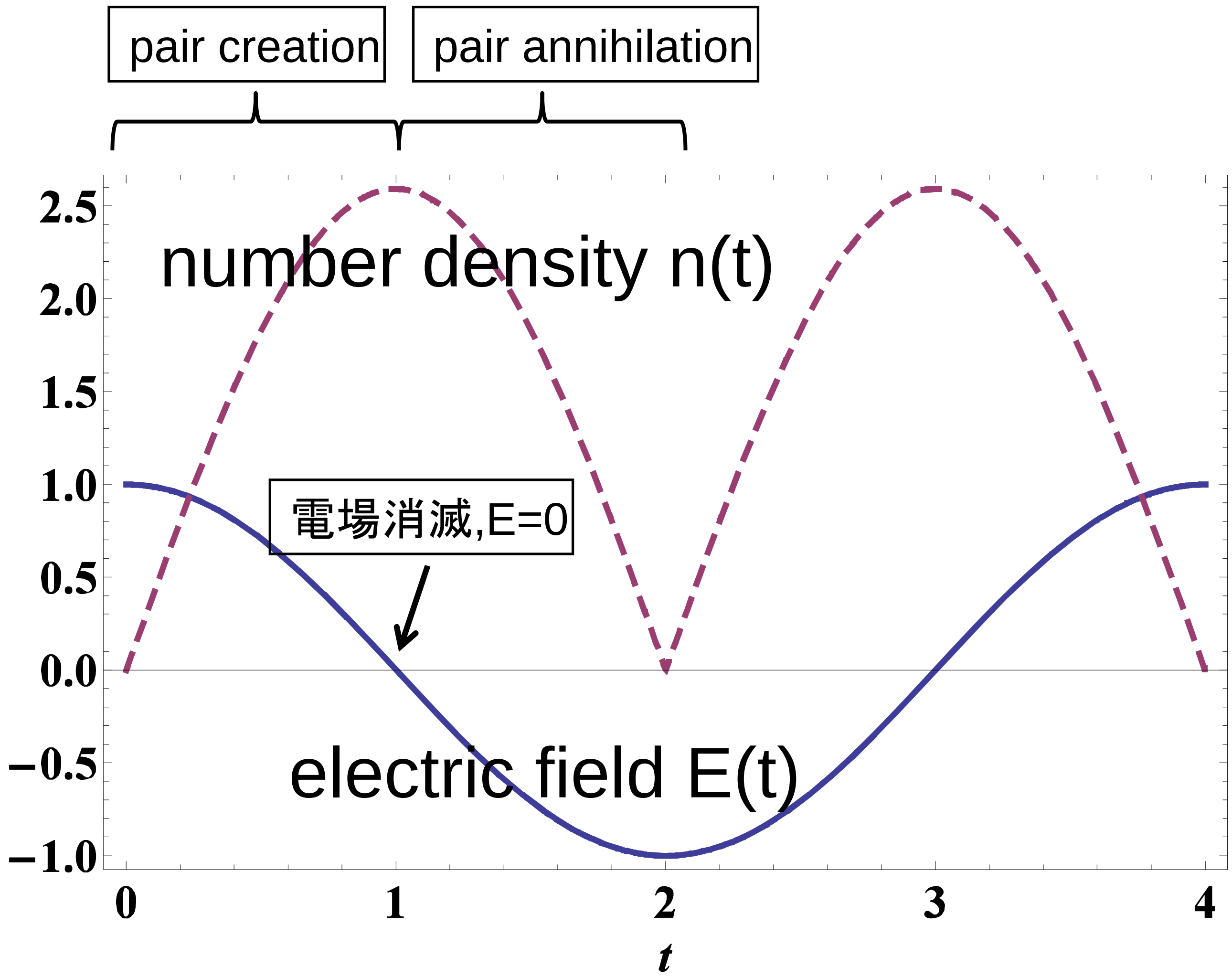
"independent on r"

$$n(t,r) = \frac{\alpha B E_0}{2\pi\sqrt{\alpha eB/\pi}}\left|\sin\left(t\sqrt{\alpha eB/\pi}\right)\right|$$

（α=e^2/4π）

$$B_\theta(t,r) = 0$$





pair creation
pair annihilation
number density n(t)
電場消滅,E=0
electric field E(t)
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
t

Conclusion

We have obtained exact analytic solutions $E(t,r)$, $n(t,r)$, and $B_\theta(t,r)$ in Schwinger mechanisı of massless charged fermions in the limit of B>> when an electric flux tube is switched on.

They are derived only by using the chiral anoma without addressing explicit forms of fermion's wavefunctions.