電子正孔系における
励起子モット転移と量子凝縮

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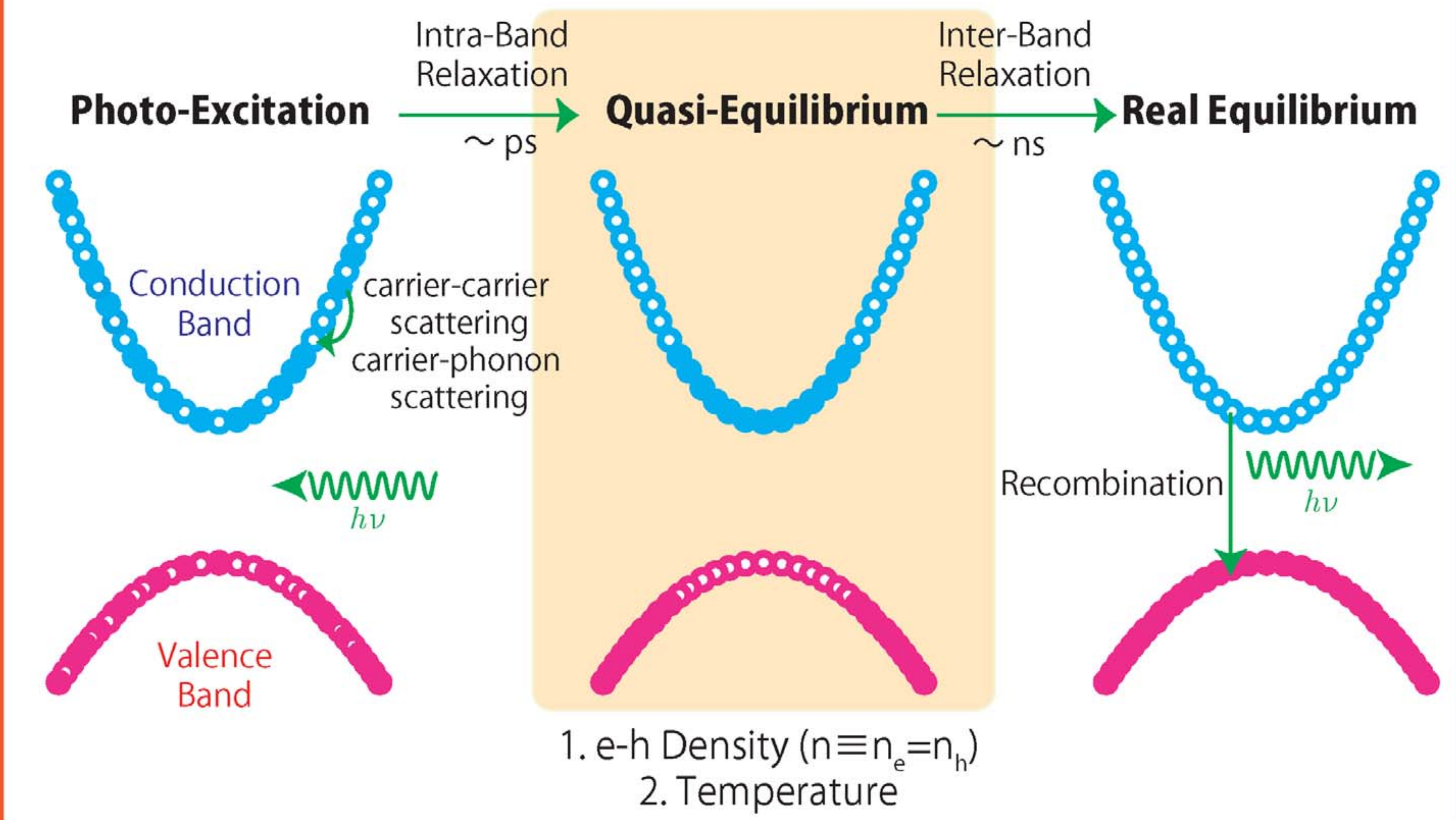
目次

Optical Science of Dynamically Correlated Electrons

Concept of Quasi-Equilibruium



Exciton

Exciton (Bound state of 1e and 1h) ⇒ Analog of H atom

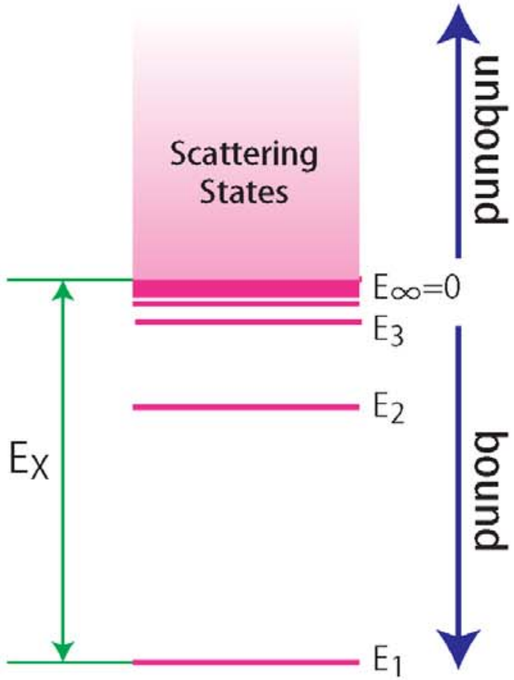
Relative motion between an electron and a hole
⇒ Bound state induced by the attractive Coulomb interaction

"quasi-Boson"

Exciton Bohr radius: $a_B = \dfrac{\hbar^2 \epsilon}{m_r e^2}$

Exciton energy levels: $E_n = -E_X \dfrac{1}{n^2} \quad (n = 1, 2, 3, \cdots)$

Exciton binding energy: $E_X = \dfrac{e^4 m_r}{2\epsilon^2 \hbar^2}$



Exciton in semiconductors v.s. H atom

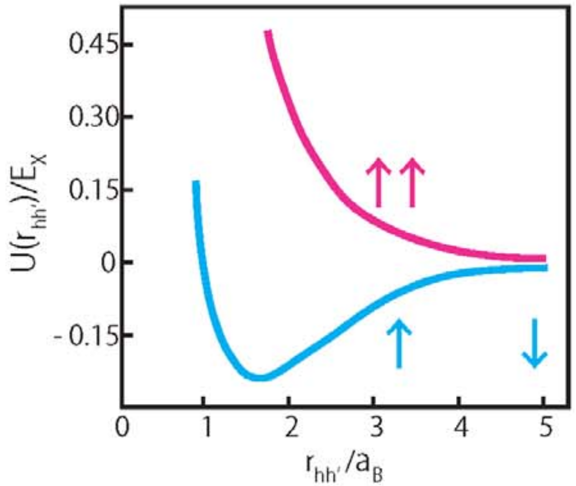
Reduced mass ～×1/10
Dielectric const.～×10
⇒

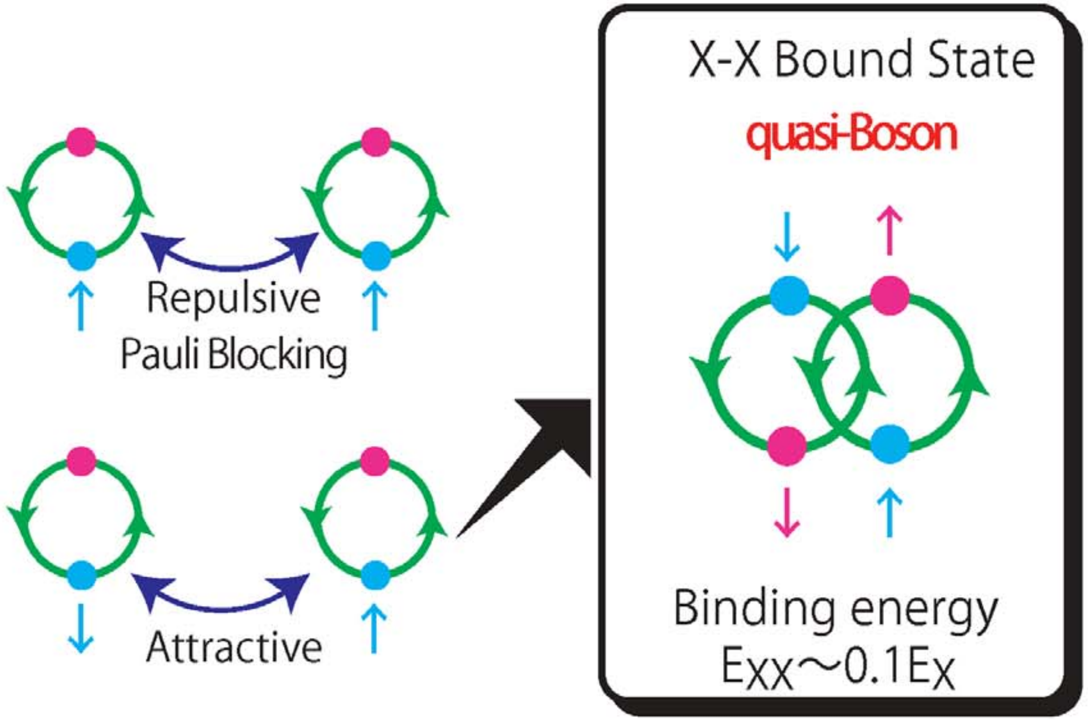
Binding Energy	Bohr radius
～×1/1000	～×100
～10meV	～10nm

Biexciton

Biexciton ⇒ Analog of H_2 molecule

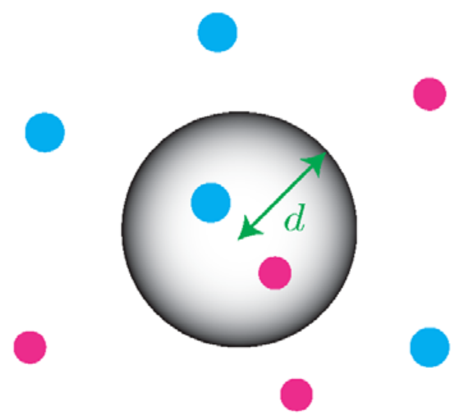
Exciton-Exciton Interaction Potential (Heavy hole mass limit)





Two electrons and two holes form spin-singlets
→ Orbital wave function without node

Parameters of Many Electron-Hole Systems



Energy Scales:

Kinetic energy: $K = \frac{p_F^2}{2m_e} + \frac{p_F^2}{2m_h} \sim \frac{(\hbar/d)^2}{m_r} \propto d^{-2}$

Interaction energy: $U = \frac{e^2}{\epsilon d} \propto d^{-1}$

Temperature: $k_B T$

Volume per a pair

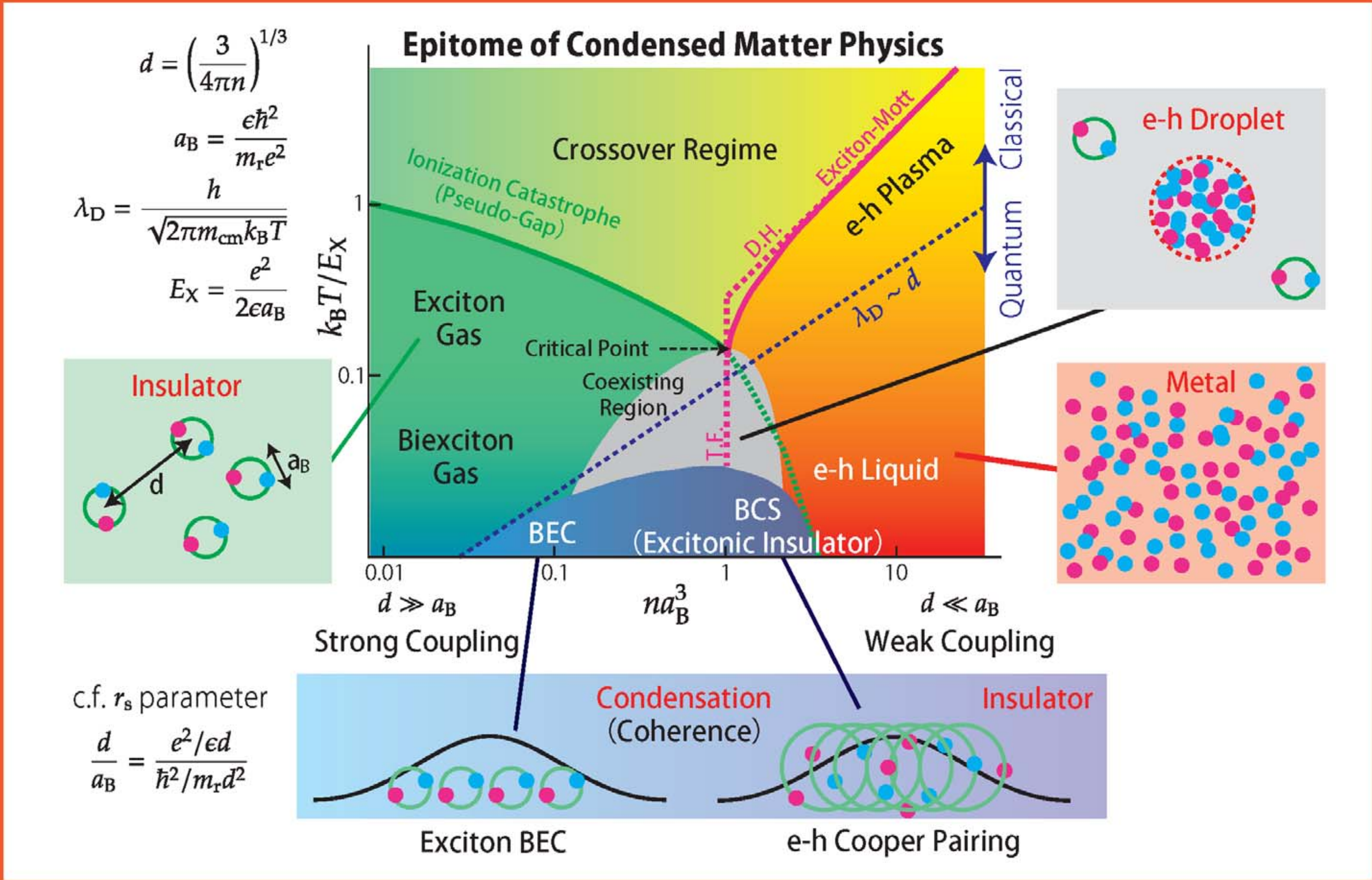
$$= \frac{1}{n_{eh}} = \frac{4\pi}{3} d^3$$

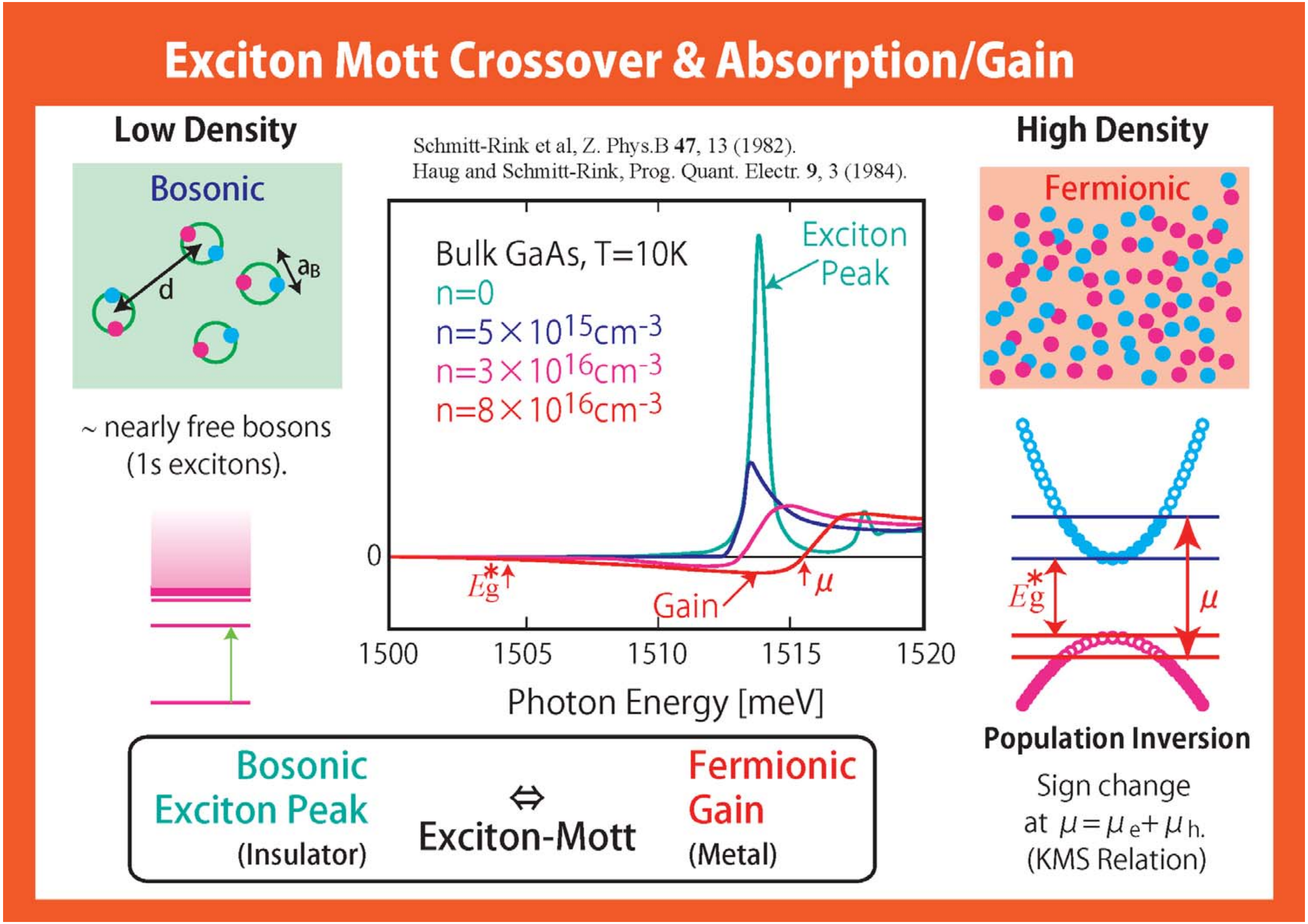
r_s parameter: $r_s = \frac{d}{a_B} \sim \frac{U}{K}$

Temperature: $\frac{k_B T}{E_X}$

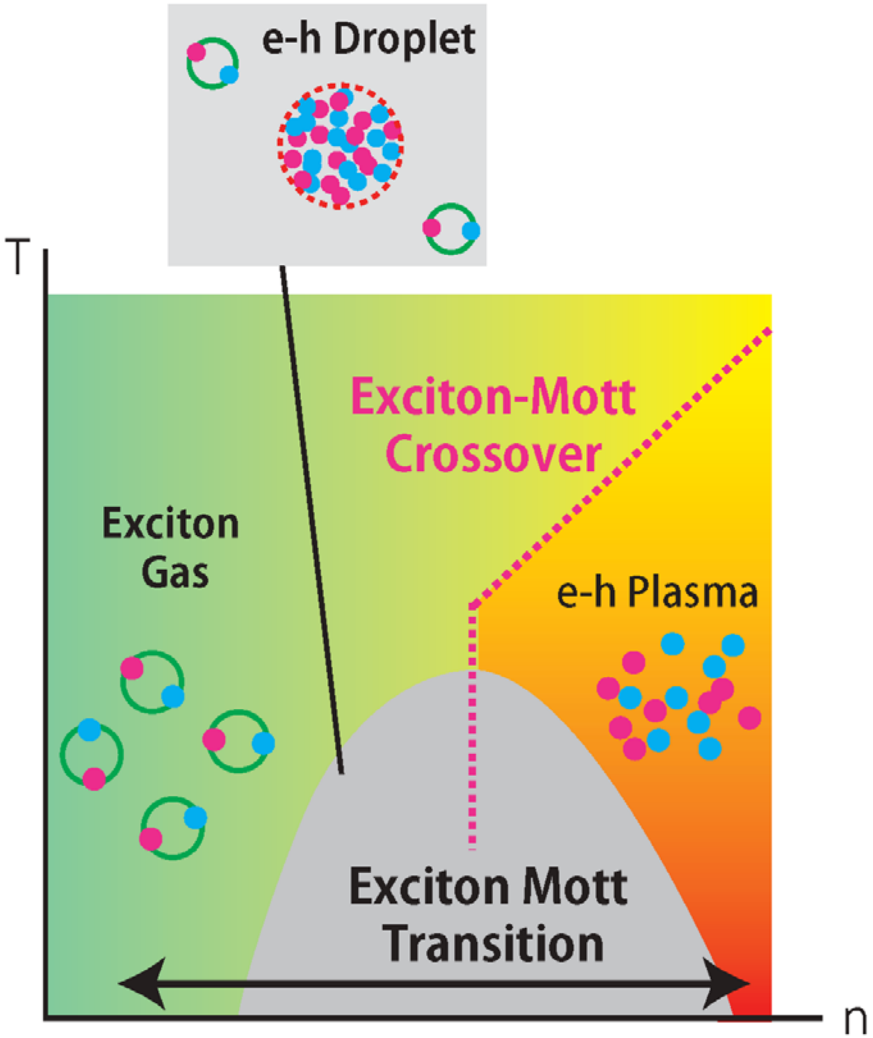
1. Interaction strength ⇔ e-h density
2. Low density = Strong Coupling
   High density = Weak Coupling

Phase Diagram of 3D e-h Systems (Schematic)

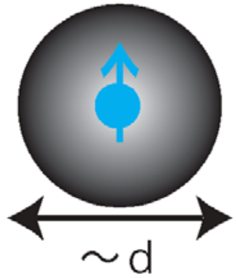



Exciton Mott Crossover & Absorption/Gain
Low Density
Bosonic
d
aB
~ nearly free bosons
(1s excitons).
Schmitt-Rink et al, Z. Phys.B 47, 13 (1982).
Haug and Schmitt-Rink, Prog. Quant. Electr. 9, 3 (1984).
Bulk GaAs, T=10K
n=0
n=5×10^15cm^-3
n=3×10^16cm^-3
n=8×10^16cm^-3
Exciton
Peak
0
Eg*
Gain
μ
1500
1505
1510
1515
1520
Photon Energy [meV]
High Density
Fermionic
Eg*
μ
Population Inversion
Sign change
at μ=μe+μh.
(KMS Relation)
Bosonic
Exciton Peak
(Insulator)
⇔
Exciton-Mott
Fermionic
Gain
(Metal)

Exciton-Mott Transition/Crossover



Exchange Hole (HF Approximation)

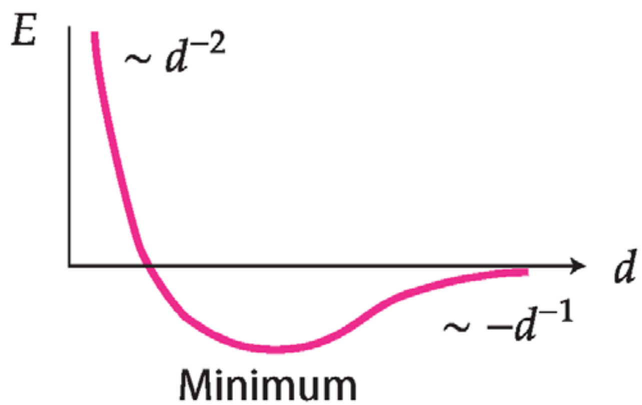


Energy decrease

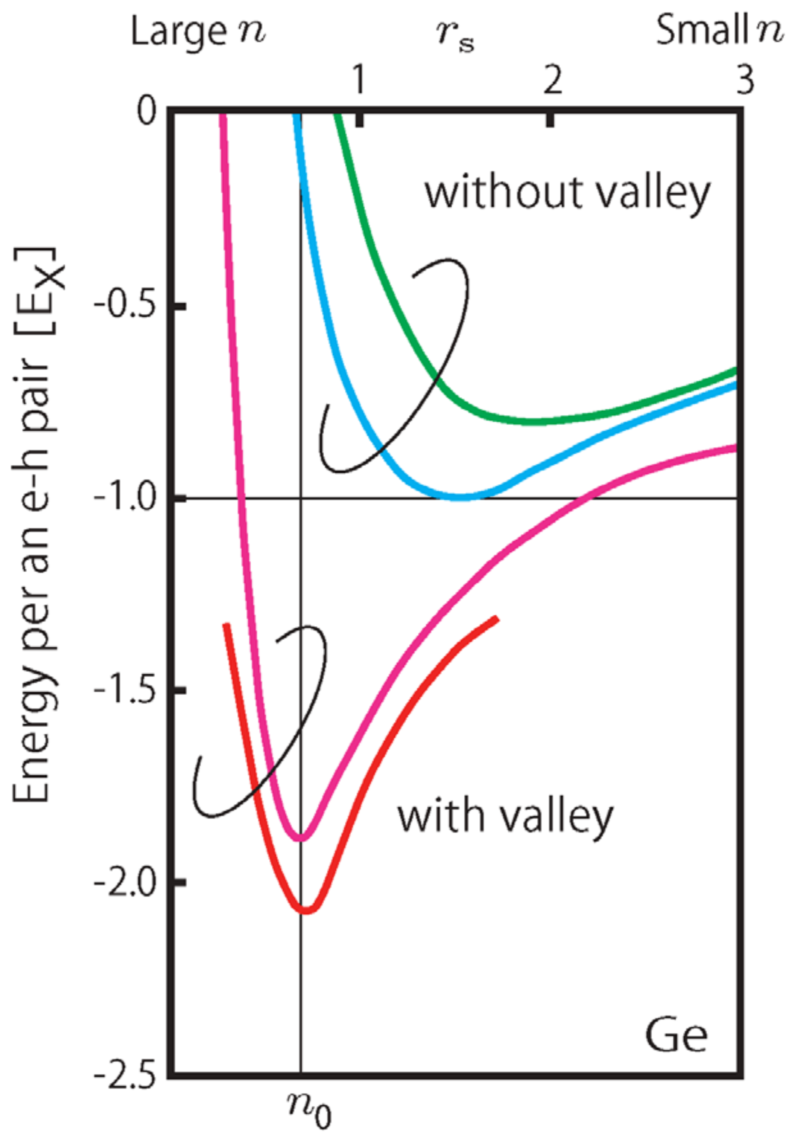
$\sim e^2/\epsilon d$

Energy per an e-h pair

$$K \sim \frac{(\hbar/d)^2}{2m_r} \propto d^{-2} \qquad U \sim -\frac{e^2}{\epsilon d}$$



Electron Hole Droplet

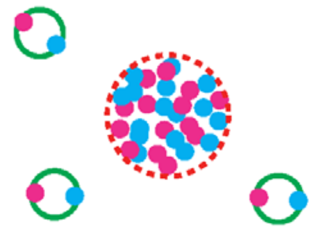


RPA Calculation

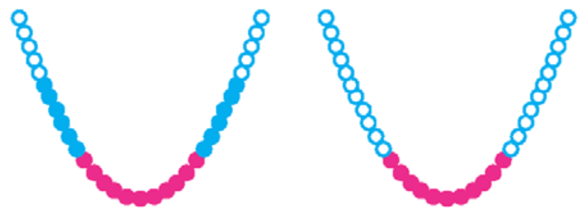
Brinkman and Rice, PRB **7**,1508 (1973)
Combescot and Nozieres, J. Phys **C5**, 2369 (1972)

If $E_{min} < -E_X$, $E(n)$ cannot be downward convex

→ Formation of **e-h droplet**



Valley Degeneracy



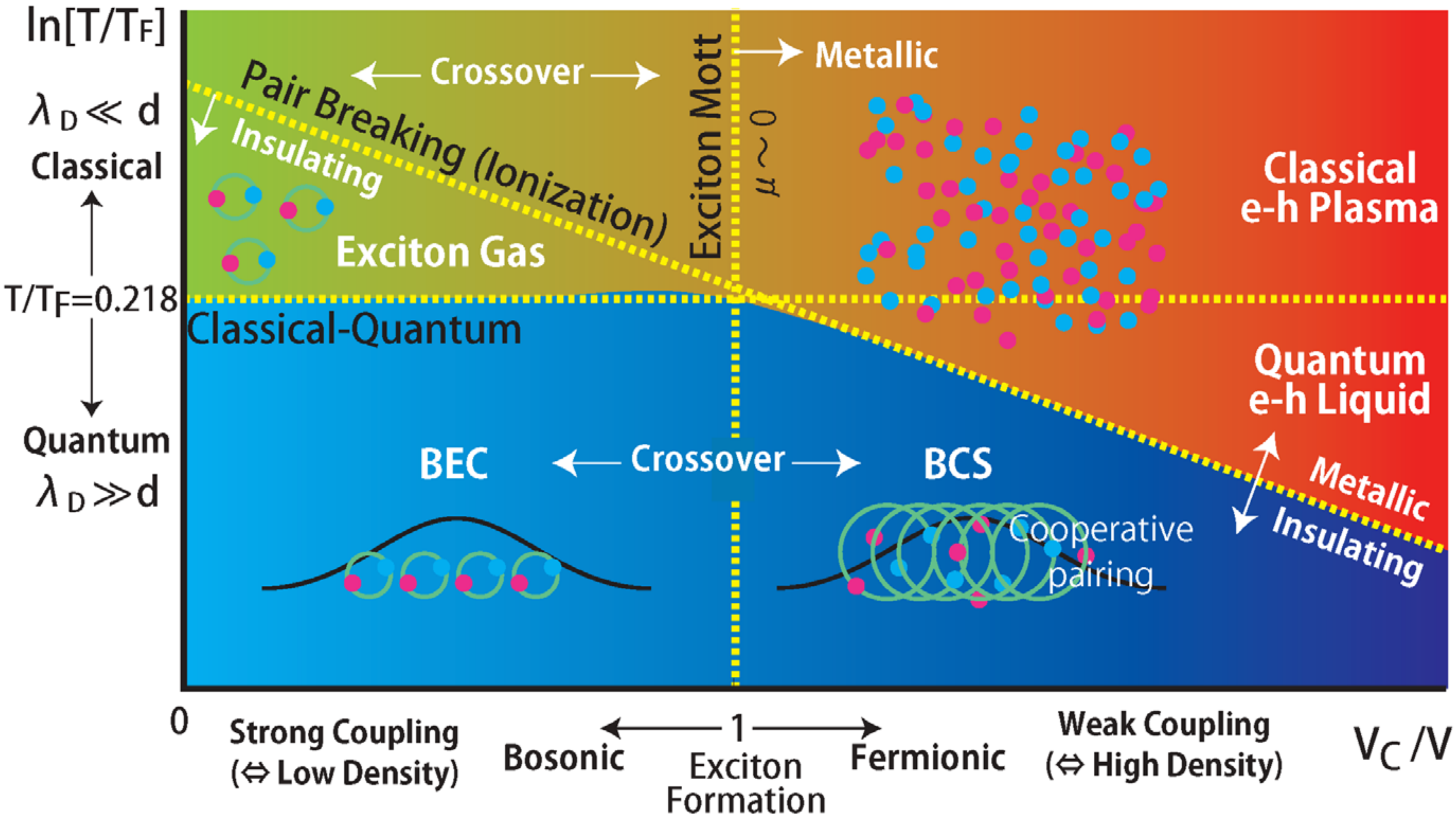
Decrease in kinetic energy

BCS-BEC Crossover (Quantum Condensation)

Thouless Criterion : Divergence of pair susceptibility

Nozieres and Schmitt-Rink, J. Low. Temp. Phys. **59**, 195 (1985).

Pair susceptibility (Ladder)
Thermodynamic Potential



Our Research Interest

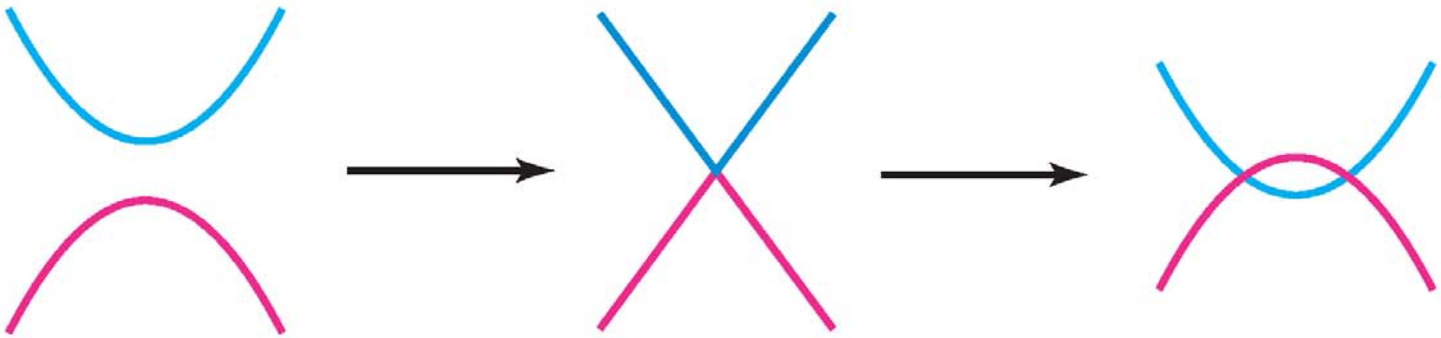
① Low Dimensional e-h Systems

2D (Quantum Well) 1D (Quantum Wire) 0D (Quantum Dot)

Magnetic field → quantum Hall systems

② Band Gap Control : Go back to the original Mott' s idea !

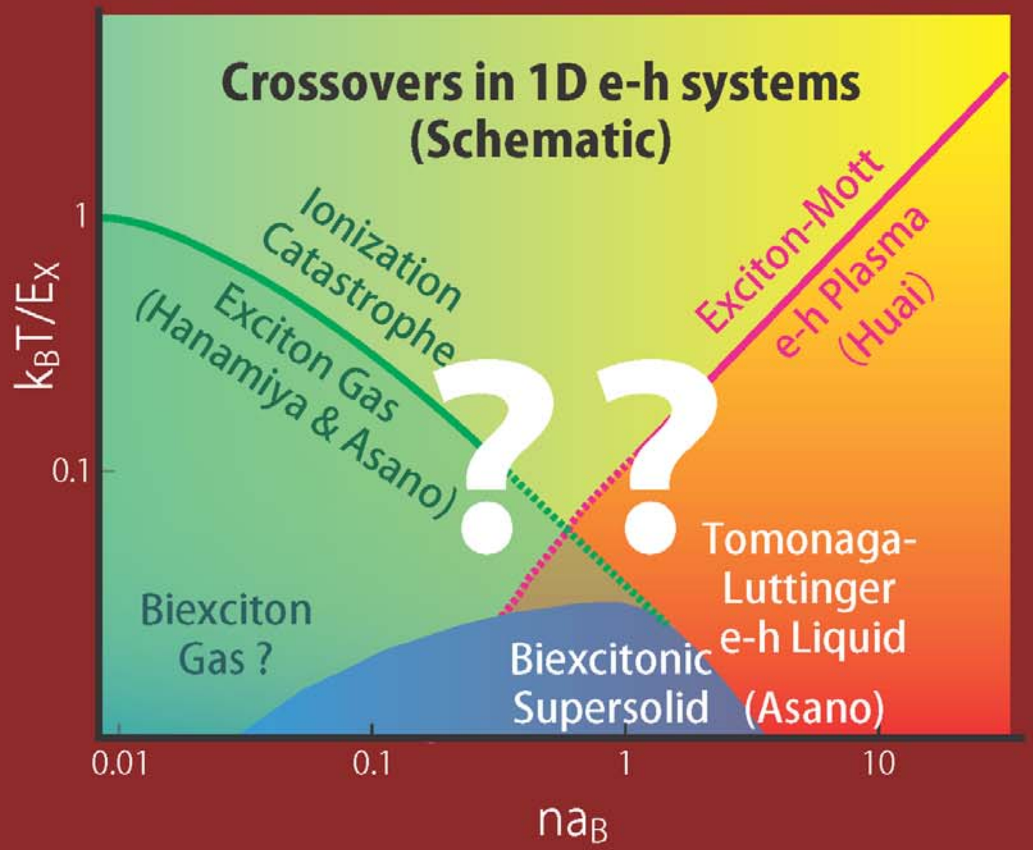
Semiconductor Dirac Semimetal



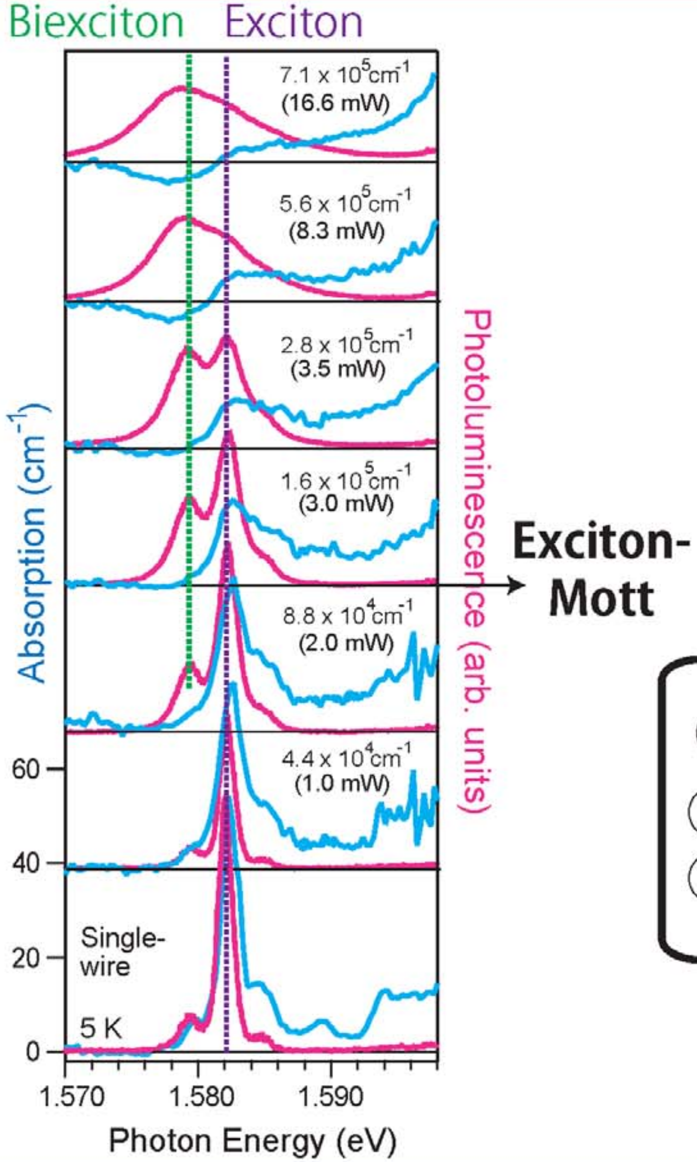
Band Gap > 0 Band Gap = 0 Band Gap < 0

Topic 1

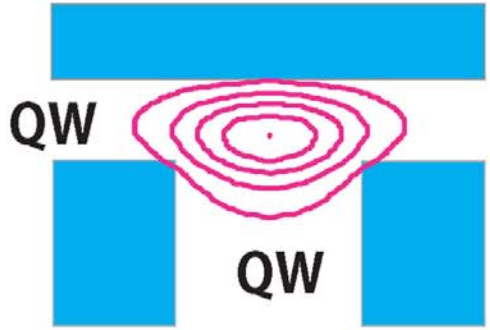
One-Dimensional Electron-Hole Systems



Experiments on T-shaped Quantum Wire



Hayamizu et al., PRL **99**, 167403 (2007).



① Highly Clean
② Long-Range Coulomb
(No gate structure)

??

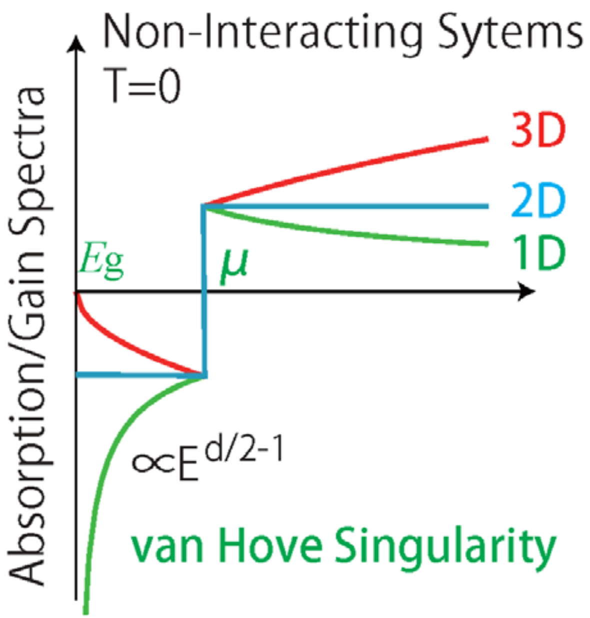
Coexistence of

① Gain in abs. spectra
② Biexciton peak in PL spectra

Optical Gain (Laser Application) & Dimensionality

Advantage

1. Large DOS at Band-Edge

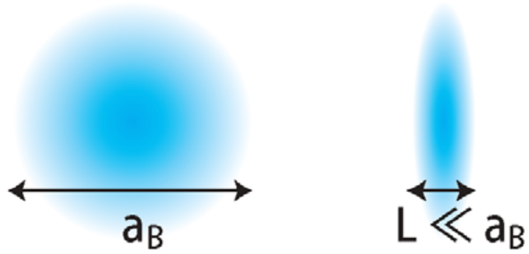


2. Strong Phase Space Filling Effect
3. Strong Excitonic Enhancement near $E \sim \mu$

Disdvantage

(Enemies of Exciton-Mott Transition)

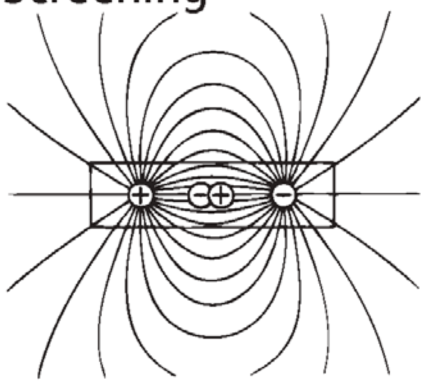
1. Huge Exciton Binding Energy



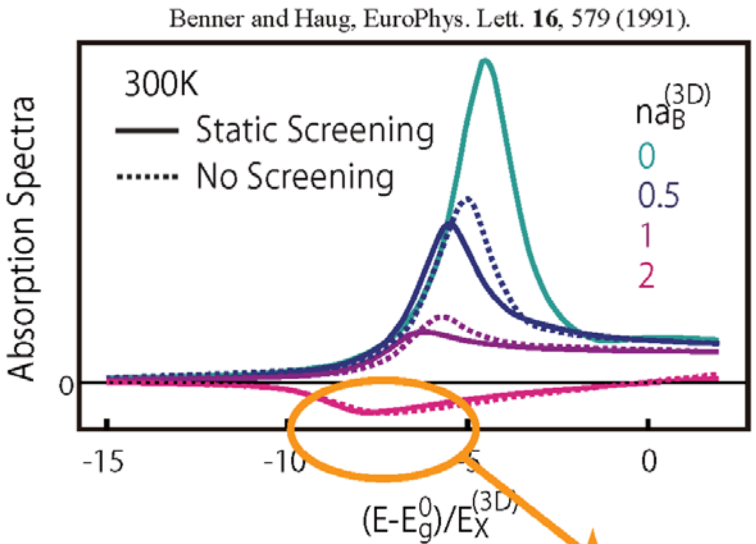
① $E_X \sim$ 100K (Ideal 1D → +∞)

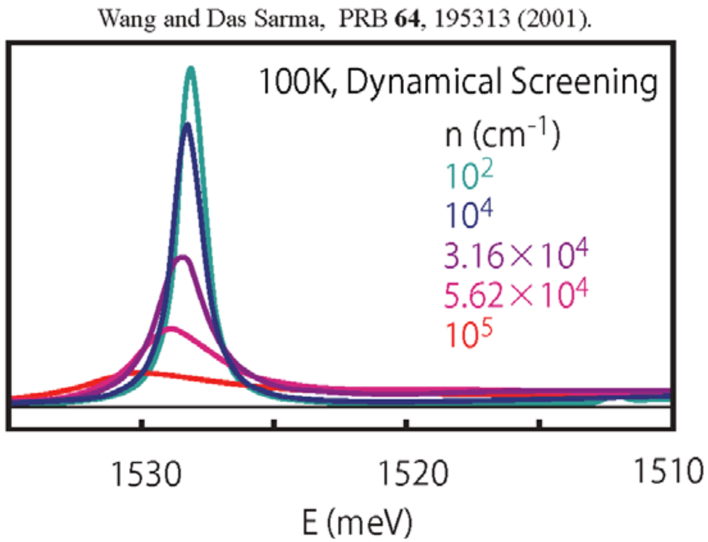
② Infinitesimal attraction ⇒ Bound

2. Small Screening

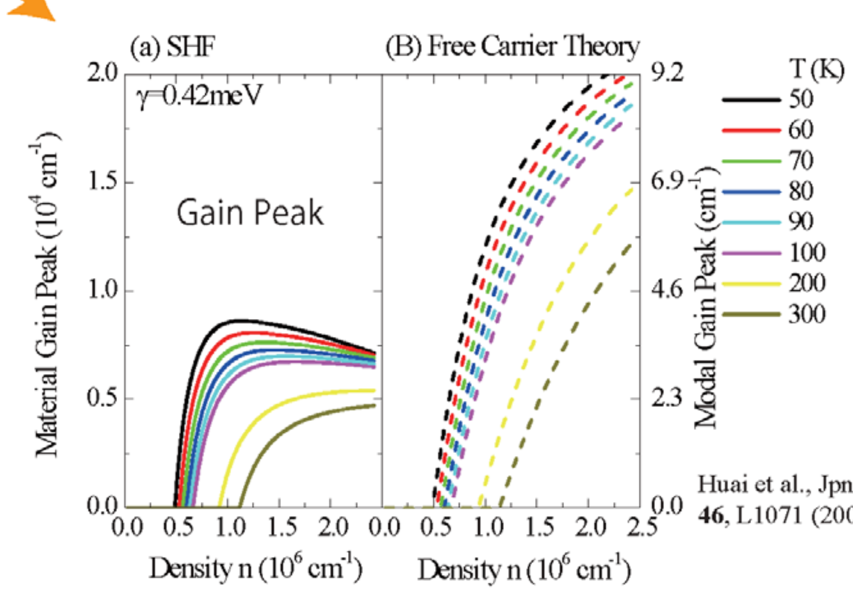


Theory for High n & High T Regime





Traditional Approach
Screened Hartree Fock
+ Ladder Approx.



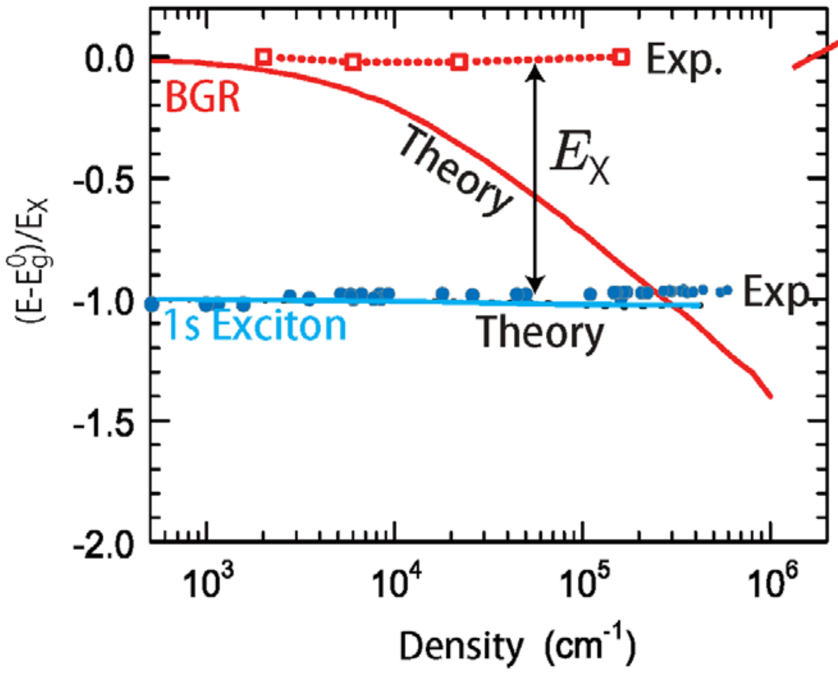
Huai et al., Jpn. J. Appl. Phys. **46**, L1071 (2007).

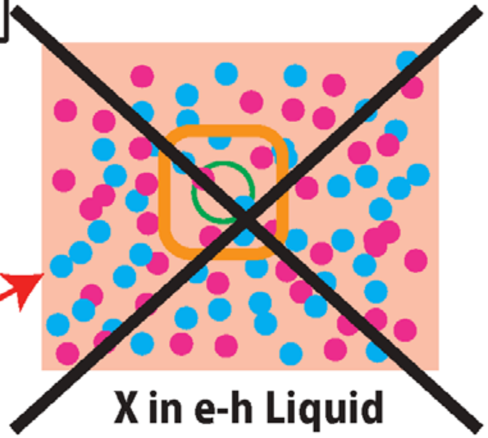
Theory for Low n & Low T Regime

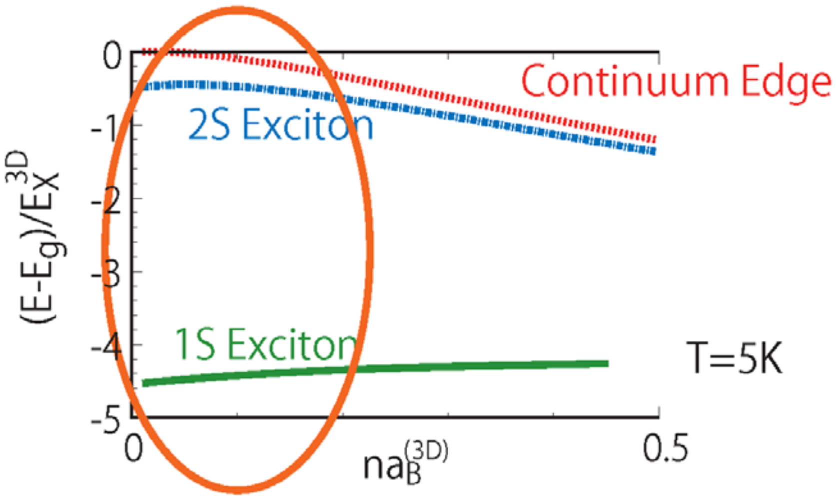
Self Consistent Theory of Exciton Gas

In 1D system, $E_X \sim$ 100K !

Failure of traditional theory



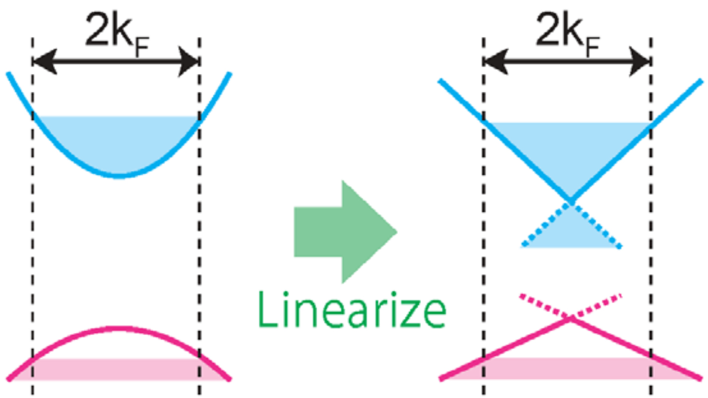




T. Hanamiya et al, physica E **40**, 1401 (2008).

Theory for High n & Low T Regime

Bosonization Approach



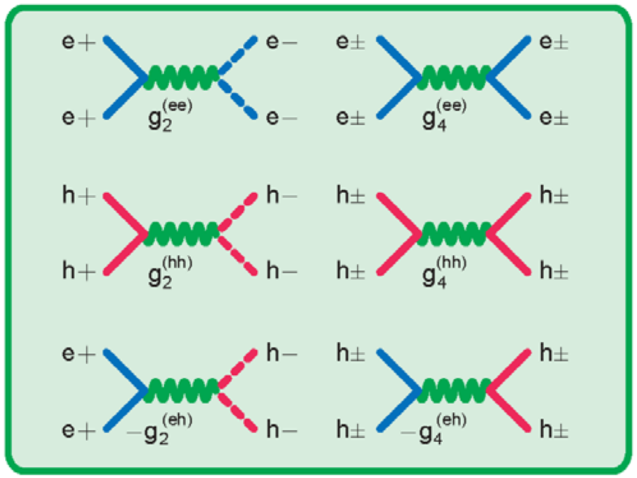
~~Fermi liquid~~ ~~quasi-electrons~~ ~~quasi-holes~~ ~~quasi-excitons~~

Importance of collective modes

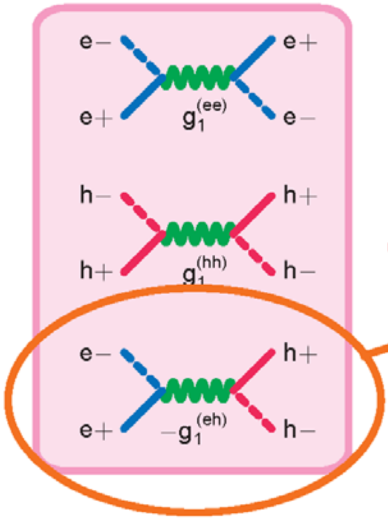
Charge	**Massive**
Mass	**Massless**
e-Spin	**Massive**
h-Spin	**Massive**

C1S0 Phase

Forward ⇒ Solvable



Backward ⇒ RG



e-h Backward Scattering is relevant!

Always insulating at T=0.

Algebraic Order of Ground State

Low energy physics is dominated by the mass density mode.

$$\mathcal{H}_\rho^{(\mathrm{m})} = \frac{v^{(\mathrm{m})}}{2\pi} \int dx \left[K^{(\mathrm{m})} \left(\partial_x \Theta_\rho^{(\mathrm{m})} \right)^2 + \frac{1}{K^{(\mathrm{m})}} \left(\partial_x \Phi_\rho^{(\mathrm{m})} \right)^2 \right]$$

$2k_F$ Mass Density Wave (Biexcitonic Crystal) $\sim x^{-K_\mathrm{m}/2}$

Biexcitonic Condensation $\sim x^{-2/K_\mathrm{m}}$

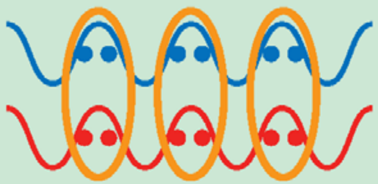
⇒ Biexcitonic Supersolid

c.f. Andreev and Lifshitz (1969).

1 (non-int.) 2 ∞ $K^{(\mathrm{m})}$

$2k_F$ Mass Density Wave

(Biexcitonic Crystal)



"Acoustic Mode" ⇔ Mass

"Optical Mode" ⇔ Charge

Biexcitonic Condensation

Phase Seperation ?

Asymmtery of e and h mass

⇒ Stabilize the biexcitonic crystal order

K. Asano and T. Ogawa, J. Lumin. **112**, 200 (2005).

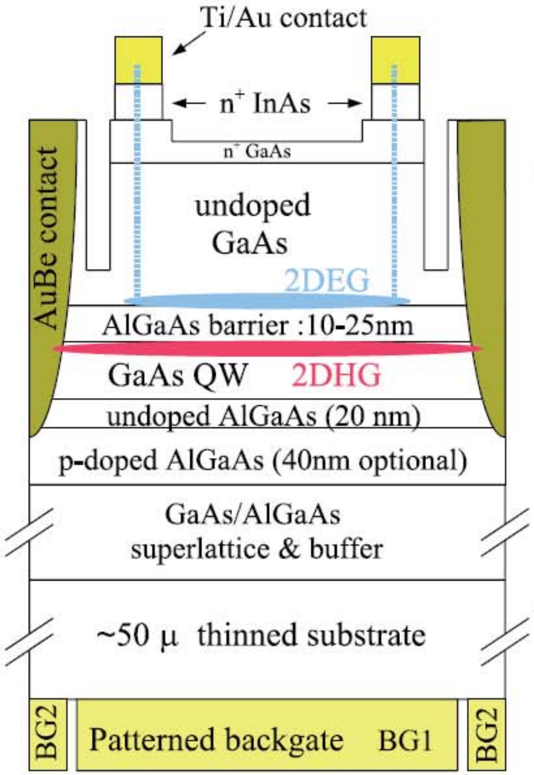
Topic 2

Fulde-Ferrell Phase in Electron-Hole Systems with Density Imbalance

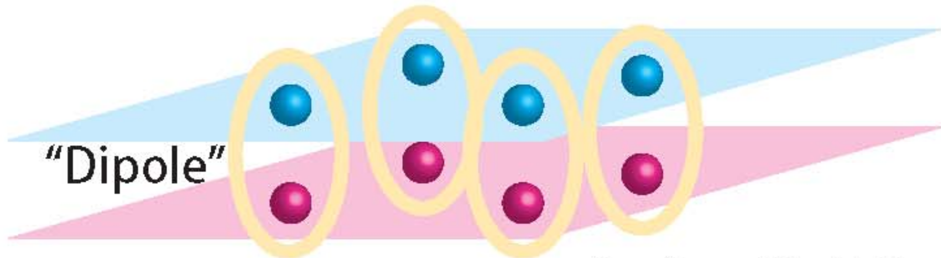
K. Yamashita, K. Asano and T. Ohashi

cond-mat/arXiv:0908.2492

Electron-Hole Bilayer Systems

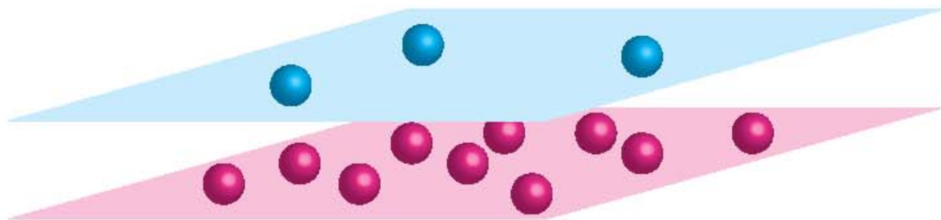


Density Balanced Case



Exciton Mott Transition
BCS-BEC Crossover

Density Imbalanced Case



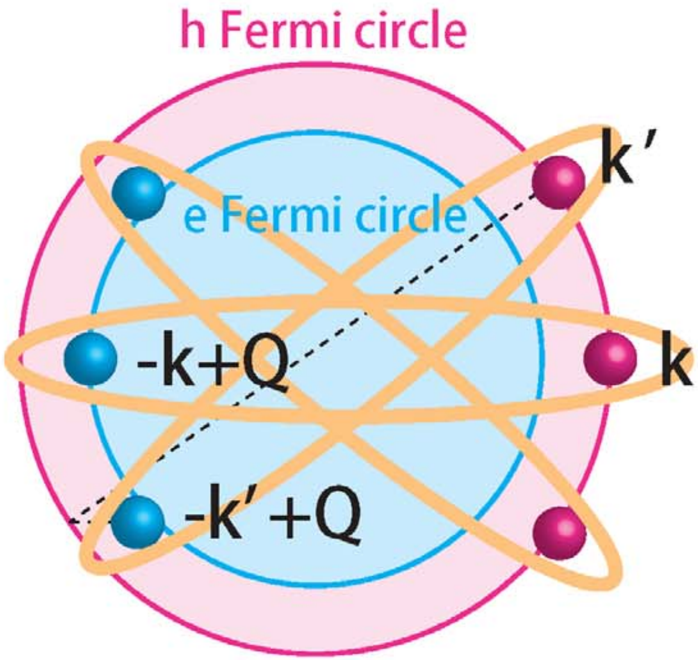
e & h densities
→ Independently controlled.

Optical spectra
Transport (Coulomb drag)

Trions ?
Deformation of Fermi circles ?
Phase Seperations ?
Exotic Quantum Condensations ?

Quantum Condensations in Imbalanced e-h Systems

Fulde-Ferrell Phase



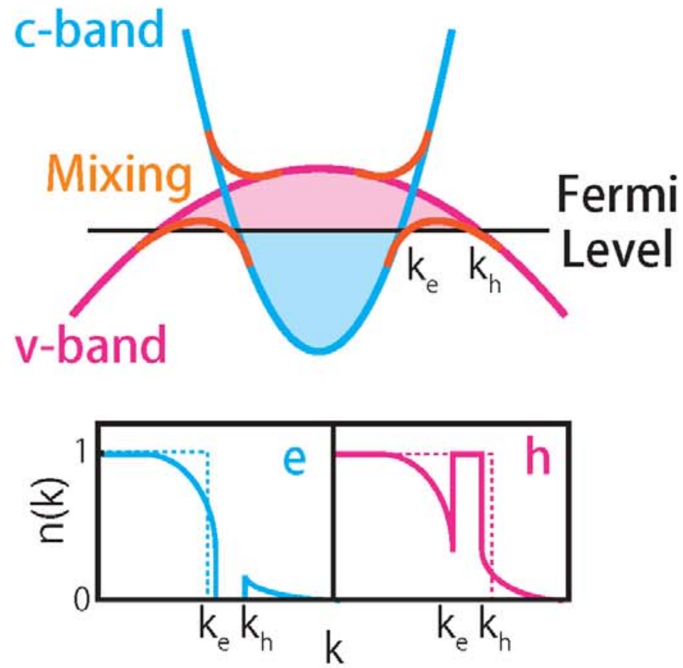
e-h pair with CM momentum Q

Fulde and Ferrell: PR **135**, 705(1964).

c.f. Inhomogeneous solution:
A. I. Larkin and Y. N. Ovchinnikov,
Sov. Phys. JETP **20**, 762 (1965).

Sarma Phase

(Breached pair phase)



Condensation of e-h pair with Q=0
+ Normal hole liquid

Sarma: J. Phys. Chem. Sol. **24**, 1029 (1963).
W. V. Liu and F. Wilczek: PRL **90**, 047002 (2003).

BCS Mean Field Approximation

Model Hamiltonian

$$H = \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}}^{(\mathrm{e})} e_{\boldsymbol{k}}^{\dagger} e_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}}^{(\mathrm{h})} h_{\boldsymbol{k}}^{\dagger} h_{\boldsymbol{k}} - \sum_{\boldsymbol{k} \neq \boldsymbol{k}', \boldsymbol{q}} V_{\boldsymbol{k}\boldsymbol{k}'} e_{\boldsymbol{k}+\boldsymbol{q}/2}^{\dagger} h_{-\boldsymbol{k}+\boldsymbol{q}/2}^{\dagger} h_{-\boldsymbol{k}'+\boldsymbol{q}/2} e_{\boldsymbol{k}'+\boldsymbol{q}/2}$$

$$V_{\boldsymbol{k}\boldsymbol{k}'} = \frac{1}{S} \int v(\boldsymbol{r}) e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} d\boldsymbol{r} = \frac{1}{S} \cdot \frac{e^2}{2\epsilon|\boldsymbol{k}-\boldsymbol{k}'|} e^{-|\boldsymbol{k}-\boldsymbol{k}'|d}$$

~~Spin~~
~~e-e & h-h interactions~~
~~Interlayer charging energy~~

Long-range Coulomb

BCS Mean Field Approximation

$$\Omega = \sum_{\boldsymbol{k}} (\eta_{\boldsymbol{k}}^{+} - E_{\boldsymbol{k}}) + \sum_{\boldsymbol{k}\boldsymbol{k}'} \Delta_{\boldsymbol{q}}(\boldsymbol{k}) [V_{\boldsymbol{k},\boldsymbol{k}'}]^{-1} \Delta_{\boldsymbol{q}}(\boldsymbol{k}') + \sum_{\boldsymbol{k}} E_{\boldsymbol{k}}^{+} f(E_{\boldsymbol{k}}^{+}) + \sum_{\boldsymbol{k}} E_{\boldsymbol{k}}^{-} f(E_{\boldsymbol{k}}^{-})$$

$$E_{\boldsymbol{k}}^{\pm} = E_{\boldsymbol{k}} \pm \eta_{\boldsymbol{k}}^{-}, \quad E_{\boldsymbol{k}} = \sqrt{(\eta_{\boldsymbol{k}}^{+})^2 + \Delta_{\boldsymbol{q}}(\boldsymbol{k})}, \quad \eta_{\boldsymbol{k}}^{\pm} = \frac{1}{2}(\epsilon_{\boldsymbol{k}+\boldsymbol{q}/2}^{(\mathrm{e})} - \mu^{(\mathrm{e})}) \pm \frac{1}{2}(\epsilon_{-\boldsymbol{k}+\boldsymbol{q}/2}^{(\mathrm{h})} - \mu^{(\mathrm{h})})$$

Numerical optimization:

CM momentum of e-h pair $\boldsymbol{q}$
Order parameter $\Delta_{\boldsymbol{q}}(\boldsymbol{k})$
→ Minimize thermodynamic potential Ω

FF and Sarma phases are considered on an equal footing !

Thermodynamical stability is automatically considered.

Phase Diagram at Zero Temperature

Parameters

Mass ratio $\frac{m^{(h)}}{m^{(e)}} = 4.3$

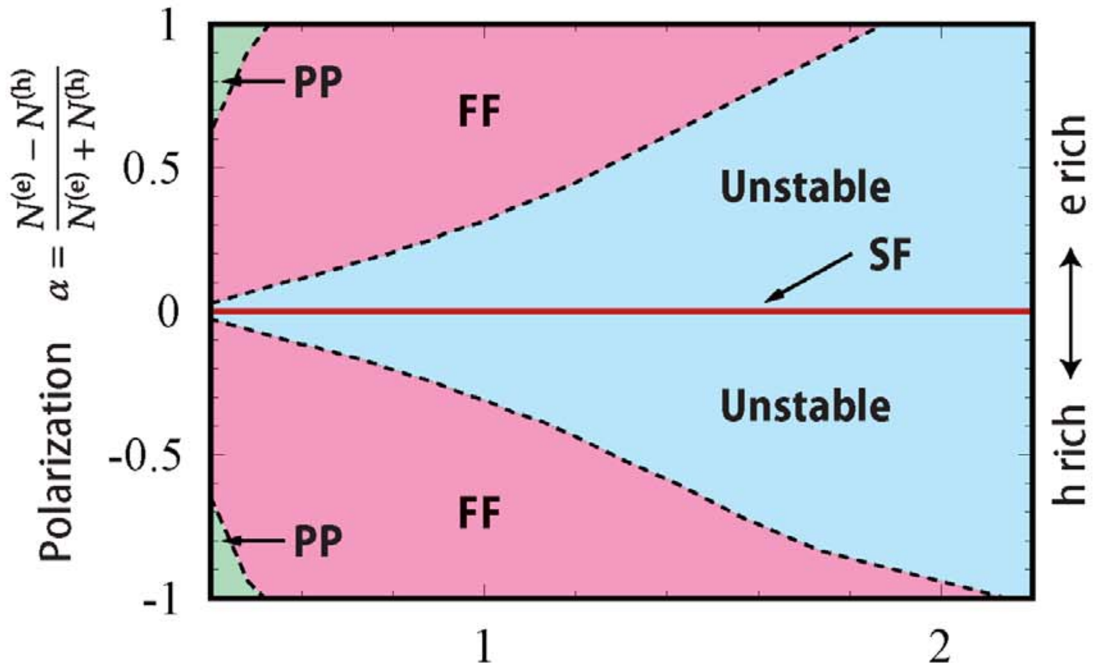
Interlayer distance $d = 2a_B^{(2D)} = \frac{\epsilon}{e^2 m_r}$

SF: superfluid phase (excitonic insulator)

FF: Fulde-Ferrell phase

PP: partially polarized normal phase

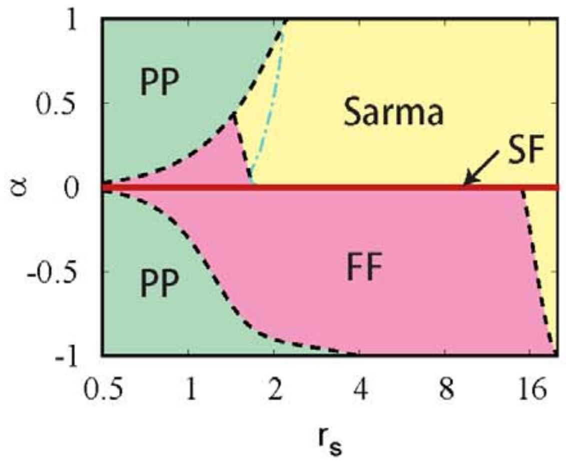
Unstable: no uniform solution



Interaction Strength $r_s = \frac{1}{a_B^{(2D)}} \sqrt{\frac{2S}{\pi(N^{(e)} + N^{(h)})}}$

c.f. Previous calculation

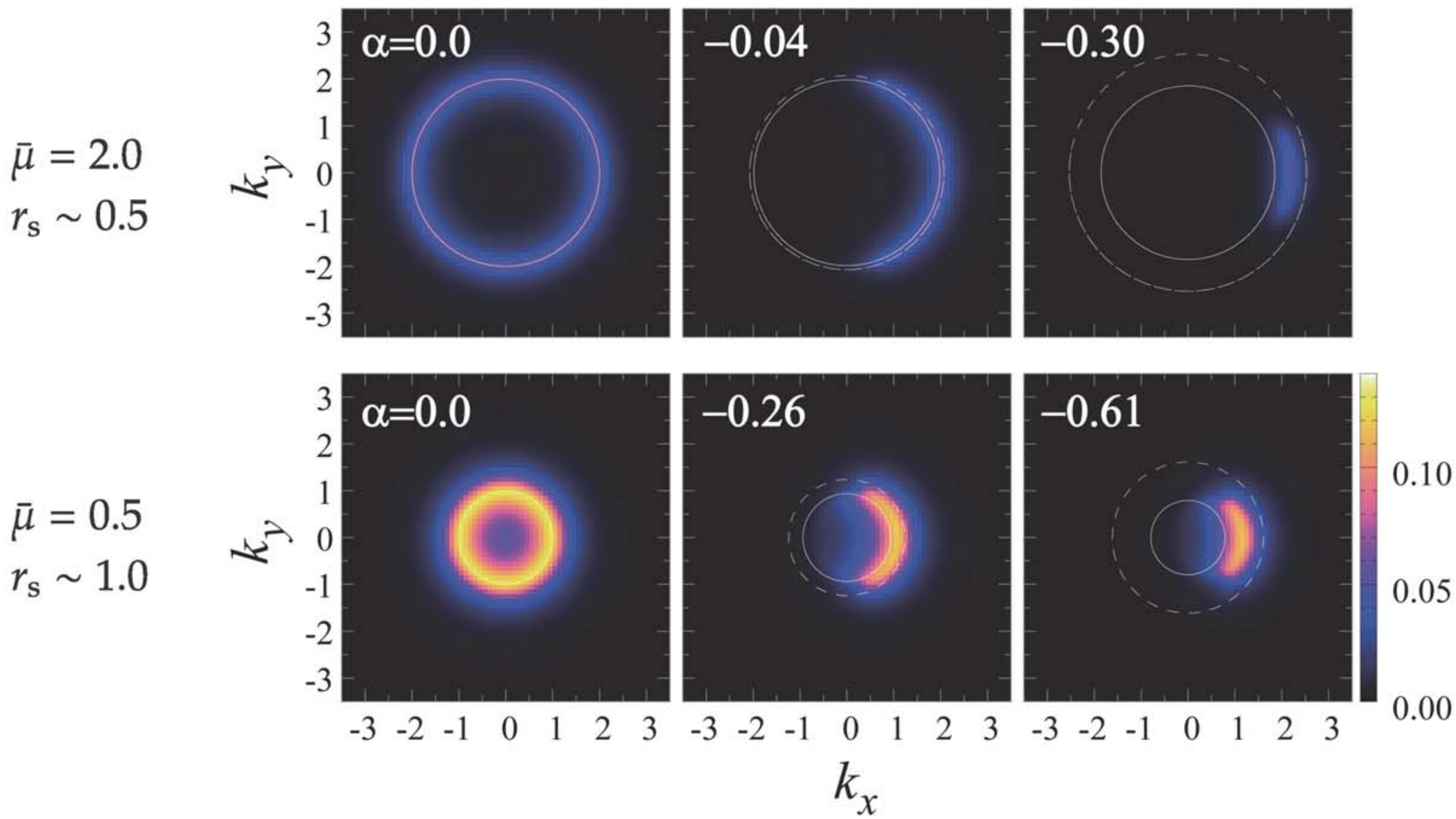
Instability of Sarma phase toward FF phase



Pieri et al. PRB75,113301 (2007).

Order Parameters

Order parameter mixing effects stabilize the FF phase.



Phase Diagram of 3D e-h Systems (Schematic)