Controlled Hawking Process by Quantum Information

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A quantum protocol can transfer energy

by Local Operations and Classical Communication (LOCC)

without breaking causality and local energy conservation.

Quantum Energy Teleportation as a science magic

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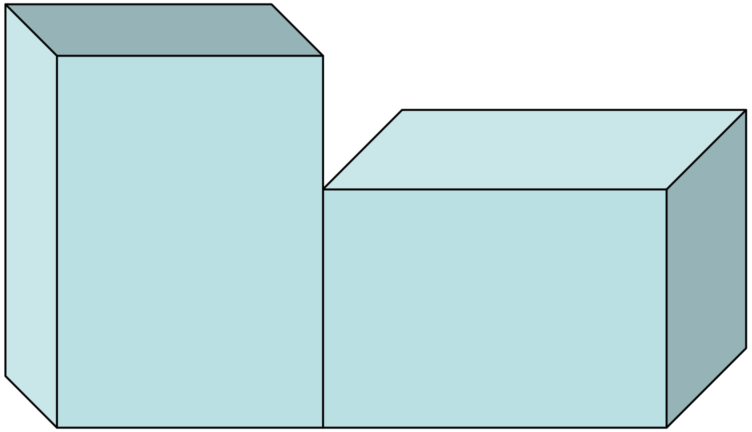
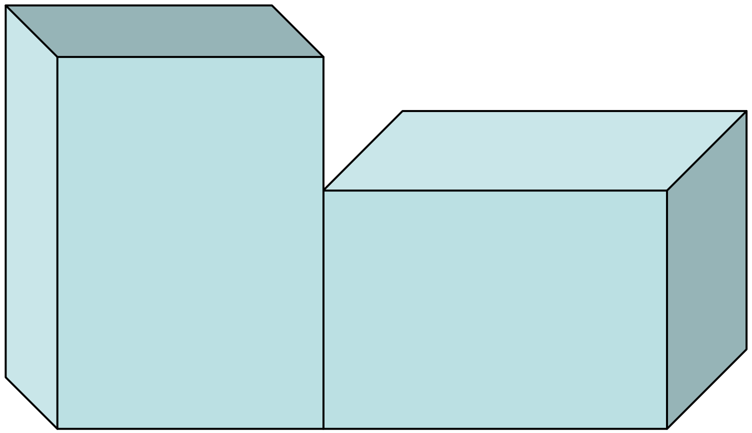
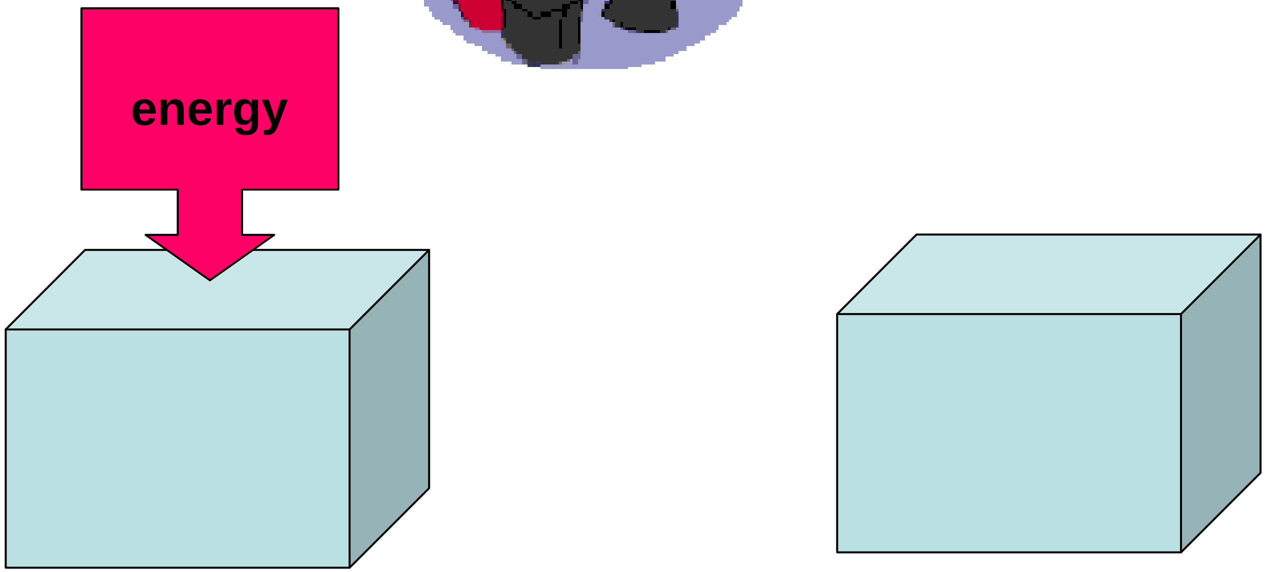
arXiv:0908.2824

Let us consider two empty boxes placed separately.

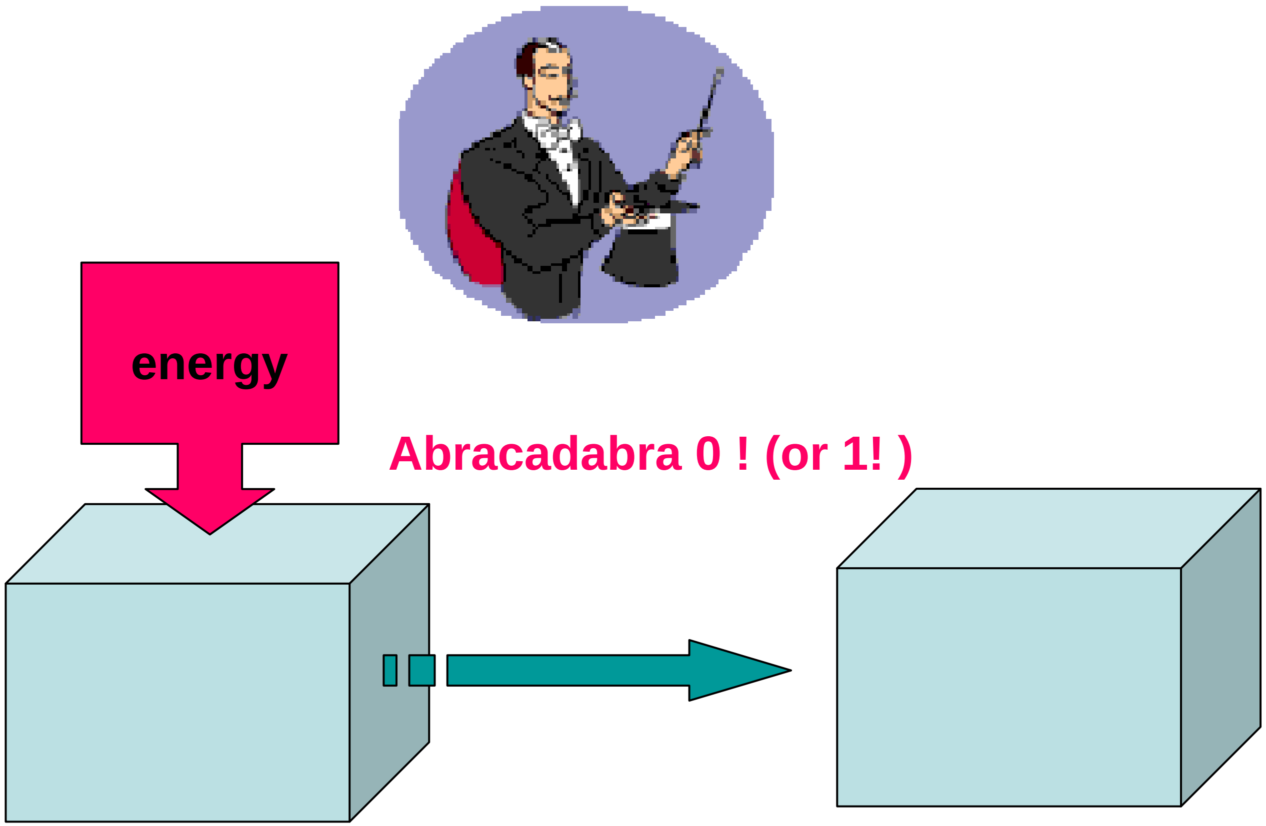




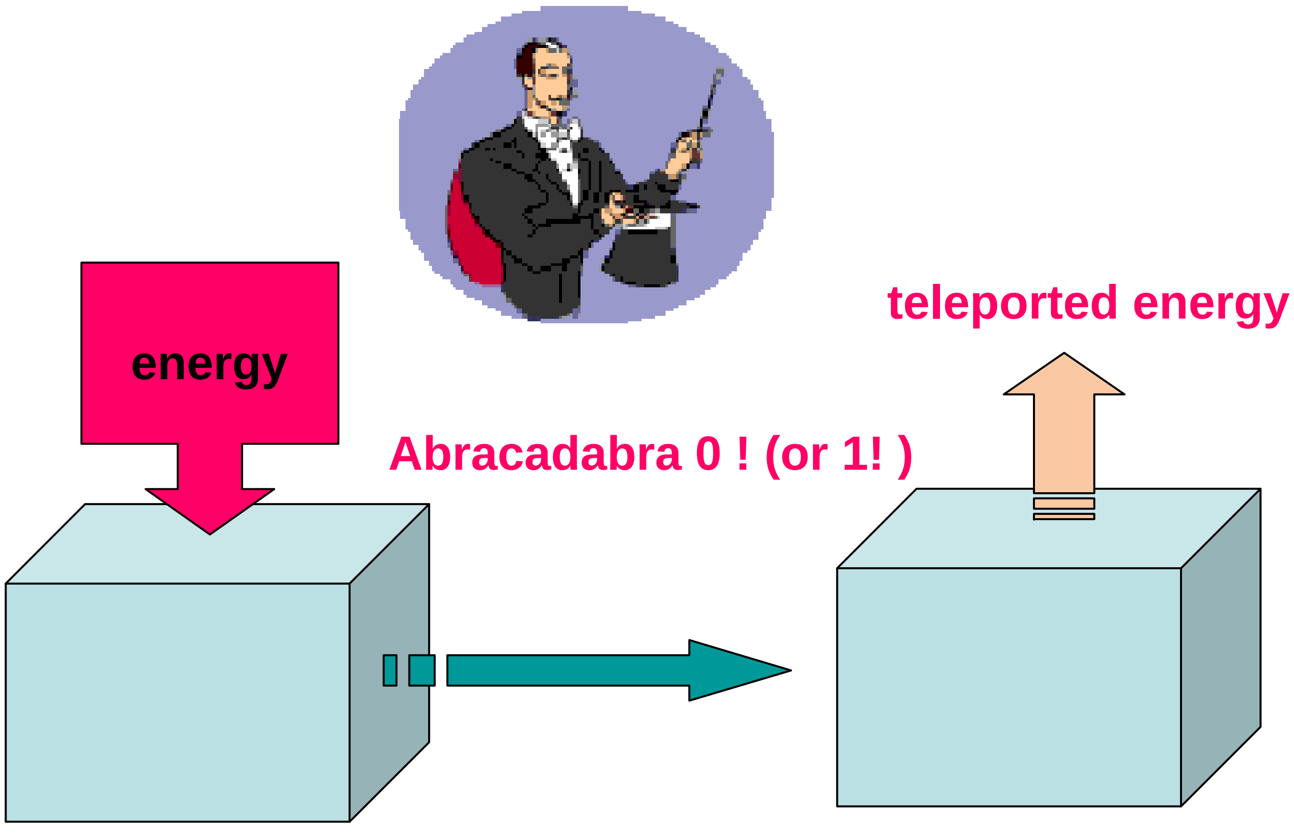
Let us infuse energy to one of the boxes.



Something happens inside the box and outputs a numerical number 0 or 1 to be announced to the other box.



In the other box, some operation is performed dependent on the announced abracadabra (0 or 1). After that, energy can be extracted from the box which was empty at the beginning.



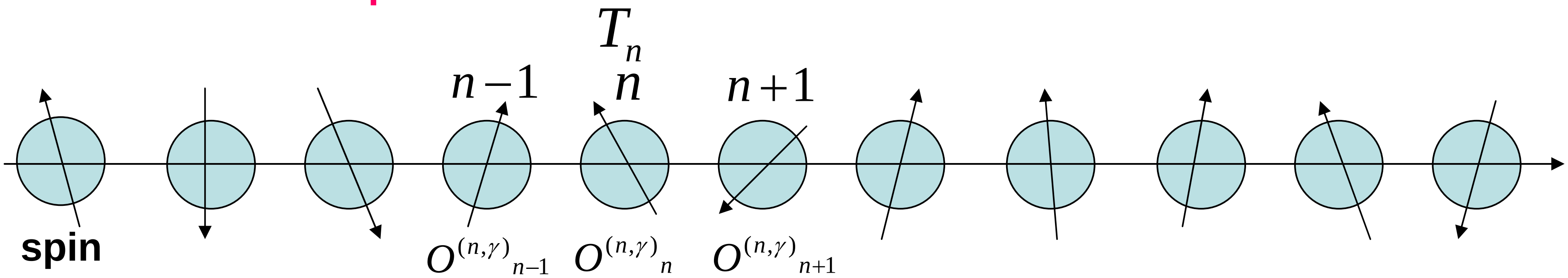
Scientific magic tricks can achieve this amazing magic !!

Quantum Energy Teleportation

The magic protocol can be considered in a lot of quantum systems including

- ○ Massless Field ⇒ Black Hole Physics
- ○ Spin Chain
- ○ Other Strongly Correlated Systems with Long Range Order
  (Trapped Ions, Quantum Hall Systems, Edge Currents,･･･)

One-dimensional Spin Chain



Spin chain is a system composed of many spins arrayed in one dimension. In this talk, we concentrate on two-level spin chains (qubit chains) and the nearest-neighbor interaction between qubits. Then general forms of the energy density operators are given by

Energy Density:

$$T_n = \sum_{\gamma} O^{(n,\gamma)}{}_{n-1} \otimes O^{(n,\gamma)}{}_{n} \otimes O^{(n,\gamma)}{}_{n+1}$$

The total Hamiltonian is given by a sum of the energy density operators:

Total Hamiltonian:

$$H = \sum_{n} T_n$$

Example: Ising Spin Chain in the presence of transversal magnetic field.

$$H = -B\sum_n \sigma^z{}_n - J\sum_n \sigma^x{}_n \sigma^x{}_{n+1} - E_g$$

$$T_n = \sum_{\gamma=1}^{3} O^{(n,\gamma)}{}_{n-1} \otimes O^{(n,\gamma)}{}_n \otimes O^{(n,\gamma)}{}_{n+1}$$

$$\boxed{\begin{aligned} &O^{(n,1)}{}_{n-1} = I_{n-1}, O^{(n,1)}{}_n = -B\sigma^z{}_n - \varepsilon, O^{(n,1)}{}_{n+1} = I_{n+1}, \\ &O^{(n,2)}{}_{n-1} = I_{n-1}, O^{(n,2)}{}_n = -\frac{J}{2}\sigma^x{}_n, O^{(n,2)}{}_{n+1} = \sigma^x{}_{n+1}, \\ &O^{(n,3)}{}_{n-1} = \sigma^x{}_{n-1}, O^{(n,3)}{}_n = -\frac{J}{2}\sigma^x{}_n, O^{(n,3)}{}_{n+1} = I_{n+1}. \end{aligned}}$$

$$\left(E_g = \sum_n \varepsilon \right)$$

ground state

(eigenstate with the lowest eigenvalue of the Hamiltonian)

$$H\left|g\right\rangle = 0 \qquad \left\langle g\left|T_n\right|g\right\rangle = 0$$

by shifting constant in the definition of T_n .

If $\left\langle g\left|T_n\right|g\right\rangle = c_n \neq 0$, we can redefine the energy density as

$\boxed{T'_n = T_n - c_n}$ in order to satisfy $\left\langle g\left|T'_n\right|g\right\rangle = 0$.

Then the Hamiltonian is also redefined as $\boxed{H' = \sum_n T'_n}$

and the eigenvalue of the ground state becomes zero

because $\left\langle g\left|H'\right|g\right\rangle = \sum_n \left\langle g\left|T'_n\right|g\right\rangle = 0$.

By this argument, we can treat the Hamiltonian as

a non-negative operator in the following discussion.

non-negativity: $\boxed{H \geq 0}$

We concentrate on short time scales in which dynamical evolution induced by the Hamiltonian is negligible.

$$\exp\left[-itH\right] \approx I$$

On the other hand, it is assumed that LOCC for the spins can be repeated many times even in a short time interval.

$$\frac{a_{lattice}\Delta n}{c} << t << \frac{1}{E_{input}}$$

E_{input} : **typical energy input in the protocol**

c : **light velocity**

$a_{lattice}$: **lattice spacing**

Δn : **site distance between two parties who perform classical communication**

Entanglement and Correlation Functions

Definition of Separable Ground State

$$|g\rangle = \prod_n |g_n\rangle$$

For the separable ground states, factorization properties of two point functions are automatically satisfied:

$$\langle g|T_n \mathrm{O}_m|g\rangle = \langle g|T_n|g\rangle\langle g|\mathrm{O}_m|g\rangle \qquad (|n-m| \geq 2)$$

Entangled Ground State

$$|g\rangle \neq \prod_n |g_n\rangle$$

If we have local operators O_m **such that**

$$\langle g|T_n \mathrm{O}_m|g\rangle \neq \langle g|T_n|g\rangle\langle g|\mathrm{O}_m|g\rangle$$

with certain n and m satisfying $|n-m| \geq 2$,

the ground state is an entangled state.

If $\langle g | T_n \mathrm{O}_m | g \rangle \neq \langle g | T_n | g \rangle \langle g | \mathrm{O}_m | g \rangle$,

the ground state cannot be an eigenstate of T_n .

Proof: If the ground state was the eigenstate of the operator:

$$T_n | g \rangle = \varepsilon | g \rangle$$

the factorization property would be satisfied as follows.

$$\langle g | T_n \mathrm{O}_m | g \rangle = \varepsilon \langle g | \mathrm{O}_m | g \rangle = \langle g | T_n | g \rangle \langle g | \mathrm{O}_m | g \rangle$$

$$\varepsilon = \langle g | T_n | g \rangle$$

This contradicts the assumption and thereby, the ground state is not an eigenstate.

Let us introduce a spectral decomposition of the energy density operator.

spectral decomposition:

$$\boxed{T_n = \sum_{\nu,k_\nu} \varepsilon_\nu \left|\nu, k_\nu\right\rangle\left\langle\nu, k_\nu\right|}$$

ε_ν **are eigenvalues and** $\left|\varepsilon_\nu, k_\nu\right\rangle$ **are corresponding eigenstates.**

k_ν **denotes the degeneracy index of the operator.**

Because $\left\{\left|\varepsilon_\nu, k_\nu\right\rangle\right\}$ **is a complete set of basis vectors in the total Hilbert space, we can expand the ground state uniquely as**

$$\boxed{\left|g\right\rangle = \sum_{\nu,k_\nu} g_{\nu,k_\nu} \left|\nu, k_\nu\right\rangle}$$

Then the definition of the origin of energy density gives the following.

$$\boxed{0 = \left\langle g\left|T_n\right|g\right\rangle = \sum_{\nu} \varepsilon_\nu \sum_{k_\nu} \left|g_{\nu,k_\nu}\right|^2}$$

Let the lowest eigenvalue be denoted by ε_- . Then let us consider the equation:

$$\sum_{\nu} \varepsilon_{\nu} \sum_{k_{\nu}} \left| g_{\nu,k_{\nu}} \right|^2 = 0.$$

If ε_- is positive,

the l.h.s. is always positive and the above relation does not hold.

If ε_- is zero,

$g_{\nu,k_{\nu}}$ vanish except g_{-,k_-} in order to satisfy the above equation.

However, this means that the ground state is an eigenstate of T_n and contradicts our entanglement assumption.

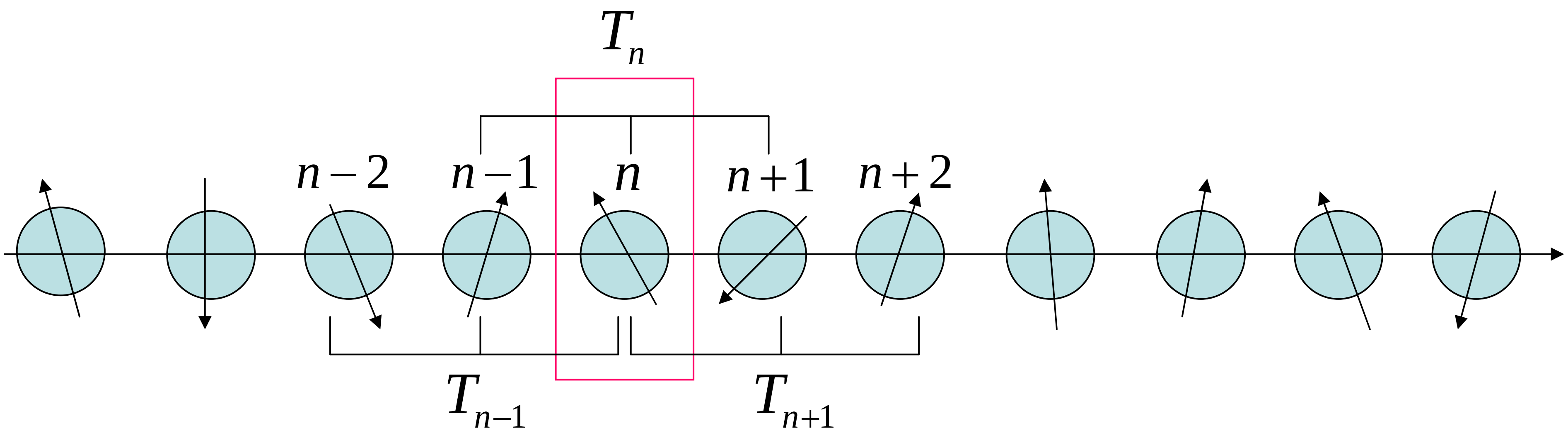
Hence, the lowest eigenvalue of T_n must be negative !

Groud-state entanglement generates negative energy.

a protocol of quantum energy teleportation

for qubit chains

For later convenience, let us introduce an operator which describes localized energy around site n.

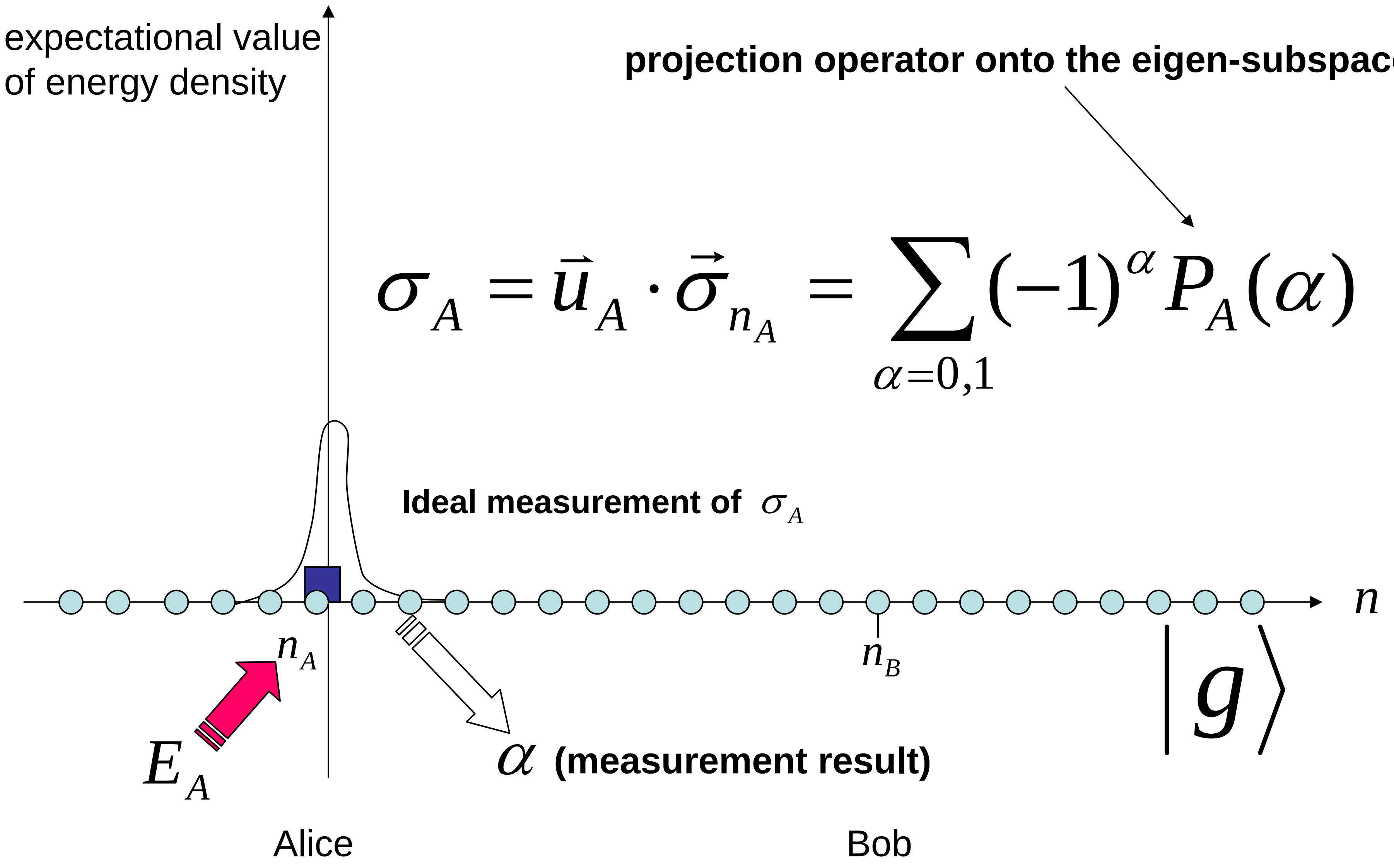


localized energy operator around site n

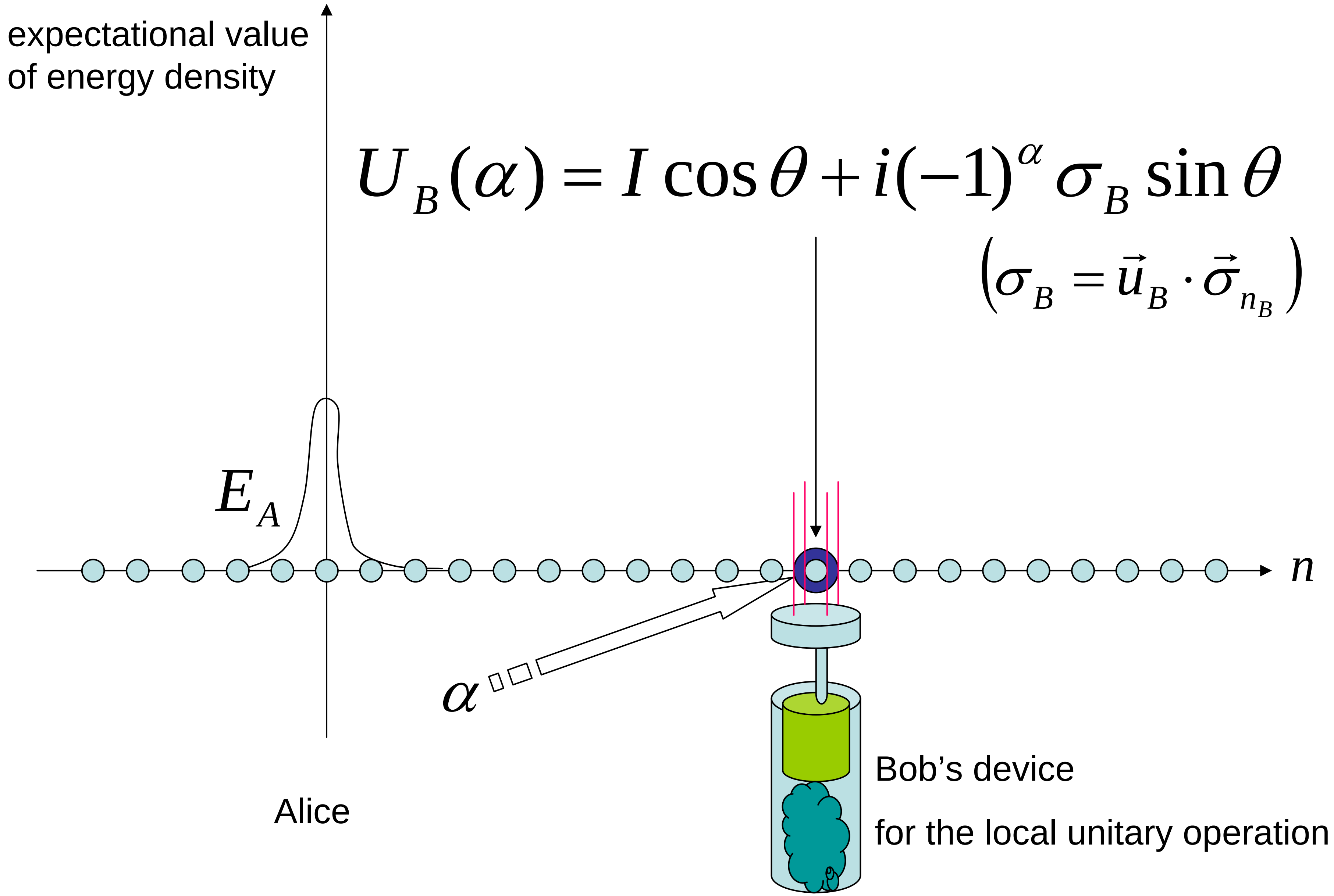
a sum of T_n **including contributions of a qubit at site** n

$$\boxed{H_n = T_{n-1} + T_n + T_{n+1}}$$

$$\sigma_A = \vec{u}_A \cdot \vec{\sigma}_{n_A} = \sum_{\alpha=0,1} (-1)^{\alpha} P_A(\alpha)$$



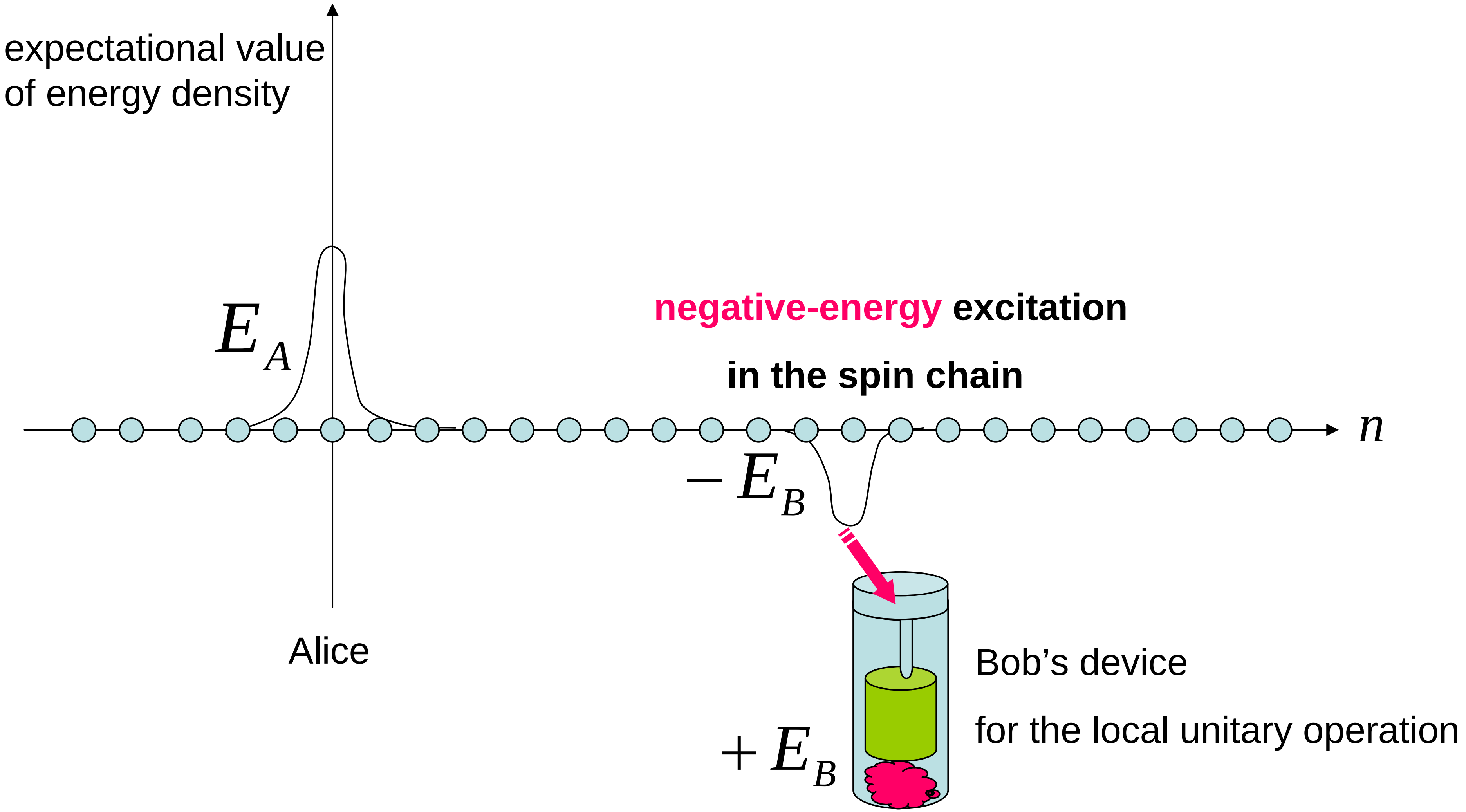
Step 1: The qubit chain is in the ground state. Perform an ideal measurement of σ_A on the qubit of Alice. The result is denoted by α .The measurement needs energy input E_A .



$$U_B(\alpha) = I\cos\theta + i(-1)^{\alpha}\sigma_B \sin\theta$$

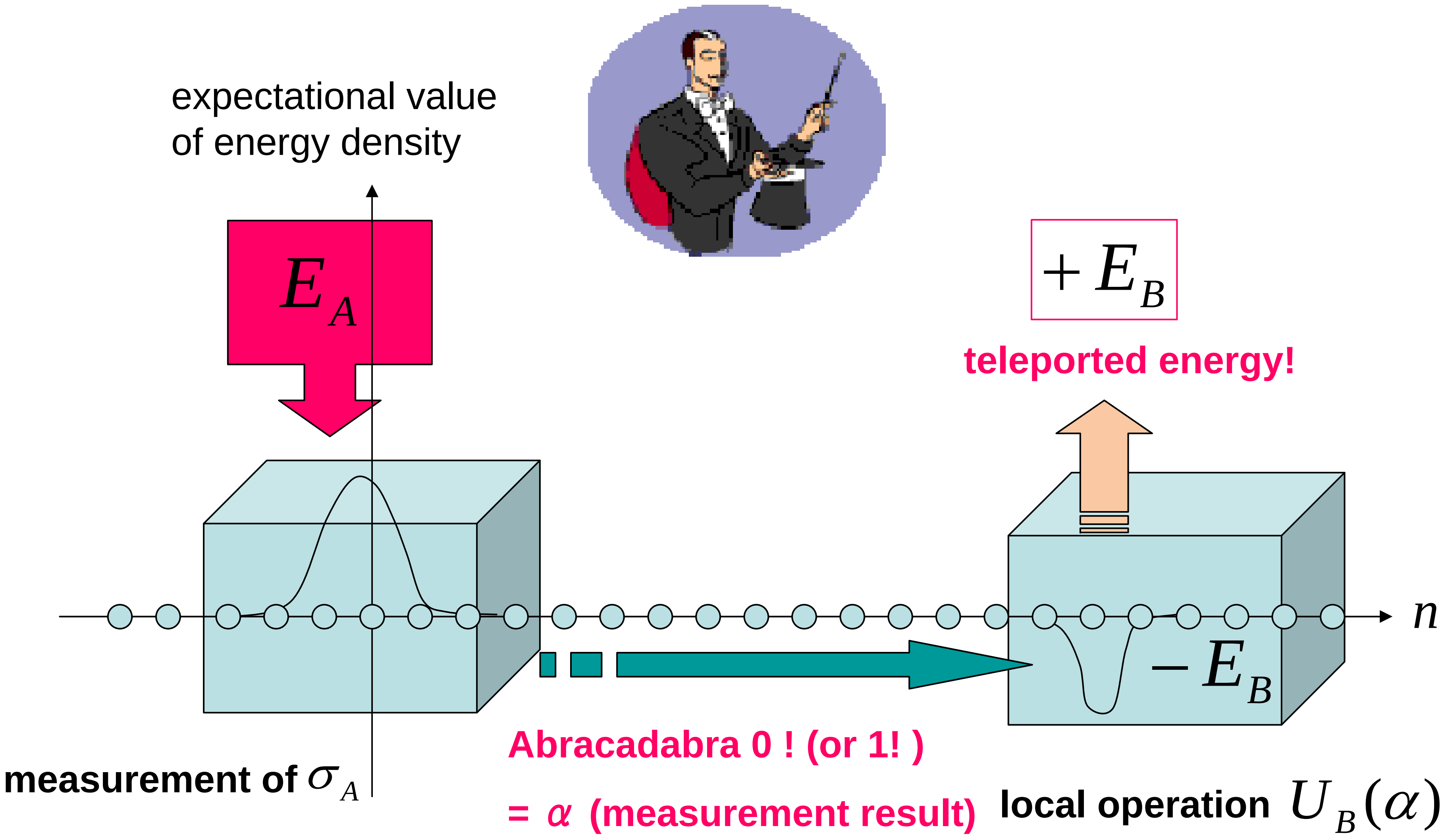
$$\left(\sigma_B = \vec{u}_B \cdot \vec{\sigma}_{n_B}\right)$$

Step 2: Alice announces the measurement result α to Bob by classical communication. Bob performs a unitary operation to his qubit dependent on the result α.



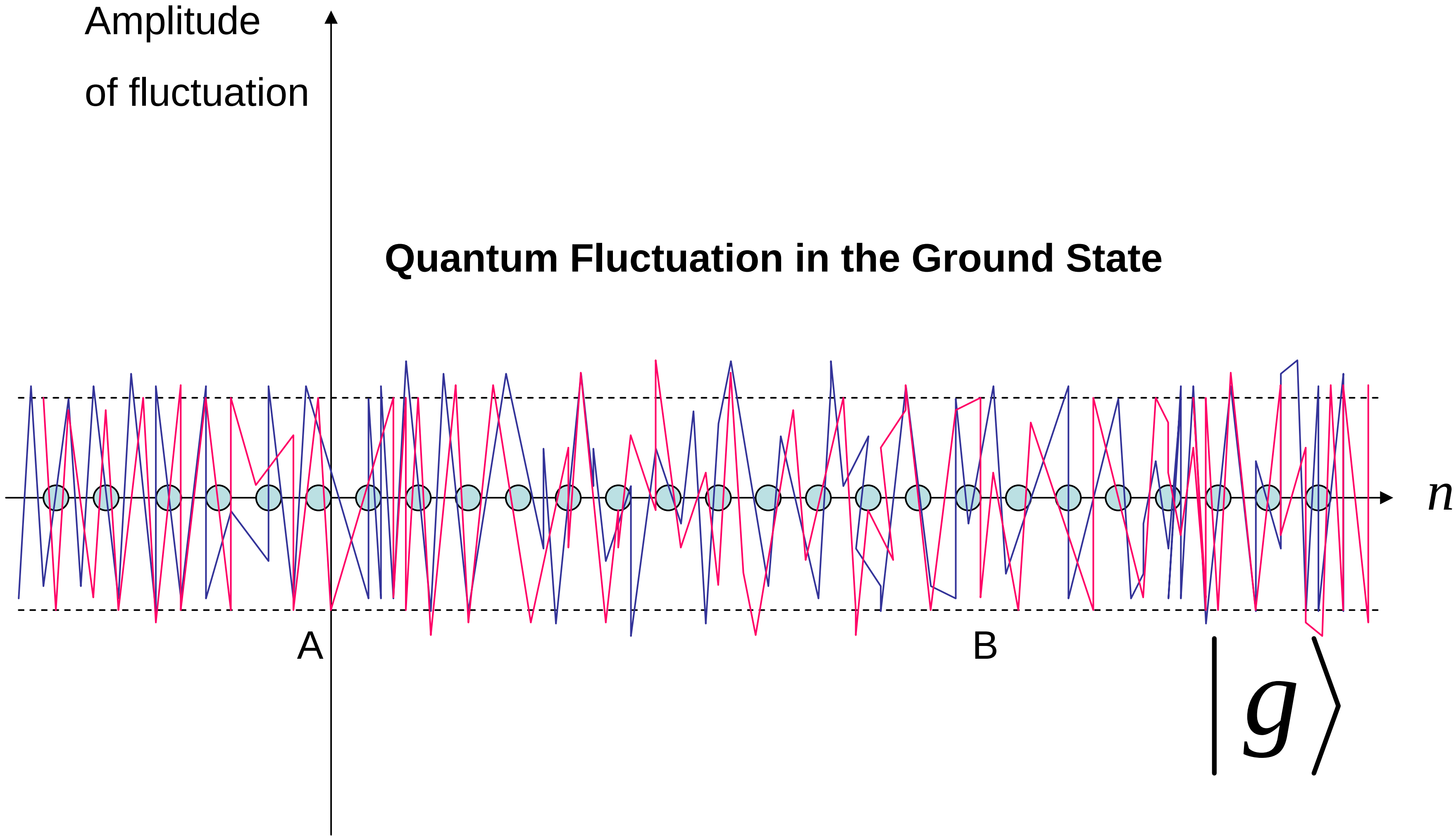
Step 3: After Bob's operation, negative energy density appears around Bob's qubit. The local energy conservation law tells us that opposite amount of energy (positive energy) is moved to Bob's device during the local unitary operation.

Trick of the Magic: **negative energy which is allowed in quantum mechanics is hidden in Bob's box.**



Why does the protocol work

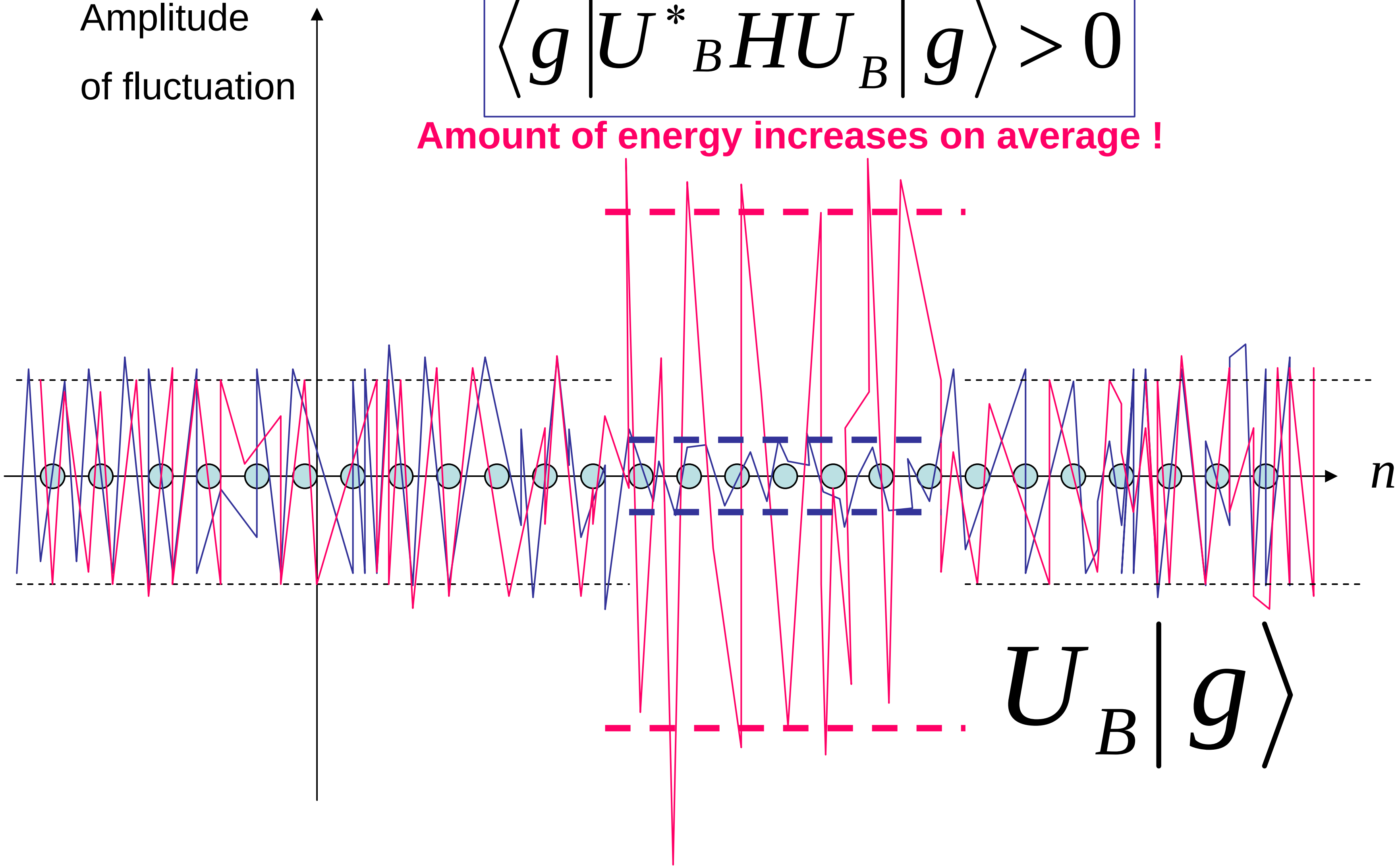
without physical energy carriers ?



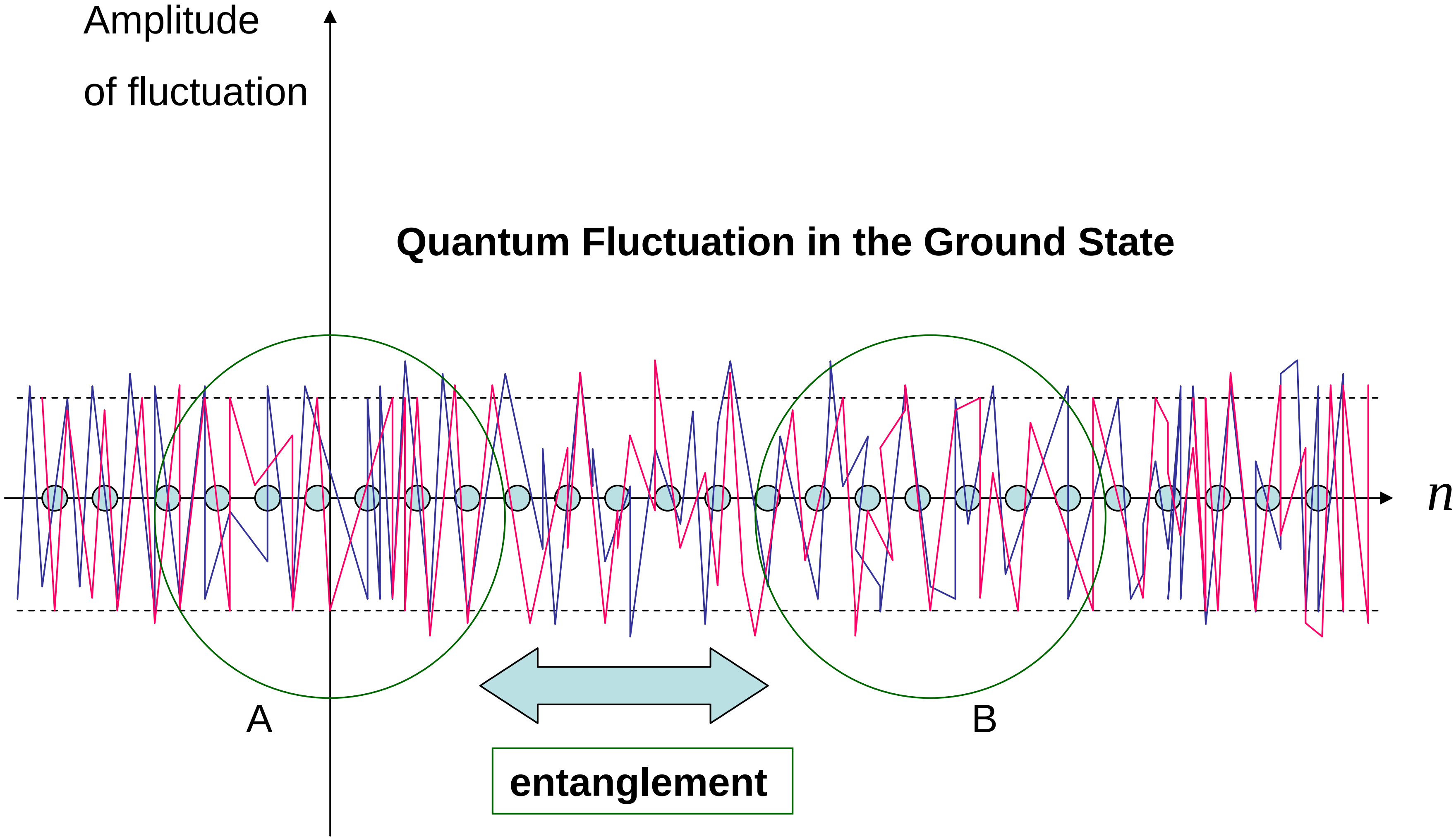
The ground state has many components of quantum fluctuation as superposition of states. In the above figure, red and blue lines simply describe those different components.

$$\langle g | U^*{}_B H U_B | g \rangle > 0$$

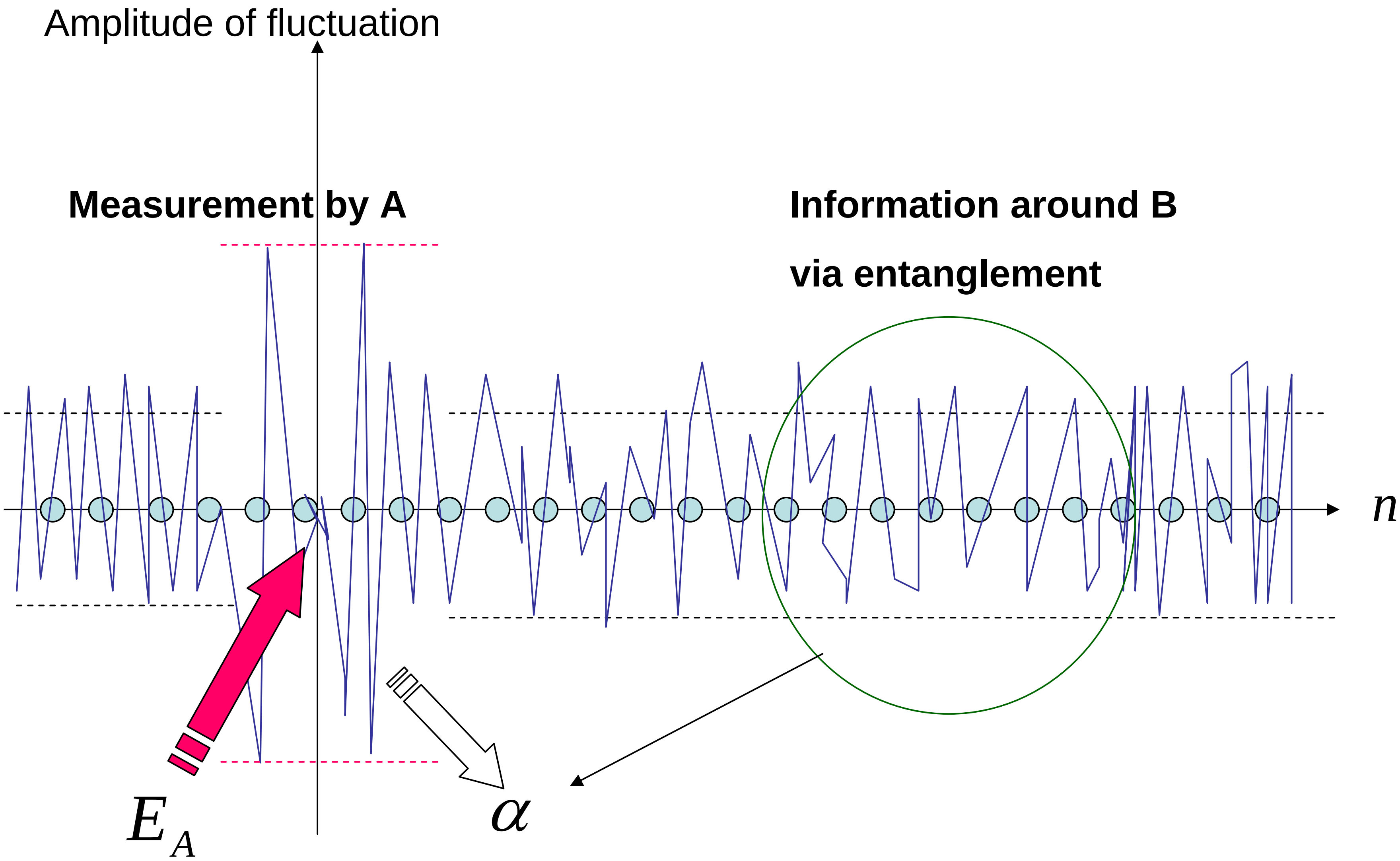
Amount of energy increases on average !



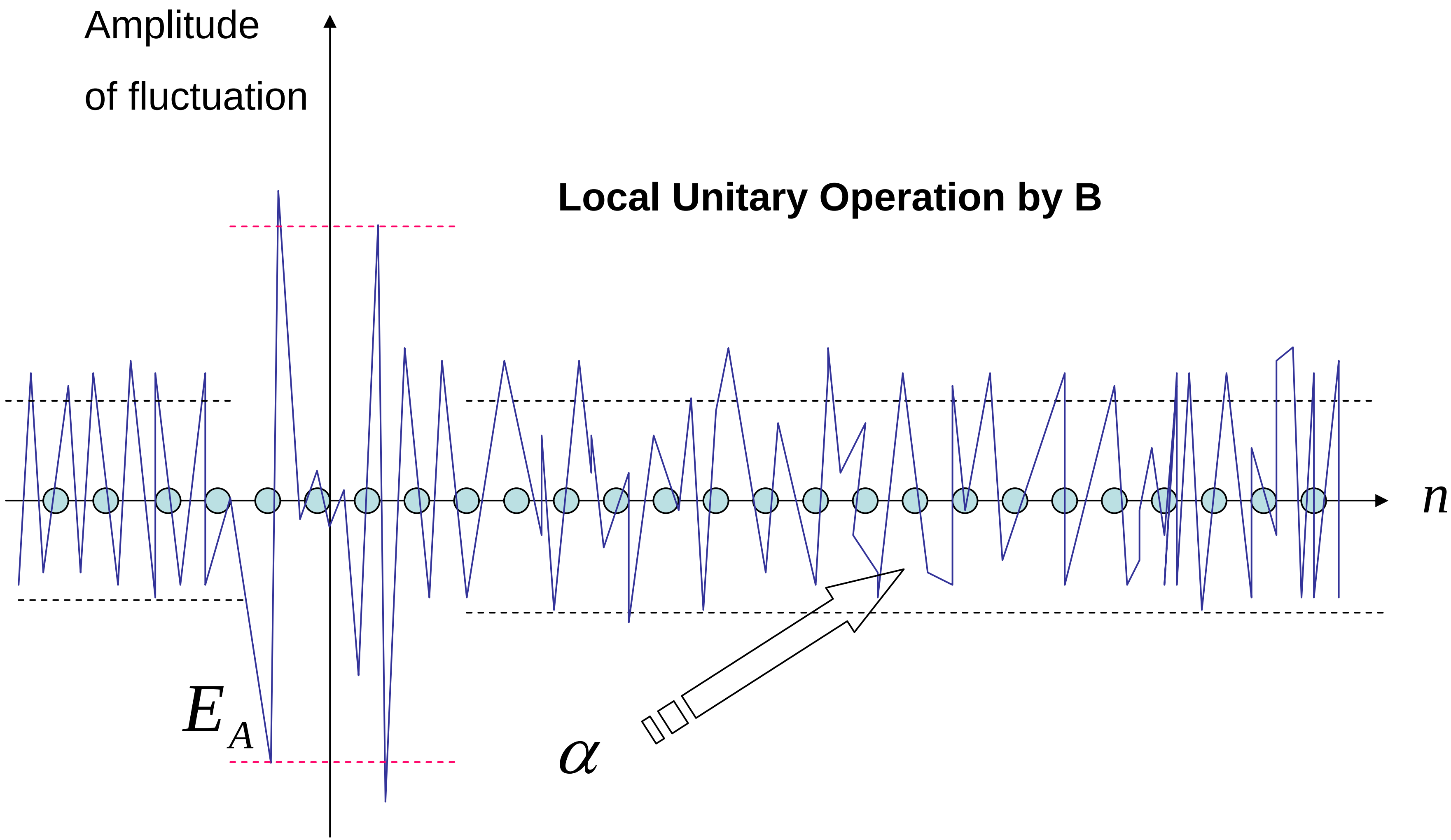
If an unitary operation independent of Alice's measurement result acts on Bob's qubit, the blue-lined component may become suppressed, but the red-lined component becomes large. Thus, on average, positive amount of energy must be infused in the spin chain.



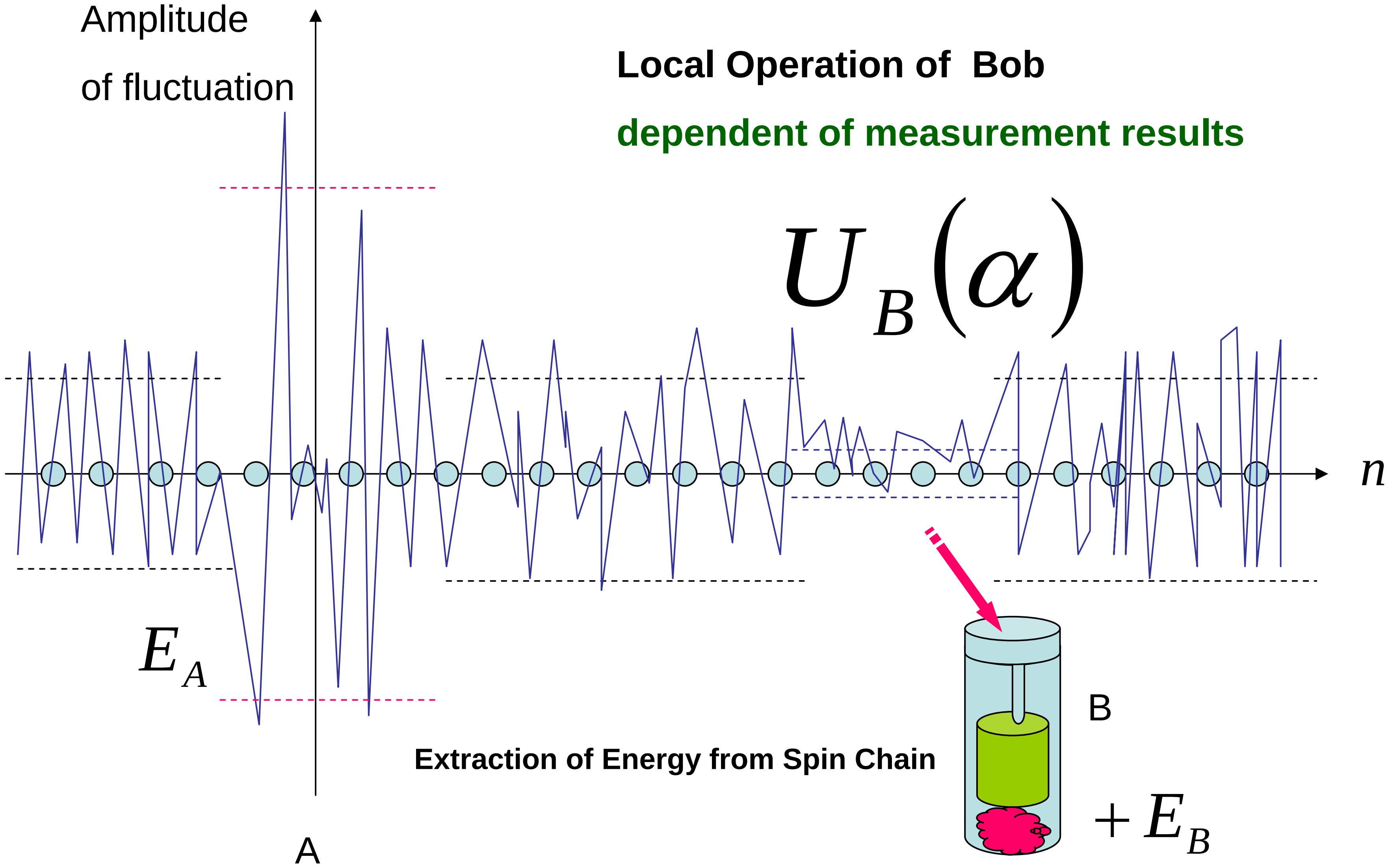
In QET case, we use the ground-state entanglement between fluctuation around Alice and fluctuation around Bob .



Alice's measurement specifies the value of α and its corresponding fluctuation component. In the figure, the blue-lined component is selected and the red-lined component vanishes . Because of the entanglement, Alice's measurement result α includes information about fluctuation around Bob.



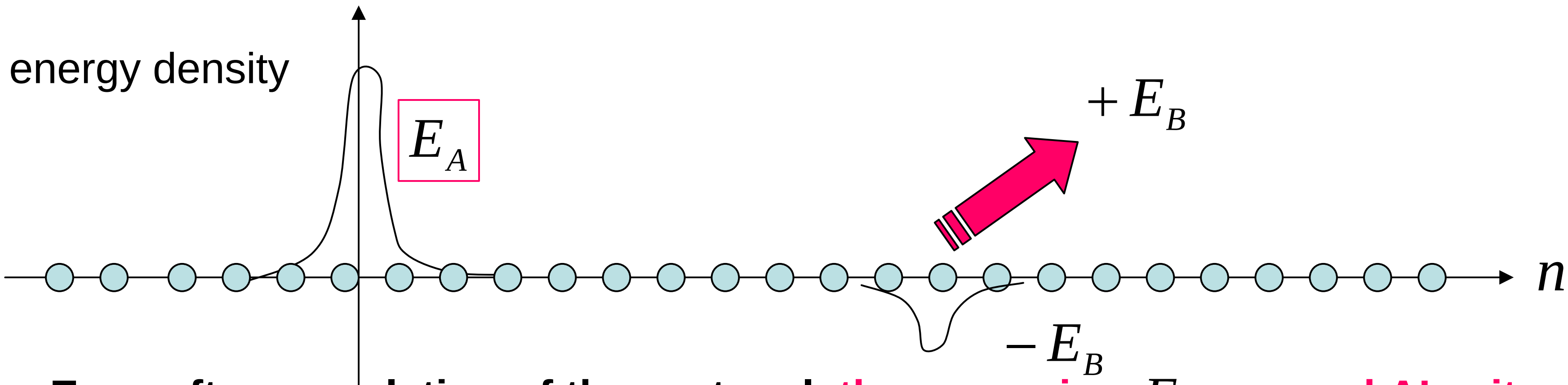
By getting information about α from Alice, Bob knows how the fluctuation in front of Bob behaves. Bob can choose an appropriate unitary operation in order to suppress the fluctuation.



By squeezing the fluctuation locally, Bob can obtain energy from the spin chain. The extracted energy was hidden in Bob's region from the start !

Thus no energy carrier is hired in the QET protocol !!

Even after B's operation, $\boxed{E^{(f)}{}_A = Tr\left[\rho H_{n_A}\right] = Tr\left[\rho' H_{n_A}\right] = E_A}$



Even after completion of the protocol, there remains E_A around A's site in the spin chain which A had to first deposit for the measurement.

If A could withdraw the energy completely by her local operations after completion of the protocol, energy gain of B needed no cost.

However, if so, the total energy of the spin chain became $-E_B$ and negative. This is inconsistent with non-negativity of the total Hamiltonian.

Hence, it is impossible that A extracts energy larger than $E_A - E_B$ from the spin chain only by her local operations.

$$\boxed{E_A = 0 \Rightarrow E_B = 0}$$

$$\left[\partial^2{}_t - \partial^2{}_x\right]\phi(t,x) = 0$$

$$x^{\pm} = t \pm x$$

$$\partial_+\partial_-\phi = 0$$

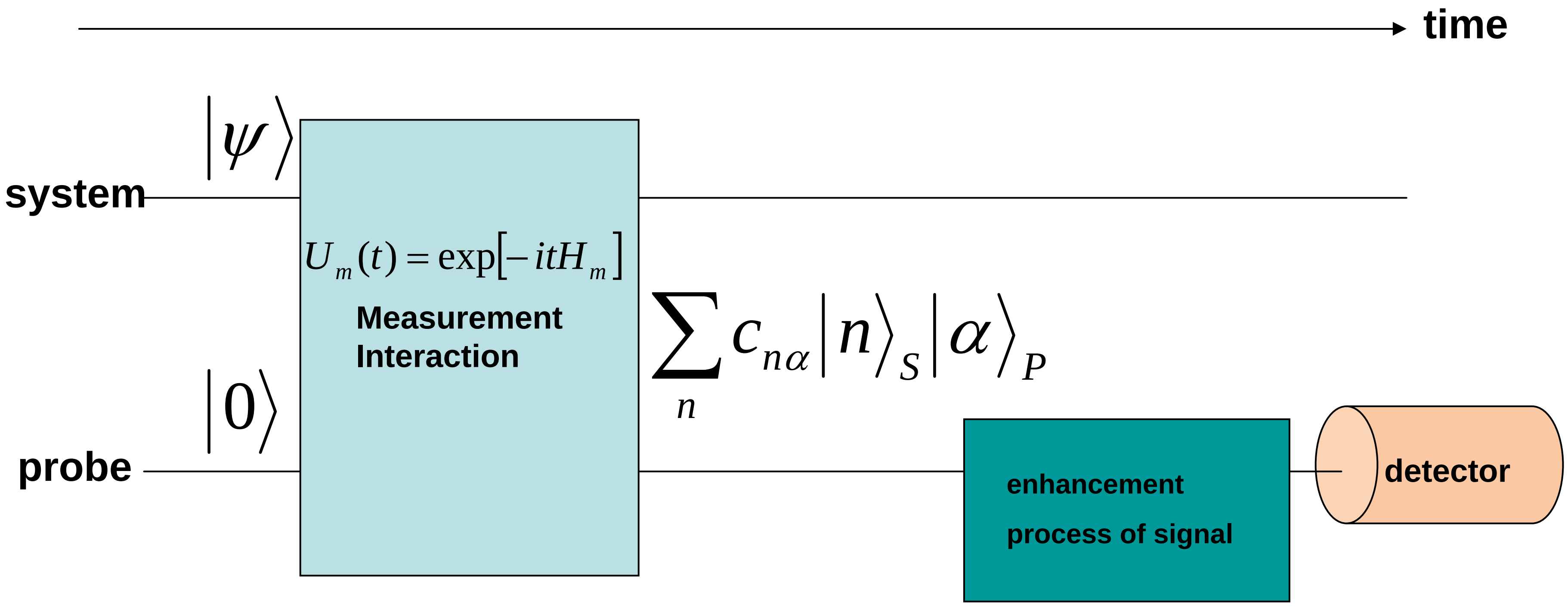
$$\phi = \phi_R\left(x^-\right) + \phi_L\left(x^+\right)$$

right-mover component

left-mover component

Preparation: POVM measurements

(Generalized measurement, Indirect measurement)



ideal measurement output

$$\sum_n c_n |n\rangle_S |n\rangle_P$$

$$U_m(t) = \exp\left[-itH_m\right]$$

$$Tr_P\left[\left(I_S \otimes |\alpha\rangle\langle\alpha|_P\right)U_m(t)\left(\rho_S \otimes |0\rangle\langle 0|_P\right)U_m{}^*(t)\right] = M_\alpha \rho_S M^*{}_\alpha$$

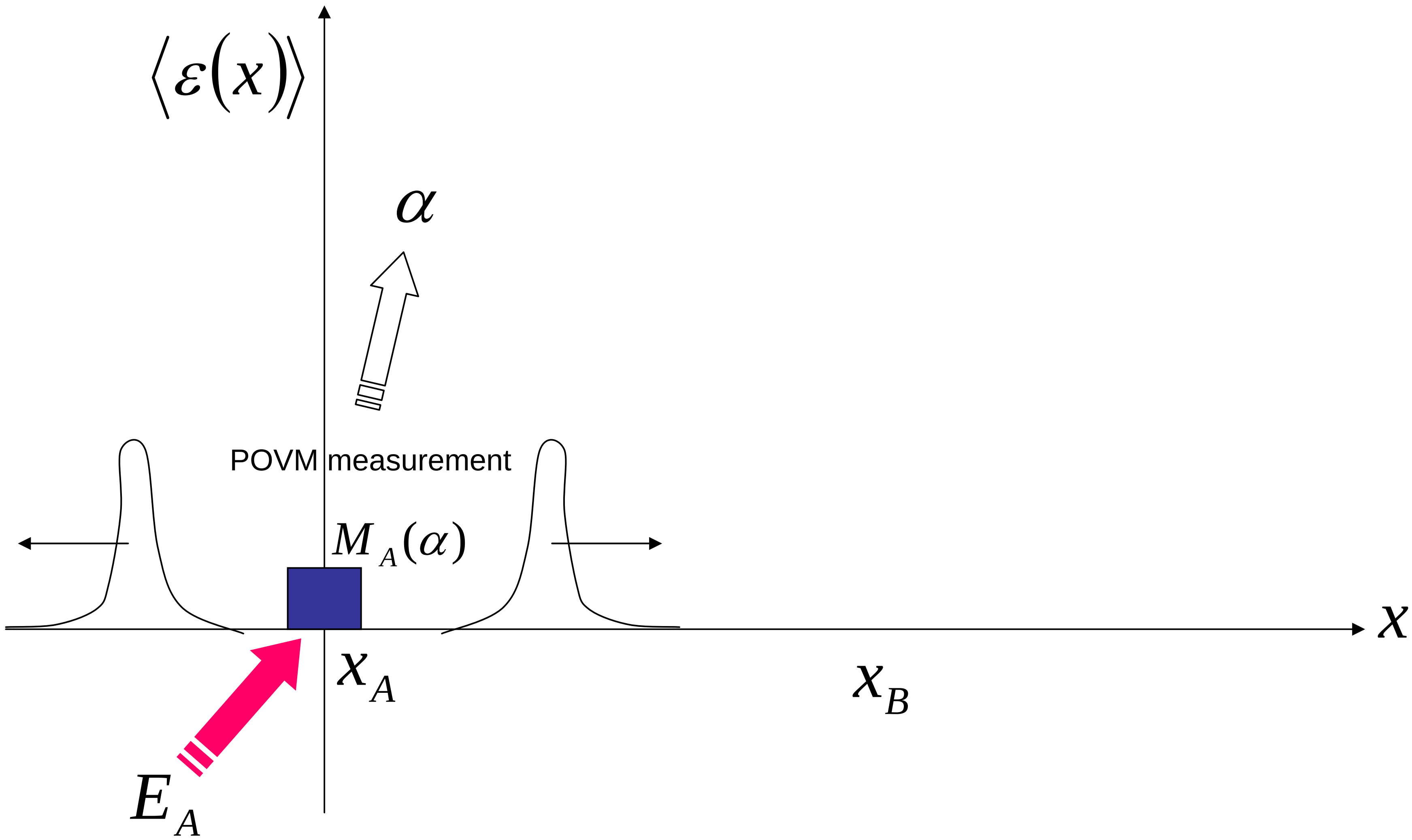
Positive Operator Value Measure (POVM)

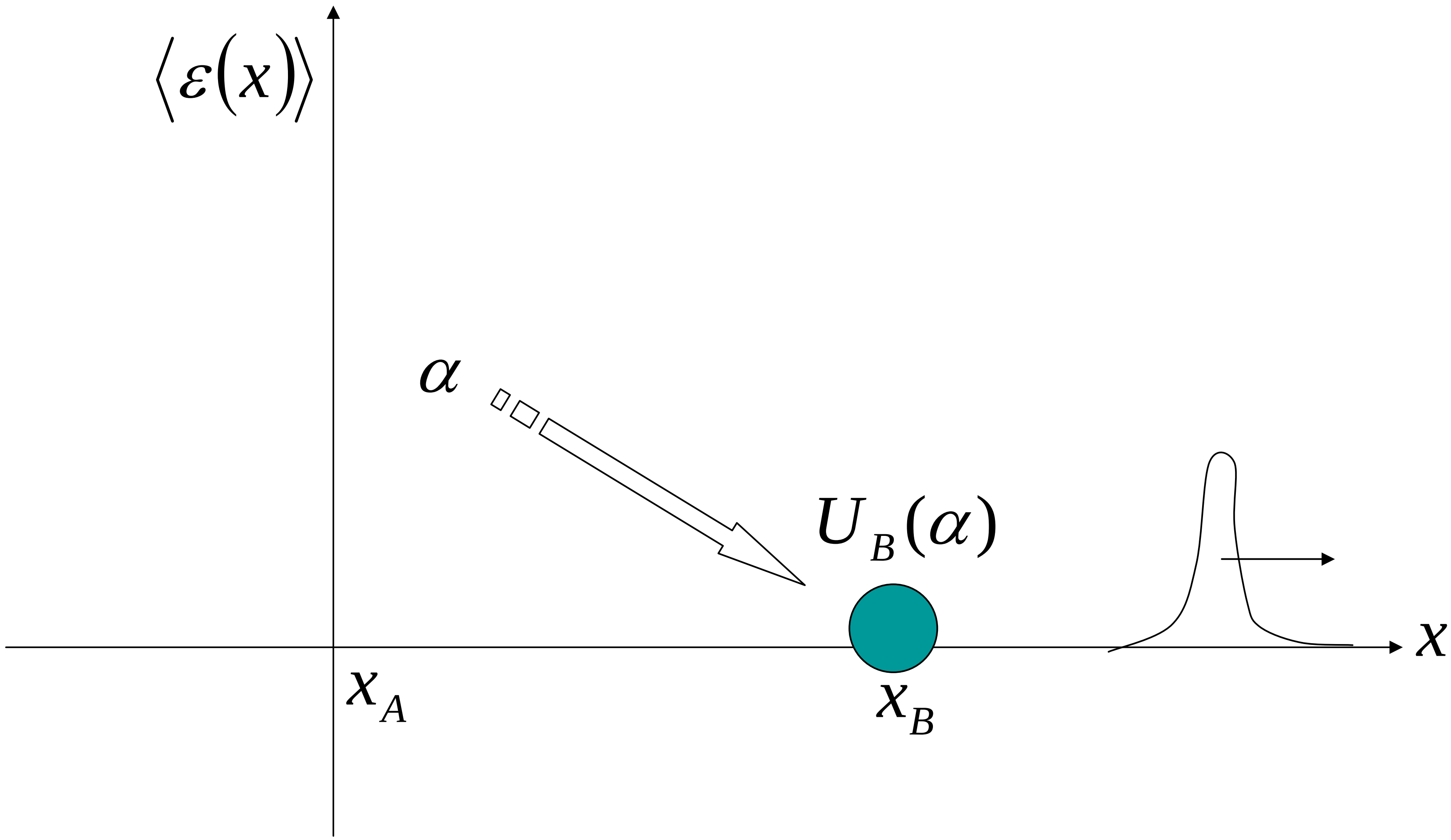
$$\left\{M_\alpha{}^* M_\alpha \mid \sum_\alpha M_\alpha{}^* M_\alpha = I\right\}$$

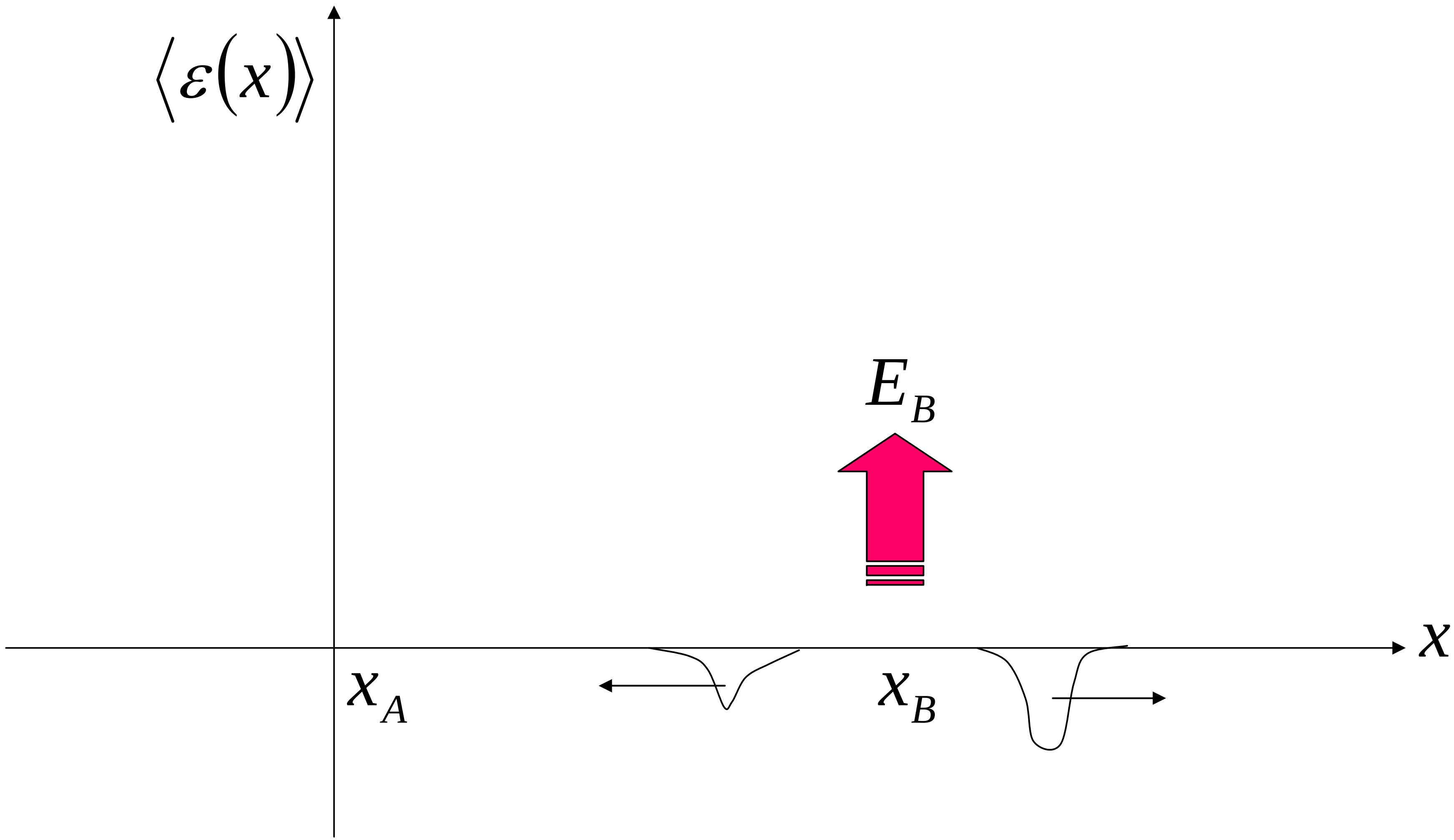
probability of α $$p_\alpha = Tr\left[|\psi\rangle\langle\psi| M_\alpha{}^* M_\alpha\right]$$

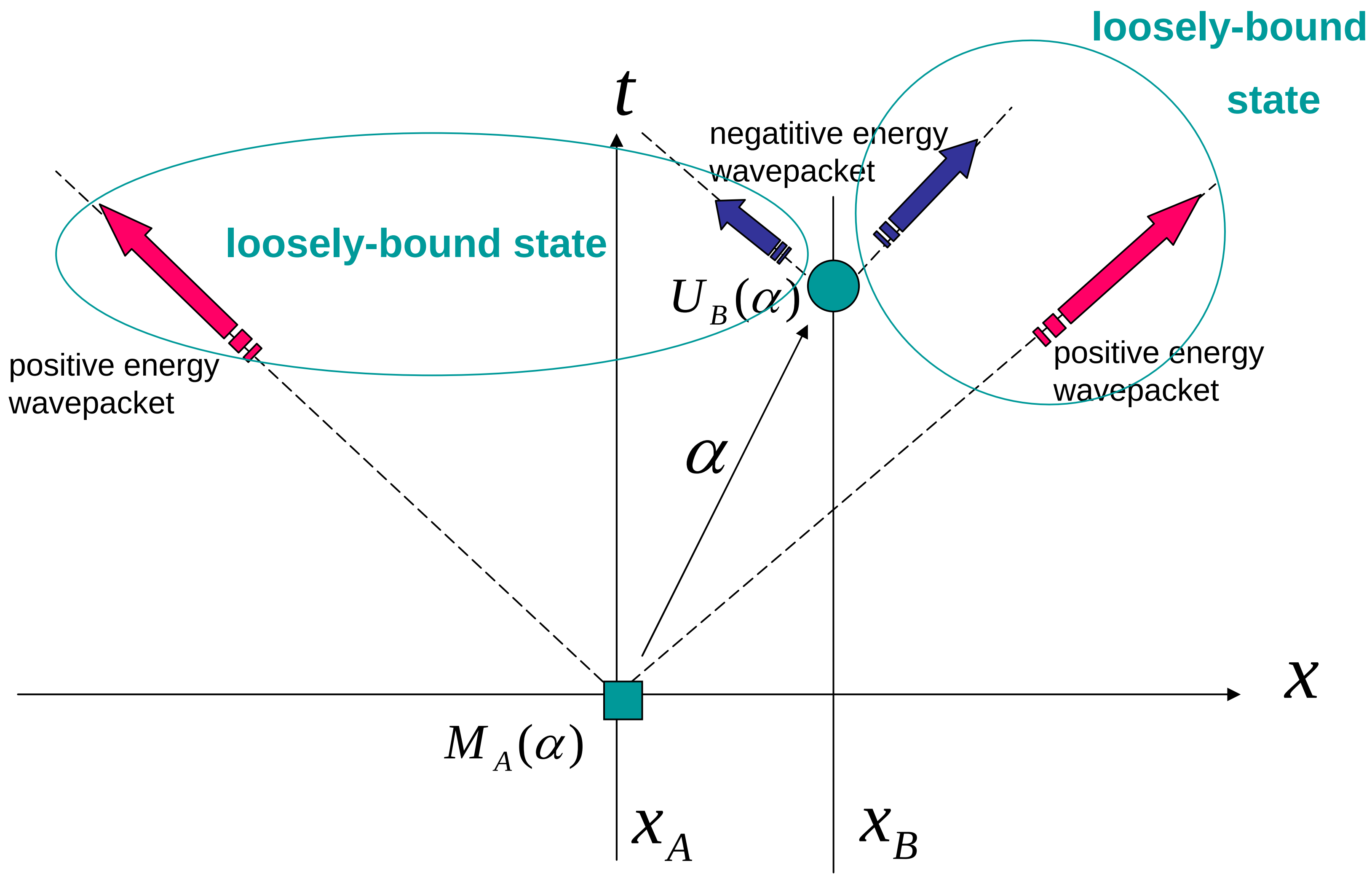
collapsed state $$\rho_\alpha = \frac{M_\alpha |\psi\rangle\langle\psi| M_\alpha{}^*}{\langle\psi| M_\alpha{}^* M_\alpha |\psi\rangle}$$

protocol


$\langle \varepsilon(x) \rangle$
α
POVM measurement
$M_A(\alpha)$
x
x_A
x_B
E_A

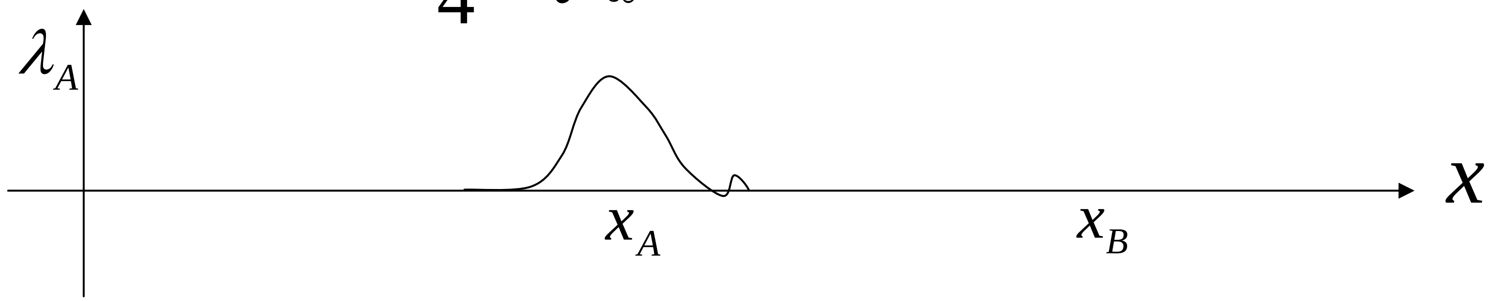

$\langle \varepsilon(x) \rangle$
α
$U_B(\alpha)$
x
x_A
x_B


⟨ε(x)⟩
E_B
x
x_A
x_B

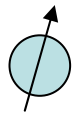


loosely-bound state
loosely-bound
state
t
negatitive energy
wavepacket
$U_B(\alpha)$
positive energy
wavepacket
positive energy
wavepacket
α
x
$M_A(\alpha)$
x_A
x_B

$$\Phi_A = \frac{\pi}{4} - \int_{-\infty}^{\infty} \lambda_A(x)\Pi(x)dx$$



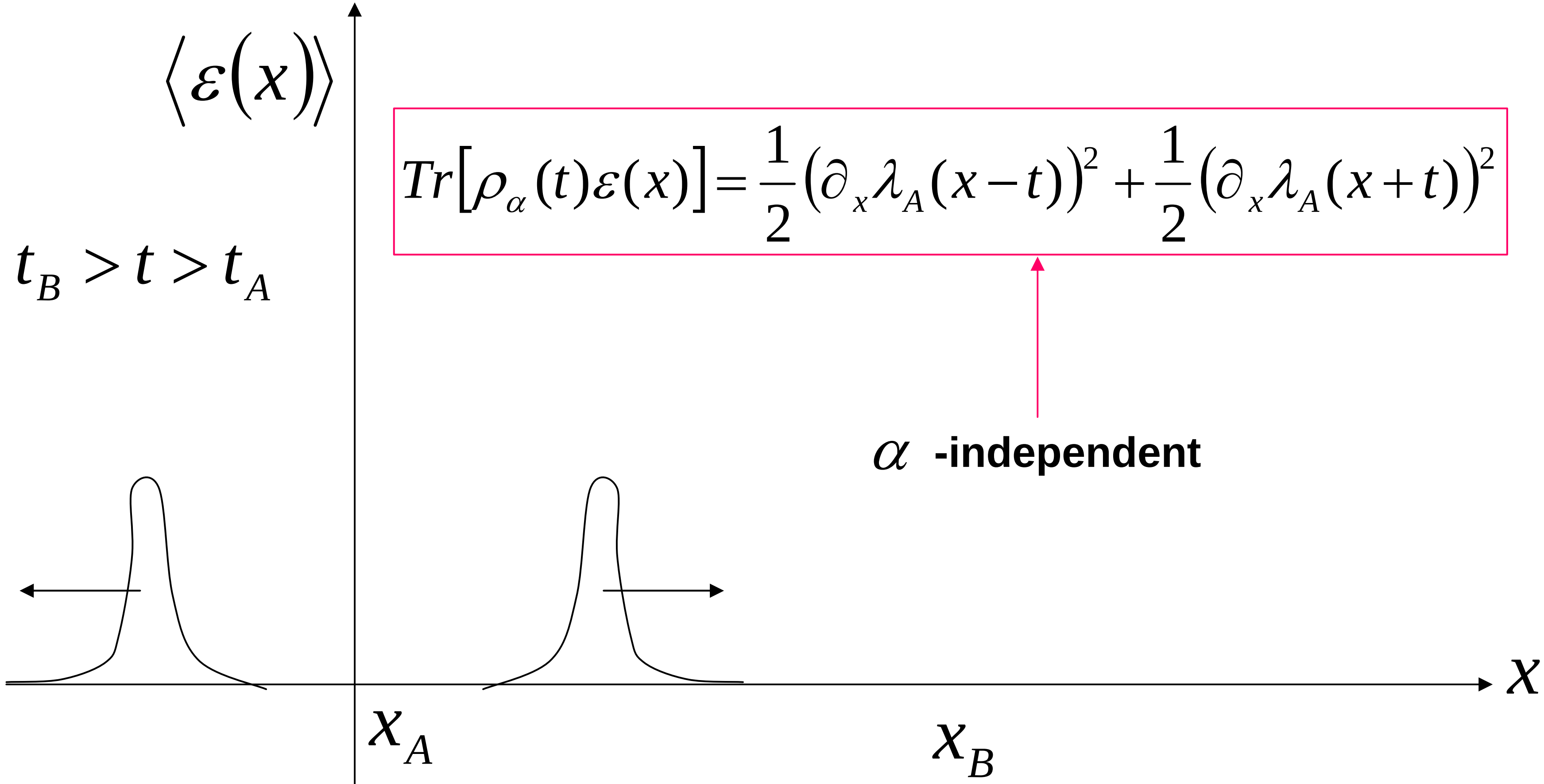
system × qubit at A

$\phi(t,x)\otimes$ 

$$V = \exp\left(-i\left(\Phi_A \otimes i\left[|1\rangle\langle 0|_P - |0\rangle\langle 1|_P\right]\right)\right)$$

$$M_A(\alpha)\rho M^*{}_A(\alpha) = Tr_P\left[\left(I \otimes |\alpha\rangle\langle\alpha|_P\right)V\left(\rho \otimes |0\rangle\langle 0|_P\right)V^*\right]$$

$$M_A(0) = \cos\Phi_A, \quad M_A(1) = \sin\Phi_A$$



$t_B > t > t_A$

$$\boxed{Tr[\rho_\alpha(t)\varepsilon(x)] = \frac{1}{2}\left(\partial_x \lambda_A(x-t)\right)^2 + \frac{1}{2}\left(\partial_x \lambda_A(x+t)\right)^2}$$

α **-independent**

Post-measurement

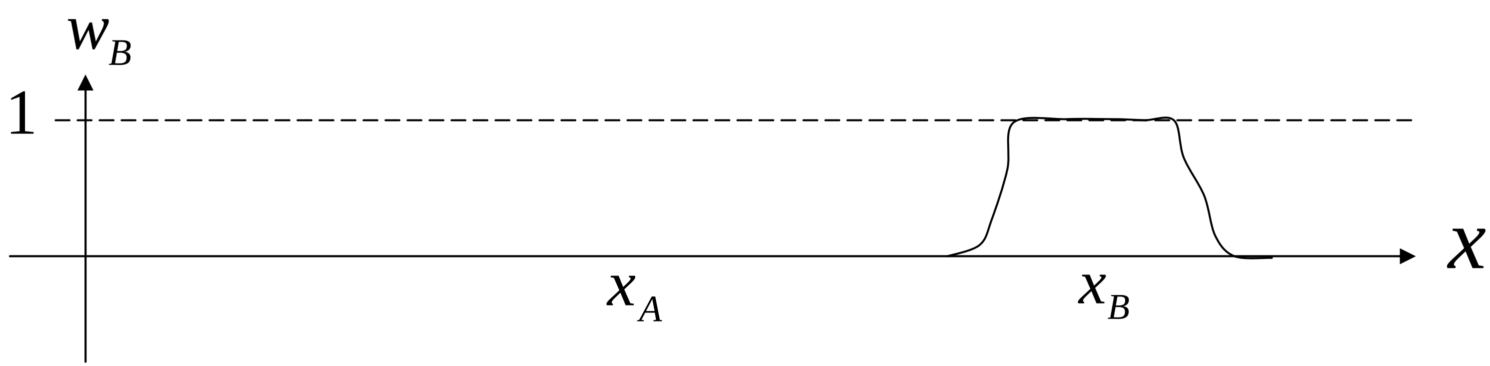
Cat States

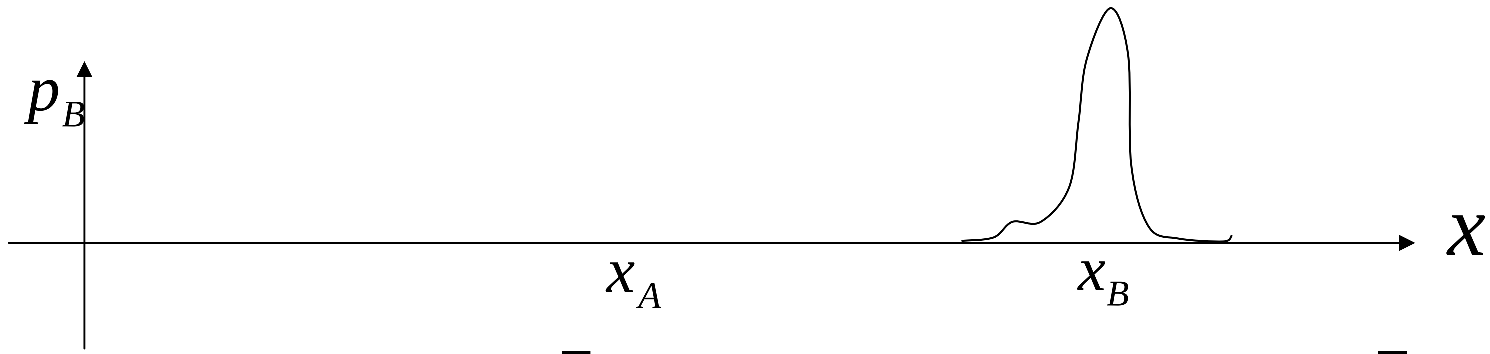
$$|\Psi_{\alpha=0}\rangle = \frac{1}{\sqrt{2}}\left(e^{\frac{i\pi}{4}}|\lambda_A\rangle + e^{-\frac{i\pi}{4}}|-\lambda_A\rangle\right)$$

$$|\Psi_{\alpha=1}\rangle = \frac{1}{\sqrt{2}}\left(e^{-\frac{i\pi}{4}}|\lambda_A\rangle + e^{\frac{i\pi}{4}}|-\lambda_A\rangle\right)$$

coherent state

$$H_B = \int_{-\infty}^{\infty} w_B(x)\varepsilon(x)dx$$





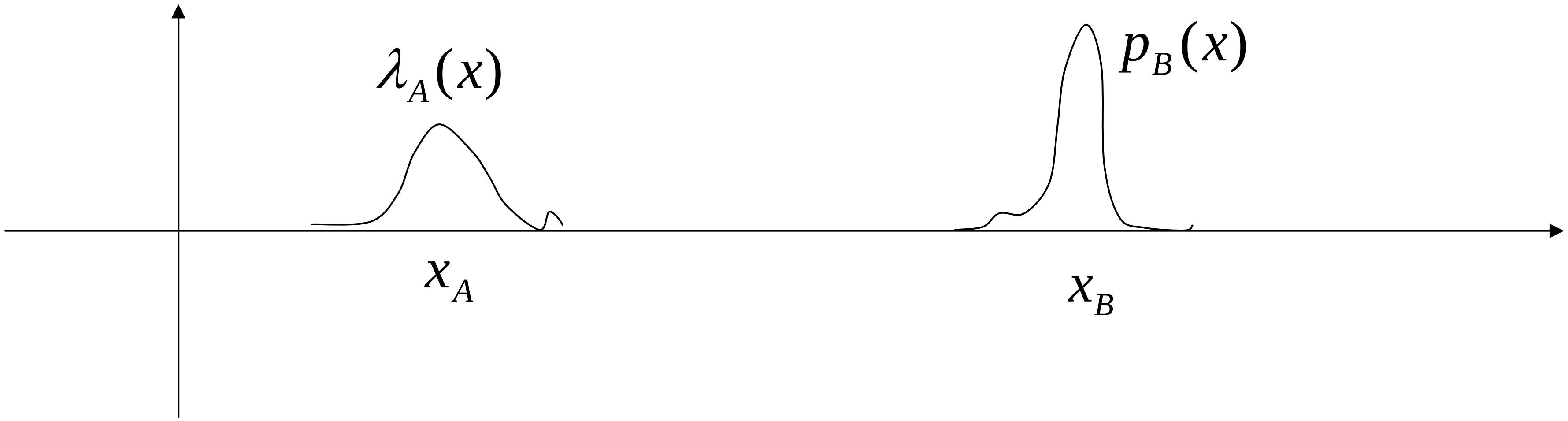
$$U_B(\alpha) = \exp\left[ig(-1)^{\alpha}\int_{-\infty}^{\infty} p_B(x)\phi(x)dx\right]$$

g **is fixed so as to extract the maximum energy.**

$$\rho = \sum_{\alpha=0,1} U_B(\alpha) U(t_B) M_A(\alpha) |0\rangle\langle 0| M^*{}_A(\alpha) U^*(t_B) U^*{}_B(\alpha)$$

$$E_B = -Tr\left[\rho H_B\right]$$

$$\boxed{E_B = \frac{\left|\langle 2\lambda_A | 0\rangle\right|^2}{2\pi^2 \int_{-\infty}^{\infty} p_B(x)^2 dx}\left[\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \lambda_A(x)\left[\frac{1}{(x-y-t_B)^2} + \frac{1}{(x-y+t_B)^2}\right] p_B(y)\right]^2}$$



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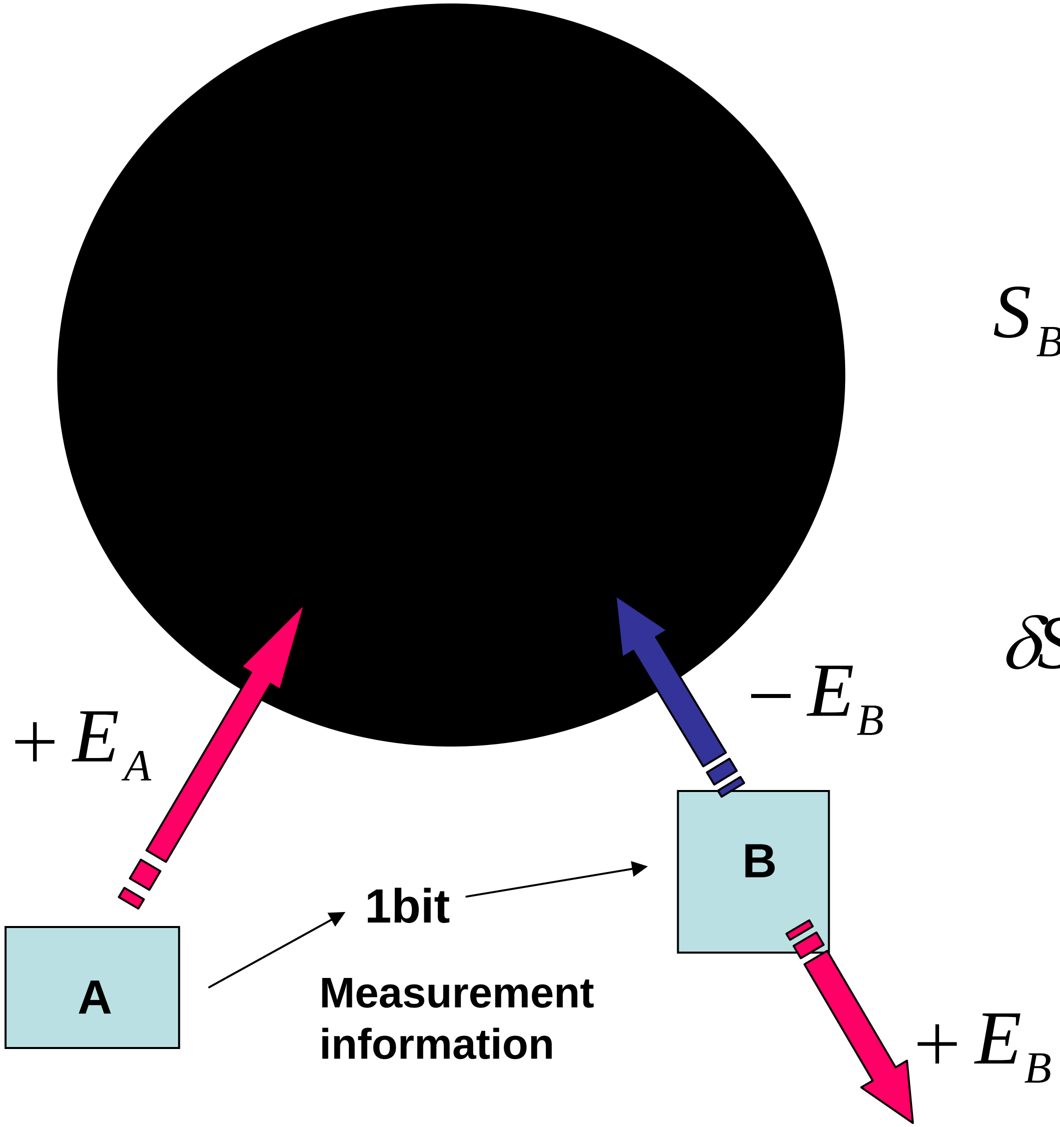
M.H. arXiv:0907.1378

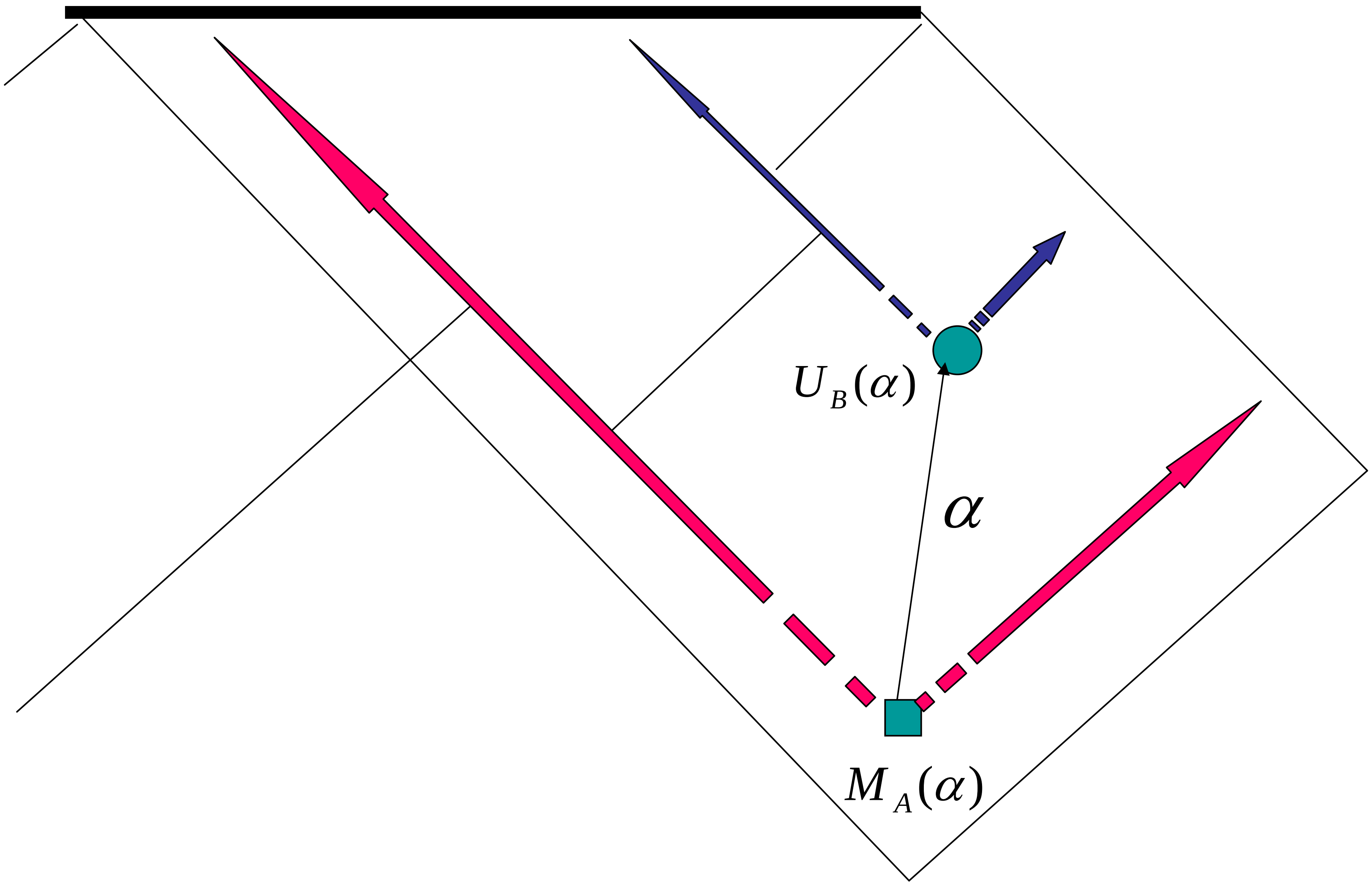


$$S_{BH} = \frac{1}{4G} A = 4\pi G M_{BH}{}^2$$

$$\delta S_{BH} = 8\pi G M_{BH} \delta M_{BH} = 8\pi G M_{BH} \left(E_A - E_B \right)$$

Detail analysis can be seen

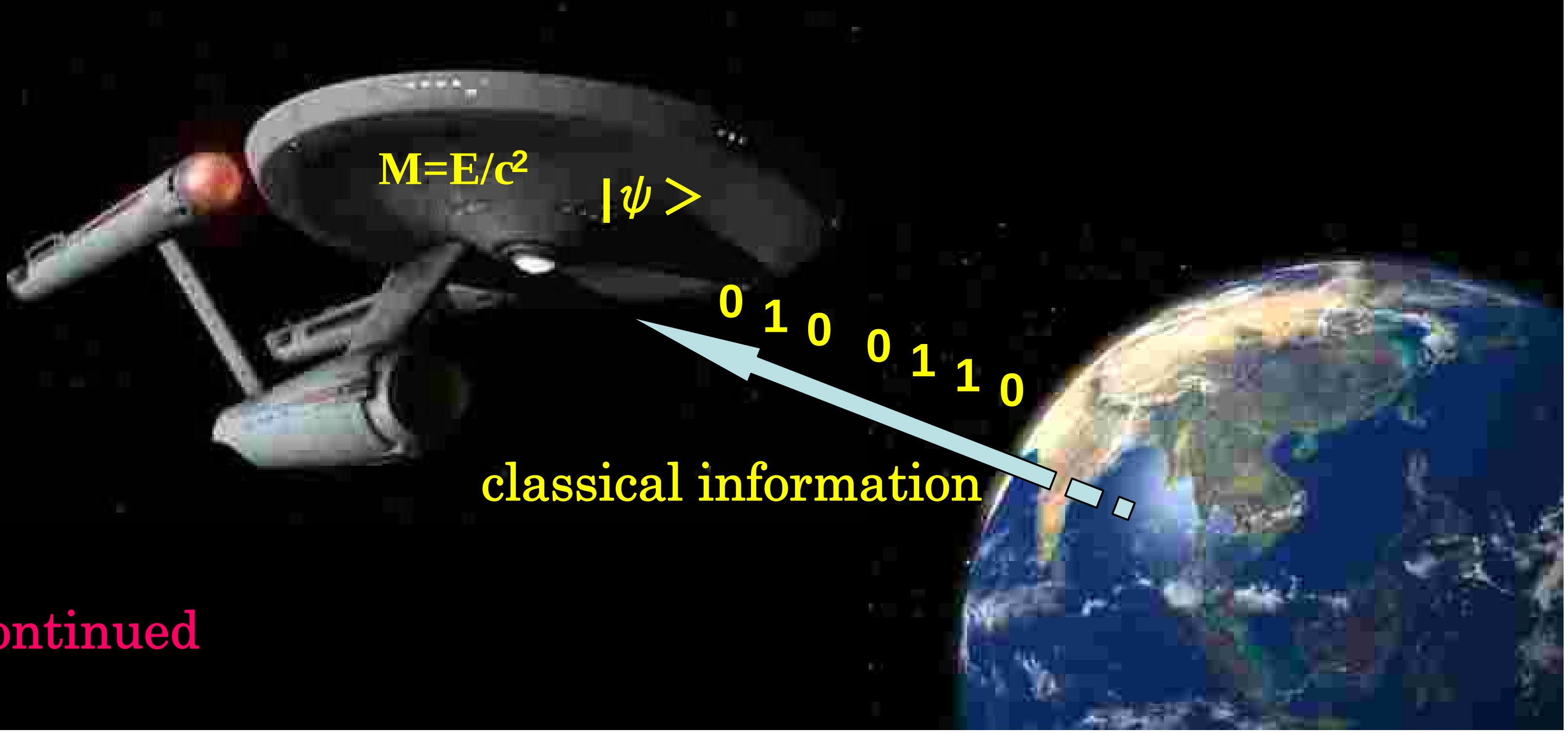
in the poster presentation !!


$U_B(\alpha)$
α
$M_A(\alpha)$

Thank you for your attention !

Transporter of U.S.S. Enterprise

"JUST JOKING !? "



To be continued