
知床

Phase Structure of unquenched (2+1)-dimensional QED and QCD by spectral function

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1 An alternative to Dyson-Schwinger eq

1 Old attemt by Bloch-Nordsieck near $p^2 = m^2$,

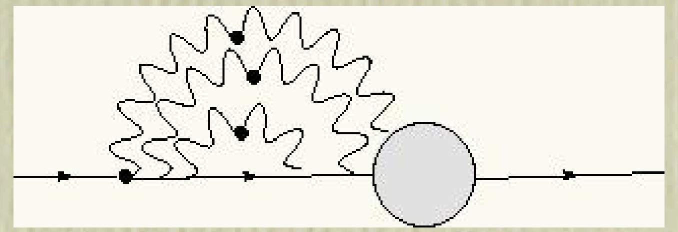
$$S_F(p) \simeq \frac{\gamma \cdot p + m}{m^2(1 - p^2/m^2)^{1-D}}, D = \frac{\alpha(d-3)}{2\pi}, \alpha = \frac{e^2}{4\pi}.$$

2 3-dim-th suggests validity in whole region.

3 Problems: severe infrared divergences with massless photon-$>$ cured by vacuum polarization,$1/N$ approximation. Is there any help in QCD ?

4 The result shows confinement: $Z_2^{-1} = 0$:absence of pole in time-like region

and super-fluidity $\left\langle \overline{\psi}\psi \right\rangle \neq 0$.



2 Soft-photon summation

Photon attached with external line is most singular by low-energy theorem

$$T_1 = -ie\frac{(r+k)\cdot\gamma + m}{(r+k)^2 - m^2}\gamma_\mu \epsilon^\mu(k,\lambda) \times \exp(i(k+r)\cdot x)U(r,s). \quad (1)$$

$O(e^2)$ spectral function F is given

$$F = \int \frac{d^3k}{(2\pi)^2}\delta(k^2)\theta(k_0)\exp(ik\cdot x)\sum_{\lambda,S} T_1\overline{T_1}. \quad (2)$$

In this case the spectral function ρ is given

$$\rho(p) = \int d^3x \exp(-ip\cdot x)\frac{\exp(-m\,|x|)}{4\pi|x|}\exp(F). \quad (3)$$

Model independent form

$$\sum_{\lambda,s} T_1\overline{T}_1 = -e^2(\frac{\gamma \cdot r}{m} + 1)[\frac{m^2}{(r \cdot k)^2} + \frac{1}{(r \cdot k)} + \frac{d-1}{k^2}].$$

To evaluate F we use photon propagator with bare mass as an infrared cut-off

$$\begin{aligned} D_F^{(0)}(x)_+ &= \int \frac{d^3k}{i(2\pi)^2}\delta(k^2-\mu^2)\theta(k^0)\exp(ik\cdot x) \\ &= \frac{\exp(-\mu\,|x|)}{8\pi i\,|x|}, \end{aligned} \qquad (4)$$

2.1 Evaluation of F

Using exponential cut-off we have

$$\begin{aligned} F &= ie^2m^2\int_0^\infty \alpha d\alpha D_F(x+\alpha r) - e^2\int_0^\infty d\alpha D_F(x+\alpha r) \\ &\quad - ie^2(d-1)\frac{\partial}{\partial\mu^2}D_F(x). \end{aligned} \qquad (5)$$

In quenced case for finite μ, F is written as

$$F = -\frac{e^2}{8\pi}\left(\frac{\exp(-\mu|x|)}{\mu} - |x|\,\mathrm{Ei}(\mu|x|)\right) - \frac{e^2}{8\pi\sqrt{r^2}}\mathrm{Ei}(\mu|x|)$$

$$-(d-1)\frac{e^2}{16\pi\mu}\exp(-\mu|x|), r^2 = m^2, \qquad (6)$$

where

$$\mathrm{Ei}(z) = \int_1^\infty \frac{\exp(-zt)}{t}dt, \qquad (7)$$

$$\mathrm{Ei}(\mu|x|) = -\gamma - \ln(\mu|x|) + (\mu|x|) + O(\mu^2). \qquad (8)$$

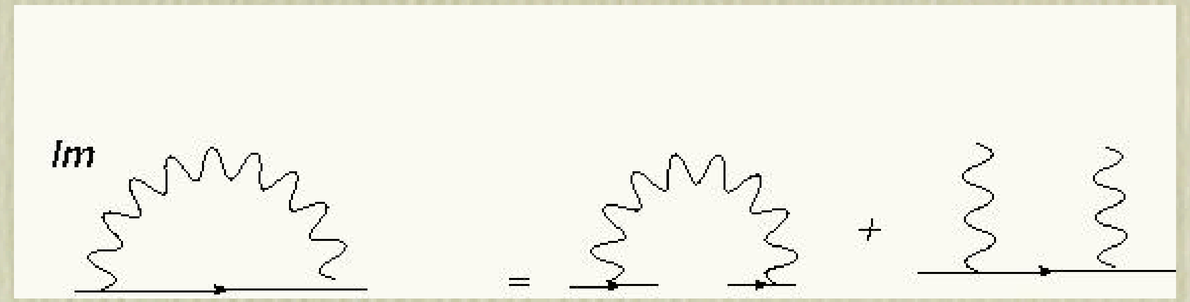
For the leading order in μ we have at short distance

$$F = \frac{(1+d)e^2}{16\pi\mu} + \frac{e^2\gamma}{8\pi m} + \frac{e^2}{8\pi m}\ln(\mu|x|) + \frac{e^2}{8\pi}|x|\ln(\mu|x|)$$

$$-\frac{e^2}{16\pi}|x|(d+1-2\gamma). \qquad (9)$$

where γ is Euler's constant and m is a physical mass.

$$m\bar{\rho}(x) = \frac{m\exp(-m|x|)}{4\pi|x|}\exp(F) \qquad (10)$$



There is mass shift and its log correction

$$\Delta m|x| = \frac{e^2}{8\pi}|x|\ln(\mu\,|x|) - \frac{e^2}{16\pi}|x|(d+1-2\gamma). \quad (11)$$

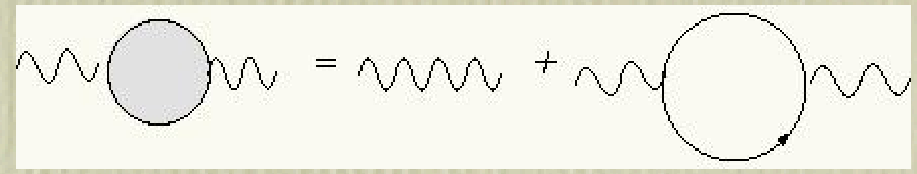
$\exp(F)$ is parametrized in the following form

$$\exp(F) = A(\mu\,|x|)^{D+C|x|}, \quad (12)$$

$$A = \exp(\frac{\gamma e^2}{8\pi m} + \frac{e^2}{16\pi\mu}(d+1)),$$

$$D = \frac{e^2}{8\pi m}, C = \frac{e^2}{8\pi}.\text{Gauge invariant.} \quad (13)$$

F **acts to change power of** $|x|$ **and mass.If** $D = 1, S_F(0) = finite.$



3 Vaccum polarization

$$\rho^{(0)}(s) = \delta(s-\mu^2) \to \rho_\gamma^F, c = \frac{e^2 N}{8}.$$

$$D_F(p) = \int_0^\infty \frac{\rho_\gamma^F(s)ds}{p^2 - s + i\epsilon} \tag{14}$$

$$\rho_\gamma^F(k) = \delta(k) + \frac{\mathrm{Im}\,\Pi(k)\theta(k^2-4m^2)}{\pi(-k^2+\mathrm{Re}\,\Pi(k))^2 + (\mathrm{Im}\,\Pi(k))^2}. \tag{15}$$

$$\begin{aligned}\Pi(k) &= -\frac{e^2 N}{8\pi}[(\sqrt{-k^2} + \frac{4m^2}{\sqrt{-k^2}})\ln(\frac{2m+\sqrt{-k^2}}{2m-\sqrt{-k^2}}) - 4m], \\ &= -\frac{e^2 N}{8} i\sqrt{-k^2}(-k^2 > 0, m = 0).\end{aligned} \tag{16}$$

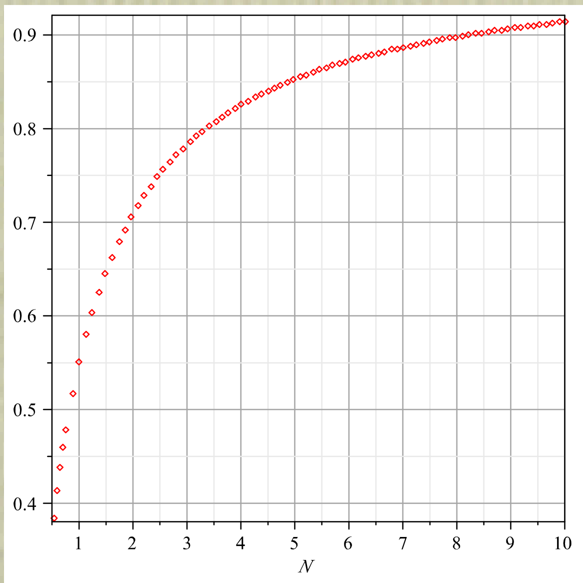


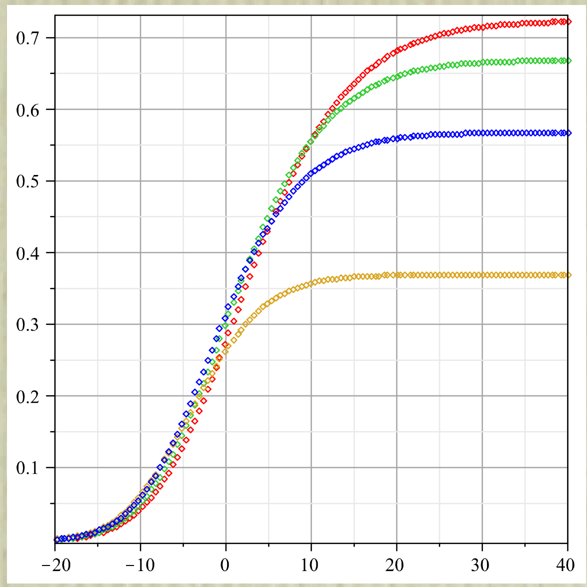
Figure 1: Z_3^{-1} for $N = 1/2..10$ in unit of $e^2 = 1$.

Assumption $\mathbf{m}_{phy} = \mathbf{c}/\mathbf{N}\boldsymbol{\pi}$.

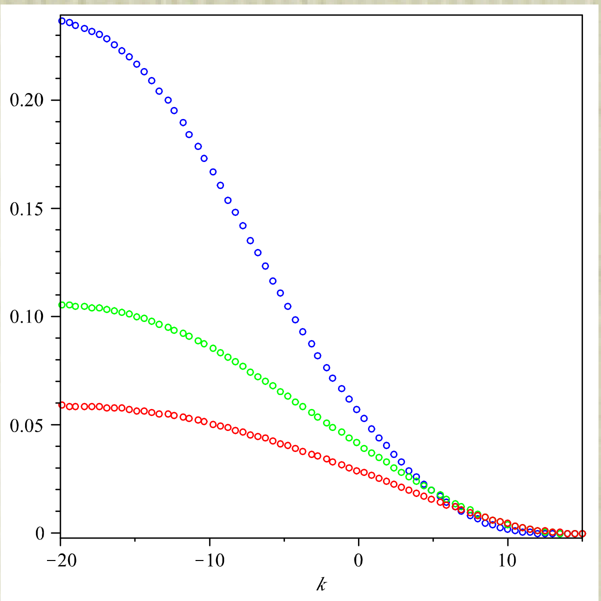
$$\exp(\widetilde{F}) = \int_0^\infty \rho_\gamma^F(s) \exp(F) ds$$

$$Z_3^{-1} = \int_0^\infty \rho_\gamma^F(s) ds. \qquad (17)$$

Coulomb force becomes weak for small N.



exp($\widetilde{F}$) for $N = 1..4, D = 1$,in unit of e^2.

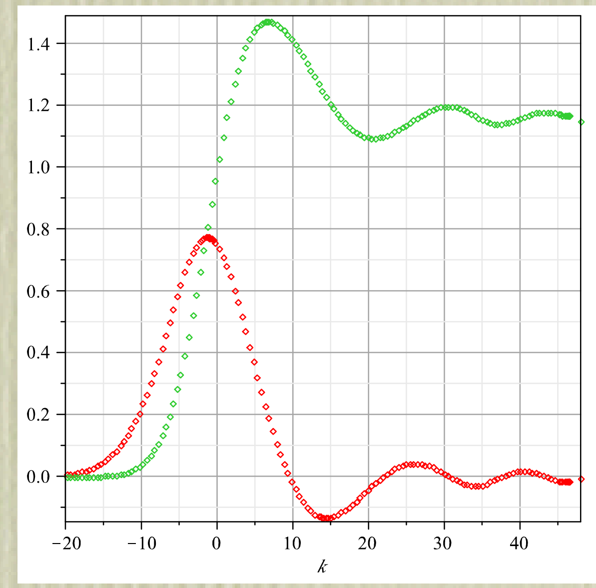


N dependence of the scalar part of $4S_F(x)$ for $D = 1$
in unit of e^2, $|x| = \ln(2/(1 - \mathrm{erf}(k/10)))$.

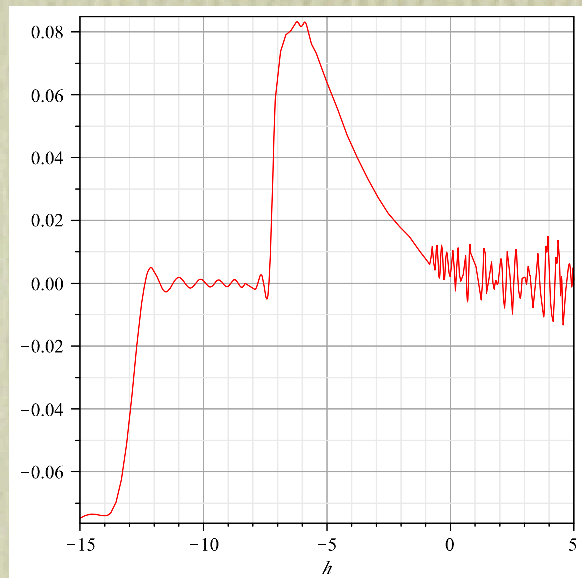
4 Minkowski region

$\sqrt{-x^2} \to iT, p^2/m^2 = s$

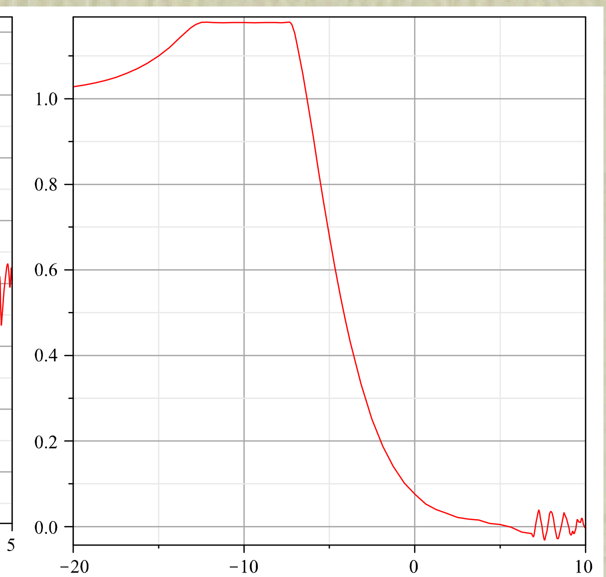
$$\rho(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dT e^{-i(s-1)T} \Im \exp(\tilde{F}(iT)) \qquad (18)$$



Real and imaginary part of $\exp(\tilde{F}(iT))$ for $N = 1, c = 1, d = -1, T = \exp(\pi/2 \sinh(k/10))$.



Spectral function $\rho(s)$ for $N = 1, c = 1, d = -1, s = \ln(2/(1 - \text{erf}(h/10)))$.

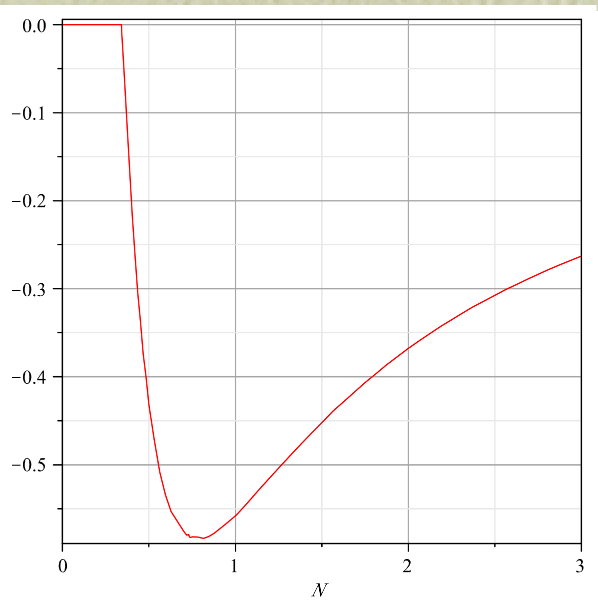


Real part of the propagator for $N = 1, c = 1, d = -1, p^2 = \ln(2/(1 - \text{erf}(h/10)))$.

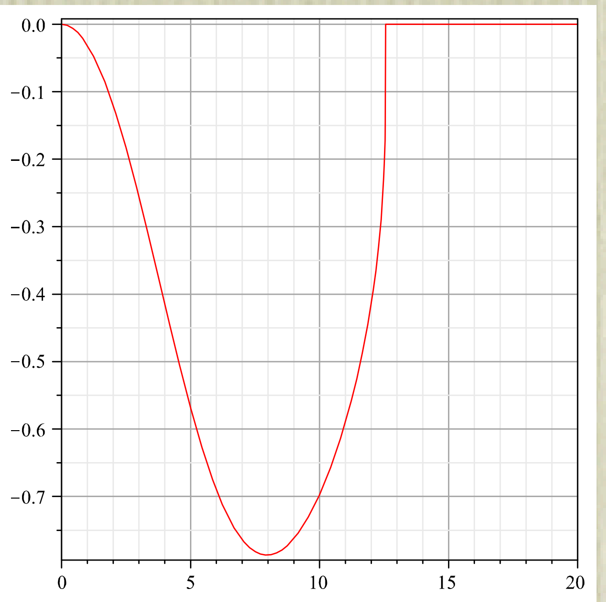
5 $\langle\overline{\psi}\psi\rangle$

$$\langle\overline{\psi}\psi\rangle = -4 \lim_{x\to 0} m\overline{\rho}(x).$$

We have $\left\langle \overline{\psi}\psi \right\rangle$ as a function of N and c in $1/N$.For small N or large c,fermion mass becomes large.In this case the photon spectral function has a quenched $\delta(s)$ which yields vanishment of $\left\langle \overline{\psi}\psi \right\rangle$.



$\left\langle \overline{\psi}\psi \right\rangle$ as a function of N for $c = 1$.



$\left\langle \overline{\psi}\psi \right\rangle$ as afunction of c for $N = 1$.Critical behavior.

For D=1, $m = c/N\pi$ we have $\left\langle \overline{\psi}\psi \right\rangle \approx -O(10^{-3})e^2$ which agrees Schwinger-Dyson analysis for small N.Chiral symmetry $U(2n)$, breaks dynamically into $SU(n)\times SU(n)\times U(1)\times U(1)$ as in QCD.

6 Summary

1 if we input mass, we have massshift,log correction and anomalous dimension which are gauge invariant.
2 $d = -1$,Including vacuum polarization we can avoid infrared divergences.
3 There may be a critical coupling for $\left\langle \overline{\psi}\psi \right\rangle \neq 0$.

4 broadning of spectral function at strong coupling.

5 in the future : massless QCD and toplogically massive QCD