Symmetry Breaking/Restoration by Wilson line in Warped or Flat Extra Dimension at Finite Temperature

(HH, In progress)

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Orbifold S^1/Z_2

- circle with identification : $y \to -y$ $(y \in [0, 2\pi R))$
- fundamental region : $[0, \pi R]$ (line element with boundary)
- fixed points : $y_0 = 0, \quad y_1 = \pi R$
- In this talk we consider
  1. $Minkowski_4 \times S^1/Z_2$ (factorizable metric)
  2. Randall-Sundrum spacetime (non-factorizable metric):

$$\begin{aligned} ds^2 &= e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 = \frac{1}{k^2 z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \\ & -\pi R \le y \le \pi R, \quad 1 \le kz \le e^{k\pi R} \end{aligned} \tag{1}$$

Gauge Theory on S^1/Z_2

Example :$SU(2)$

- Z_2 boundary conditions:

$$A_\mu(x^\mu, y_i - y) = +P_i A_\mu(x^\mu, y_i + y) P_i^{-1}, \tag{2}$$
$$A_y(x^\mu, y_i - y) = -P_i A_y(x^\mu, y_i + y) P_i^{-1}, \tag{3}$$
$$\psi_{\text{fd}}(x^\mu, y_i - y) = \eta\gamma_5 P_i \psi_{\text{fd}}(x^\mu, y_i + y), \quad \eta = \pm 1. \tag{4}$$

We choose $P_i = \text{diag}(1, -1)$

- zero modes :

$$A_\mu^{zeromode} \propto \text{diag}(1, -1) = \sigma_3, \tag{5}$$

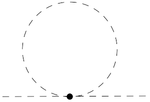
and one can set

$$A_y^{zeromode} \propto \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_2 \tag{6}$$

- Wilson line phase $\langle W \rangle = \oint g dy \langle A_y \rangle = \exp[i\theta\sigma_2]$ may break the gauge symmetry
(Hosotani mechanism)
  1. $\theta = 0$: $W = 1_2$, $U(1)_3 \to U(1)_3$
  2. $\theta = \pi$: $W = -1_2$, $U(1)_3 \to U(1)_3$
  3. otherwise : $U(1)_3 \to (broken)$

Hierarchy Problem and GHU

- Quantum correction to m_h^2 : quadratically diverges



$$\sim \Lambda^2, \quad \Lambda : \text{cutoff} \tag{7}$$

$$m_h^2 = m_{\text{bare}}^2 + \mathcal{O}(g^2\Lambda^2), \tag{8}$$

$$m_h = \mathcal{O}(100\text{GeV}), \tag{9}$$

$$m_{\text{bare}}, \Lambda = \mathcal{O}(10^{15}\text{GeV}) \gg m_h \tag{10}$$

$\rightarrow$ fine-tuning between m_{bare} and Λ.

Gauge -Higgs unification

- Gauge-Higgs unification:
  extra-dimensional component of the gauge field = the Higgs field

$$A_M = (A_\mu, A_y = h) \tag{11}$$

- gauge symmetry is broken by Hosotani mechanism.
- Effective potential and the Higgs-mass is finite.

Example - SU(3) model

Kubo-Lim-Yamashita (2001)

- $SU(3)$ on S^1/Z_2:

$$P_i = \mathrm{diag}(+1, +1, -1) \tag{12}$$

zero modes:

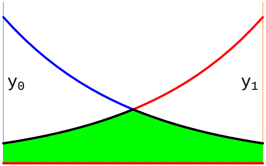
$$\begin{pmatrix} A_\mu^{3,8} & A_\mu^{1,2} & \\ A_\mu^{1,2} & A_\mu^{3,8} & \\ & & A_\mu^{8} \end{pmatrix}, \quad \begin{pmatrix} & & A_y^{4,5} = H_1 \\ & & A_y^{6,7} = H_2 \\ A_y^{4,5} = H_1^\dagger & A_y^{6,7} = H_2^\dagger & \end{pmatrix}. \tag{13}$$

$SU(2) \times U(1)$ gauge theory with doublet Higgs

Features of GHU on S^1/Z_2

- (warped and flat cases) Fermions in S^1/Z_2 extra space
  - zero-mode wave-function : domain-wall profile due to bulk mass term
  - Yukawa-coupling : overlap of wavefunctions of fermions and gauge zero modes:

$$H(x)\bar{\psi}_R(x)\psi_L(x) = \int dy \bar{\psi}_R(x,y) A_y(x,y) \psi_L(x,y) \tag{14}$$



$\rightarrow$ lightest-mode mass depends exponentially on the bulk mass parameter!

- (warped case) Higgs effective potential (and Higgs mass) are enhanced.

$$m_h \sim \mathcal{O}(100\text{GeV}) \tag{15}$$

Finite-Temperature Effects on GHU

Feature(s):

- cause first-order phase transition (Ho-Hosotani(1995), Takenaga et.al, ...)
  → may enable electroweak baryogenesis

Dynamics:

- Effective potential

$$V_{\rm eff}^{\{b,f\}} = \int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \sum_{\ell} \ln\left[\left(\frac{2\pi(n+\{0,\frac{1}{2}\})}{\beta}\right)^2 + \vec{p}^2 + m_\ell^2\right] \quad (16)$$

- When ExD=S^1 for fermion without bulk mass term,

$$m_\ell^2 = \left(\frac{\ell+\theta/2\pi}{R}\right)^2 \quad (17)$$

  → one may use many tricks (Poisson sum formula, etc...)
- However, for S^1/Z_2 the summation in m_ℓ is rather complicated.
  → Hereafter we follow the [Brevik-Odintsov-Nojiri et.al(2001)]
  ($V_{\rm eff}$ with respect to the radion VEV)

KK-mode summation

- summation in m_ℓ by means of contour-integral:

$$E^{KK} = \mathcal{F}[E(p,x)], \tag{18}$$

$$\mathcal{F}[f(p,x)] = V_{d-1} \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{i}{2\pi} \oint_C dx \frac{d}{dx} f(p,x) \ln[g(x)], \tag{19}$$

$$g(x_n) = 0, \quad m_n = \kappa x_n. \tag{20}$$

C : a set of circles surrounding $x = x_n$ in counter-clockwise.

KK conditions:

- warped (Hosotani-Noda etal, 2005)

$$\begin{aligned} 0 &= \lambda_n^2 z_1 F_{\alpha-1,\alpha-1}(\lambda_n, z_1) F_{\alpha,\alpha}(\lambda_n, z_1) - \frac{4}{\pi^2}\sin^2\frac{\theta}{2}, \\ & m_n = k\lambda_n, \quad \alpha = (M/k) + \frac{1}{2}, \\ & F_{\alpha,\beta}(\lambda, z) = Y_\beta(\lambda) J_\alpha(\lambda z) - J_\beta(\lambda) Y_\alpha(\lambda z) \end{aligned} \tag{21}$$

- flat (HH, preliminary)

$$\begin{aligned} 0 &= (\cos\theta - 1)\bar{M}^2 - \bar{m}_\ell^2 \left[\cos(2\pi\sqrt{\bar{m}_\ell{}^2 - \bar{M}^2}) - \cos\theta\right] \\ & \bar{M} = MR, \quad \bar{m}_\ell = m_\ell R. \end{aligned} \tag{22}$$

♠ Lowest mode has domain-wall like wave-function.

High-temperature expansion

We subtract zero-temperature ($\beta = \infty$) contribution from $E_d^{b,f}$:

$$\tilde{E}_d^{b,f}(\beta) = E_d^{b,f}(\beta) - E_d^{b,f}(\beta = \infty), \tag{23}$$

For $d = 4$ we have

$$\tilde{E}_4^b = \frac{V_3}{2\pi^2\beta^4}\left[\frac{\pi^4}{15}A_0(\theta) - \frac{\pi^2\beta^2}{12}A_{-2}(\theta) + \mathcal{O}(\beta^4)\right], \tag{24}$$

$$\tilde{E}_4^f = \frac{V_3}{2\pi^2\beta^4}\left[\frac{7\pi^4}{120}A_0(\theta) - \frac{\pi^2\beta^2}{12}A_{-2}(\theta) + \mathcal{O}(\beta^4)\right], \tag{25}$$

where

$$A_s(\theta) \equiv \sum_n x_n^{-s} = \frac{s}{\pi}\sin(\frac{\pi s}{2})\int_0^\infty dt\, t^{-s-1}\ln[\tilde{F}(t,\theta)] \tag{26}$$

with $\tilde{F}(t) = F(it)$

Zero-temperature results

- warped (HH(2007), Yamashita-Haba-Okada-Matsumoto (2008))
  $a \equiv e^{-k\pi R}$

$$E_4^{KK}(\beta=\infty) = \frac{k^4 a^4 V_3}{16\pi^4}\int_0^\infty dt\, t^3 \ln[\tilde{F}_{\rm warped}] \tag{27}$$

$$\tilde{F}_{\rm warped}(\theta,\nu) = 2I_\nu(t)I_{\nu-1}(t)K_\nu(at)K_{\nu-1}(at)+\cdots + 2I_\nu(at)I_{\nu-1}(at)K_\nu(t)K_{\nu-1}(t) - \frac{\cos\theta}{at^2} \tag{28}$$

- flat (HH, preliminary)

$$E_4^{KK}(\beta=\infty) \propto \int_0^\infty dt\, t^3 \ln[\tilde{F}_{\rm flat}] \tag{29}$$

$$\tilde{F}_{\rm flat} = \left[(\cos\theta-1)\mu^2 + t^2\{\cosh(2\pi\sqrt{t^2+\mu^2}) - \cos\theta\}\right] \tag{30}$$

non-zero temperature results

Preliminary...

1. warped

$$A_0(\theta) = \ln[1+\cos\theta] \quad (31)$$
$$A_{-2}(\theta) = (\theta\text{-independent}) \quad (32)$$

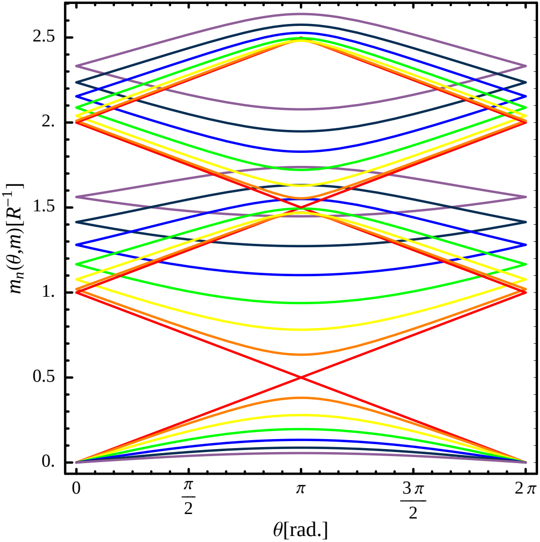
- $V_{\rm eff}$ has a singularity at $\theta = \pi$!

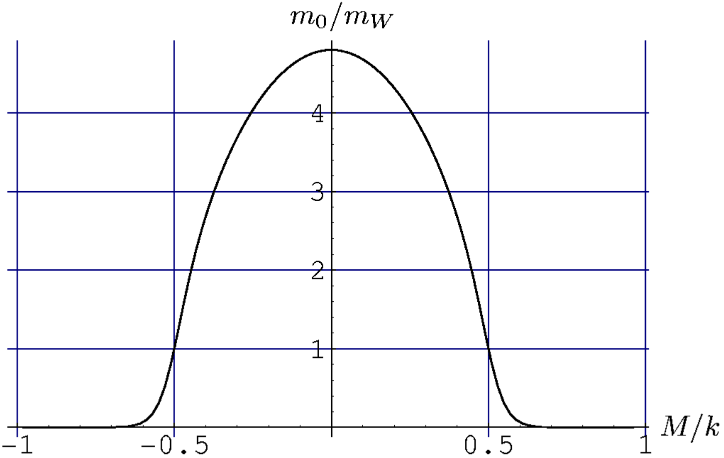
2. flat
still in progress...

$$A_0(\theta) = 1? \quad (33)$$
$$A_{-2}(\theta) = (\theta\text{-independent})? \quad (34)$$

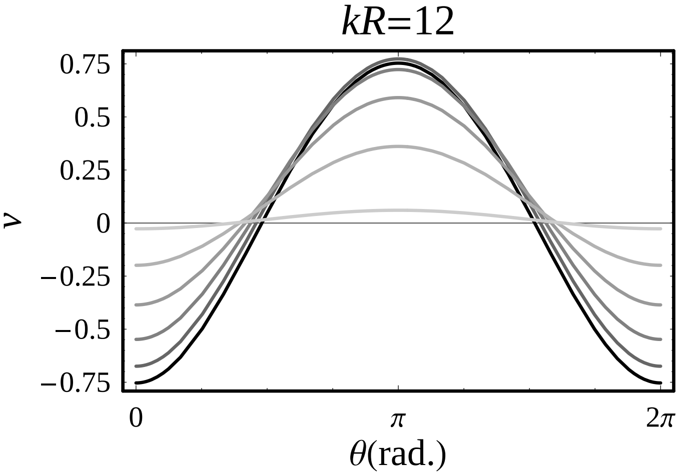
backup slides

First 5 KK masses : $m_n(\theta, m)$,
$n = 1, 2, 3, 4, 5$, red $[m = 0.0]$ orange $[m = 0.2]$ yellow $[m = 0.4]$ green $[m = 0.6]$ blue $[m = 0.8]$ indigo $[m = 1.0]$ violet $[m = 1.2]$)





[Hosotani-Noda-Sakamura-Shimasaki]



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