Phase structure in PNJL models with dimensional regularization

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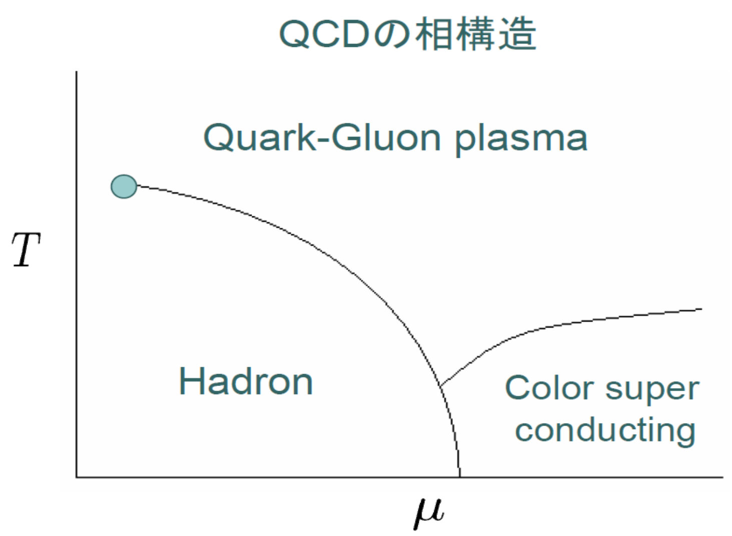
Introduction

PNJL model

NJL model (4-fermion)
＋
Polyakov loop(gluon)

4次元では繰り込み不可
↓
解析結果は正則化の方法による



Cut-off regularization

- Poincare対称性が壊れる
- Momentum cut-off: Λ?(>> μ?)

Dimensional regularization

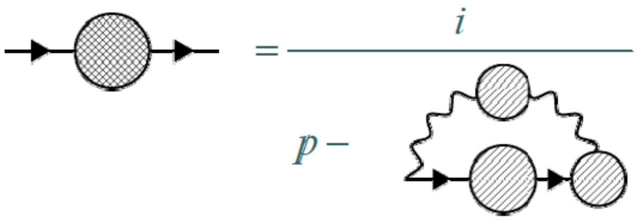
- Poincare対称性を壊さない
- Λを考えなくて良い

D=4で発散

↓

Dをパラメーターの1つと考える

D～2?



$$\int_0^\infty d^4k \frac{1}{k^2} \quad \Rightarrow \quad \int_0^\infty d^2k$$

D=4 Schwinger-Dyson equation:

ladder近似
IE(Instantaneous exchange))近似
など
↓
D=2 Gross-Noveu modelの

Model

Polyakov loop:

$$L(\vec{x}) = \mathcal{P} \exp\left\{ i \int_0^\beta d\tau\, A_4(\vec{x},\tau) \right\}$$

Effective lagrangian(1/N expansion):

$$\mathcal{L}_{eff} = \frac{\sigma^2}{2g} - V_{Polyakov}(l,\bar{l},T) - i\mathrm{Tr}\ln S^{-1}$$

$$V_{Polyakov} = -b \cdot T \left\{ 54\, e^{-a/T}\, \ell\,\bar{\ell} + \ln\left[1 - 6\,\ell\,\bar{\ell} - 3(\ell\,\bar{\ell})^2 + 4(\ell^3 + \bar{\ell}^3)\right] \right\}$$

$$S^{-1} = i\gamma_\mu\partial^\mu - \gamma_0 A^0 - \hat{M}$$

$$A^0 = -iA_4$$

$$\ell = \frac{1}{N_c}\langle \mathrm{tr} L\rangle\,, \quad \bar{\ell} = \frac{1}{N_c}\langle \mathrm{tr} L^\dagger\rangle$$

Effective Potential:

$$V_{PNJL}(\sigma, l, \bar{l}) = M_0^{4-D} V_{NJL}(\sigma, l, \bar{l}) + V_{Polyakov}(l, \bar{l})$$

$$V_{NJL} = \frac{\sigma^2}{4g} - \frac{A(D)}{D}[(m+\sigma)^2]^{D/2}$$

$$-\frac{2\sqrt{2}}{\beta(2\pi)^{(D-1)/2}\Gamma(\frac{D-1}{2})}\int_0^\infty k^{D-2}dk[ln\{1+e^{-3(\epsilon+\mu)\beta}$$

$$+3\bar{l}e^{-2(\epsilon+\mu)\beta}+3le^{-2(\epsilon+\mu)\beta}\}+ln\{1+e^{-3(\epsilon-\mu)\beta}$$

$$+3le^{-2(\epsilon-\mu)\beta}+3le^{-2(\epsilon-\mu)\beta}\}]$$

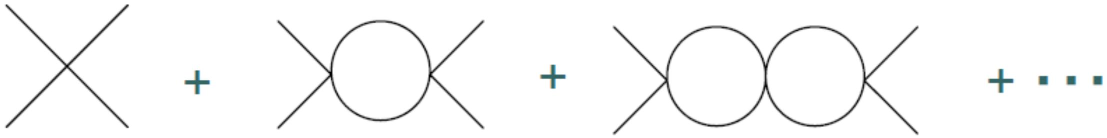
$$\varepsilon = \sqrt{k^2+(m+\sigma)^2}, m \equiv m_u = m_d$$

A(D)=2Nc Γ (1-D/2)/(2 π)$_{D/2}$

Renormalization :

In the leading order 1/N expansion,

$$G_s(p^2, \langle\sigma\rangle) =$$



$$= \frac{4g^2}{2g - \Pi_s(p)^2}$$

$\Pi_s(p^2)$:self-energy

Renormalization condition:

$$G_s(p^2 = 0, \langle\sigma\rangle = M_0) = Z_g G_s^r(p^2 = 0, \langle\sigma\rangle = M_0) = \frac{4g^r}{[(m + M_0)^2]^{D/2-1}}$$

M_0:renormalization scale, $g^r = Z_g(M_0)g$

Numerical calculation

Gap equations: $dV/d\sigma=0$, $dV/dl = 0$, $dV/dl = 0$

Parameters

NJL part:

Dimensional regularization:D=2.4, $m_u=m_d=4.5$MeV
($f_\pi=136$MeV, $m_\pi=93$MeV)

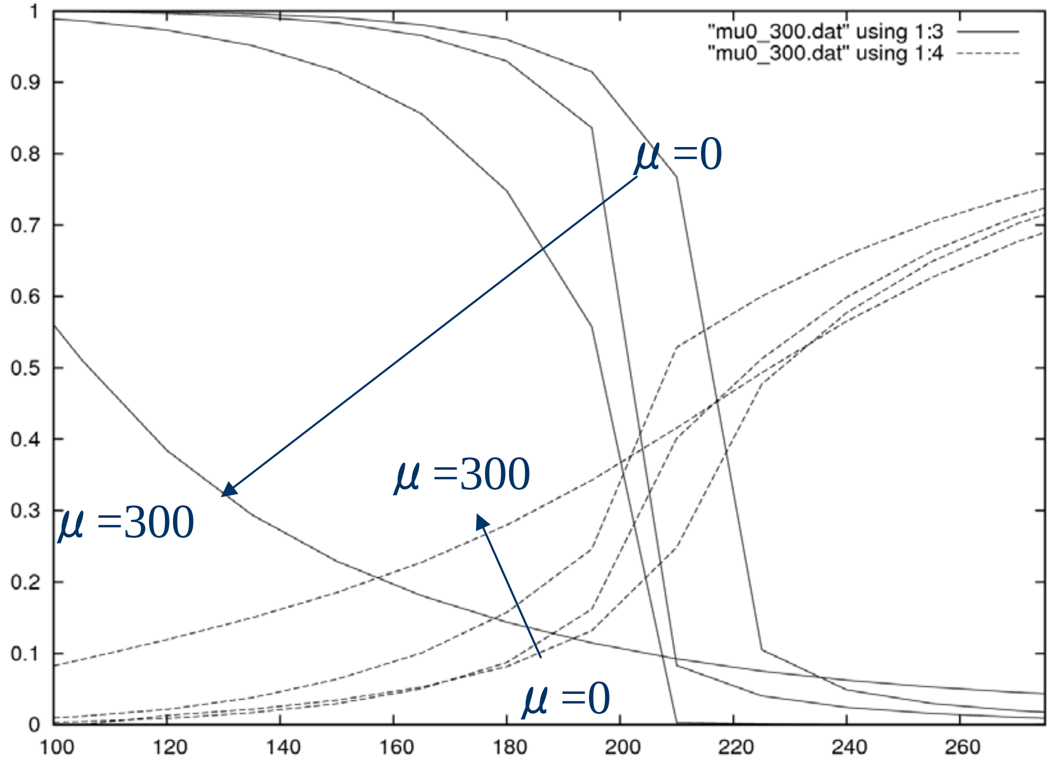
Cut-off regularization: $\Lambda=720$MeV, $m_u=m_d=4.5$MeV, $g_s\Lambda^2=3.67$[2]

Polyakovloop part:
a=664MeV, $b\ \Lambda^{-3}=0.03$[3]

[2]T.Inagaki,D.Kimura,A.Kvinikhidze,Phys,Rev.D77 116004
[3]Kenji Fukushima (Kyoto U., YukawaInst., Kyoto) . YITP-08-19, Mar 2008. 16pp. Published in Phys.Rev.D77:114028,2008, Erratum-ibid.D78:039902,2008.

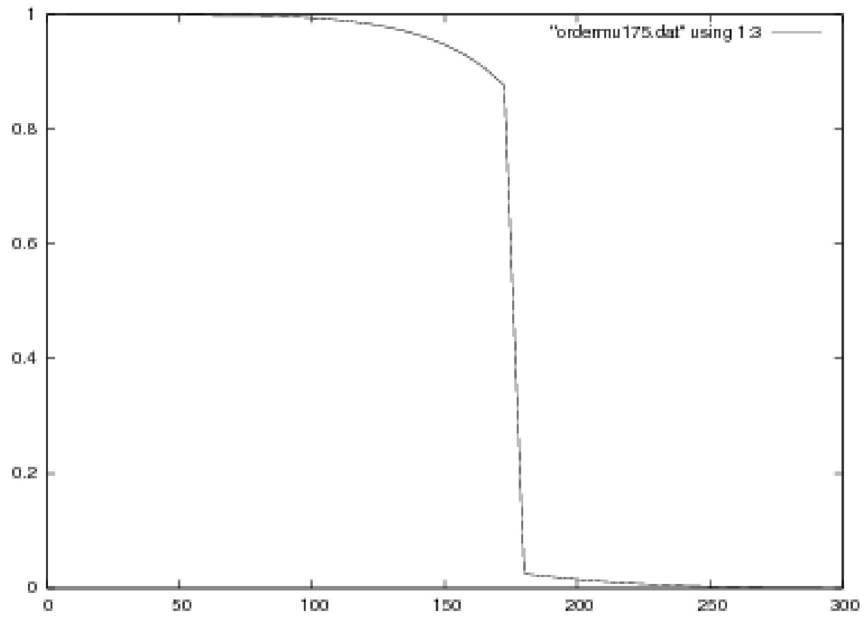
Cut-off regularization

$<\sigma>/<\sigma>_0$ <l>



μ =0,100,200,300MeV T(MeV)

$< \sigma > / < \sigma >_0$



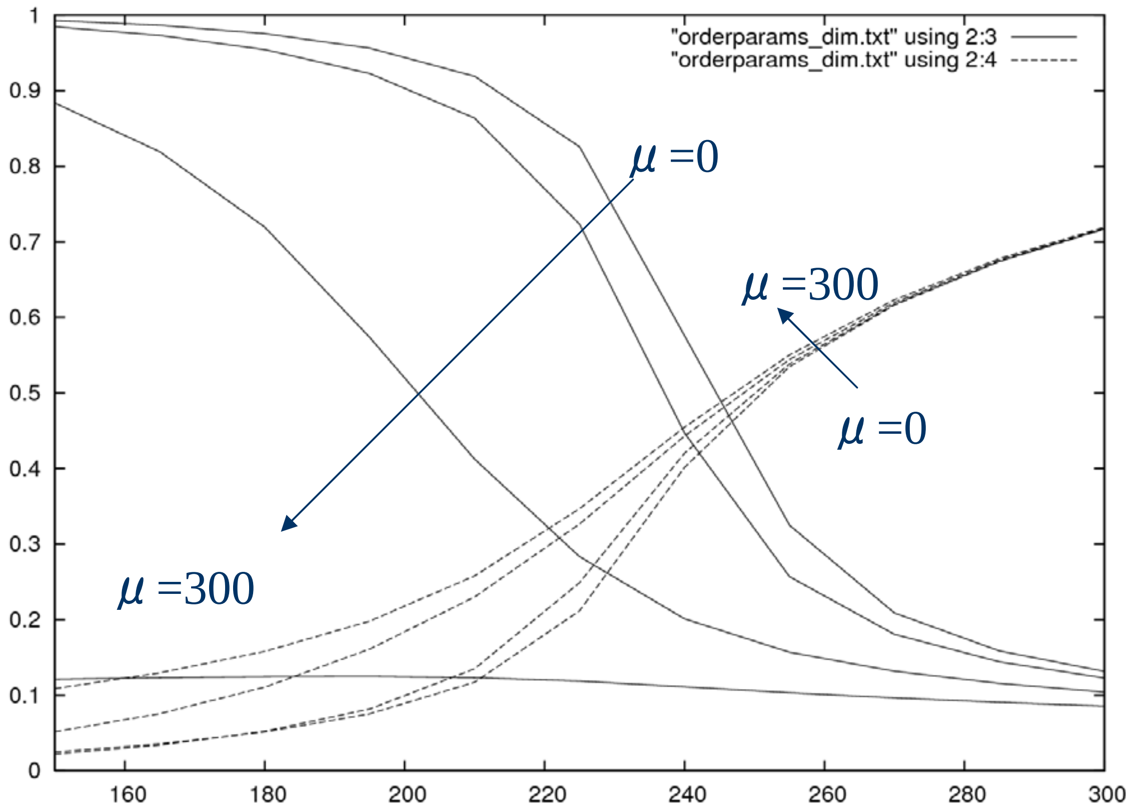
μ =175MeV

Order parameters $<\sigma>$、<l> at finite T:

Dimensional reguralization

$<\sigma>/<\sigma>_0$

<l>



T(MeV)

μ =0,100,200,300MeV

Summary and Outlook

- PNJL model(N_f=2)の解析において、正則化による結果の違いをCut-off、Dimensional regularizationの場合で比較した。
  - **Cut-off**
    - 非閉じ込めのオーダーパラメーター<l>とカイラル凝縮のオーダーパラメーター<σ>に対するTcrがほぼ一致
    - μ →largeでcross-over->1st-order->cross-over
  - **Dimensional**
    - Tcrにずれ(small μ)
    - μ ->largeで1sr-orderが現れない(?)
- これからの予定
- N_f=3(light m_u,m_d, heavy m_s)の場合
- quark mass(m_u,m_d)による相構造