非アーベリアンプラズマの非平衡ダイナミックス

奈良 寧
国際教養大学 (Akita International University)

Based on the collaboration with Adrian. Dumitru, Mike Strickland, Bjoern Schenke

Quark-Gluon Plasma (QGP) ： 平衡状態に達したクォークとグルーオンの系
非アーベリアンプラズマ(QCD プラズマ) ： 高温高密度QCD物質の一般的な状態

基研研究会「熱場の量子論とその応用」, 2008年9月3日〜5日
高エネルギー重イオン衝突の概要
ゲージ場と不安定性
非アーベリアン理論の線形ブラゾフシミュレーション
非アーベリアン理論のブラゾフシミュレーション
非アーベリアン理論のブラゾフ・ボルツマンシミュレーション
まとめと今後の課題
Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory

The 3.8 km circumference

RHIC: Au+Au at C.M. Energy of 200GeV per nucleon.

LHC: Pb+Pb at C.M. Energy of 5.5TeV per nucleon.

Creation of Hot and dense matter.
High energy heavy ion collisions

Color glass condensate

CGC  Black box  Hydro evolution  Hadron gas

$\tau = 0$, $\tau \approx 1/Q_s$, $\tau \approx 1 \text{ fm} / c$

Gluon production

Pre-equilibrium dynamics unknown

Hydrodynamics works at RHIC
Goal (motivation)

- Understanding the pre-equilibrium dynamics ($\tau<1-2\text{fm/c}$) in high energy heavy ion collisions.
- Kinetic theory approaches
- Thermalization? How? When? Which $T$?

What is the initial condition for (viscous, perfect) hydro?
Before collision

Color glass condensate

$\tau \leq 0$

Non-abelian Weizsacker-Williams filed

Non-abelian Weizsacker-Williams filed
Structure of the hadrons at high energies

QCD Linear Evolution Equations, DGLAP and BFKL predict rapid rise for $x \ll 1$.

Saturation scale

Gluon saturation

Gluon recombination becomes important

Nonlinear effect

Cloud of Small $x$ partons

Hard partons

E. Levin, A. H. Mueller, and J. Qiu, '86
A. H. Mueller and J. Qiu, '86
J. P. Blaizot and A. H. Mueller, '87

Gluon saturation

$r \sim \frac{1}{Q_s}$

$x = \frac{p_z, \text{parton}}{p_z, \text{hadron}}$

$Q^2 = 10 \text{ GeV}^2$

$xf(x,Q^2)$
Gauge field in the MV model

The gluon distribution is large:

Suggest the use of the semi-classical methods

\[ D_{\nu} F^{\nu\mu} = J^{\mu} \]

In the light-cone gauge: \( A^{+} = 0 \),
a solution can be

\[ A^{-} = 0, \quad A^{i} = \frac{i}{g} U \partial^{i} U^{\dagger}, \]

\[ U = P \exp \left[ ig \int \Lambda dz^{-} \right], -\Delta_{\perp} \Lambda = \rho \]

transverse component:

gauge transformation of vacuum: \( F_{ij}^{ij} = 0 \)

The only non-zero component of the field strength:

\( F^{+i} \neq 0 \)

\[ B \cdot E = 0, \quad B_{z} = E_{z} = 0 \]
$\tau < 1/Q_s$: Yang-Mills field dynamics

Real time evolution of the Classical Yang-Mills

Similar to Lund string model picture: but produce both color electric and magnetic fields.

Production of longitudinal color EM fields.

Instabilities of Yang-Mills field:
P. Romatschke, R. Venugopalan, K. Fukushima, Gelis, McLerran, A. Iwazaki
\[ \tau \approx \frac{1}{Q_s} : \text{(most?, some) gluons on shell} \]

- Gluons are produced at \( \tau \sim \frac{1}{Q_s} \),
  \( f \sim \frac{1}{\alpha_s} \) typical momentum \( p \sim Q_s \)

Krasnitz, Nara, Venugopalan, hep-ph/0305112

**RHIC \((Q_s=1.4 \text{ GeV})\)**

1+1D expansion

Final state interaction among produced particles

\[ \varepsilon = 3 : 1 \]
$\tau \geq 1/Q_s :$ instabilities

$\alpha_s \ll 1$

- Due to a longitudinal expansion, typical longitudinal momentum of gluons becomes
  
  \[ p_z \sim 1/\tau, \; p_x \sim p_y \sim Q_s \text{ at } \tau >> 1/Q_s \]

Free streaming limit:

\[
\frac{p_z}{p} = \frac{1}{Q_s \tau}
\]

How does this relation modified by collisions or other effects?
**Possible effect: instabilities**

In the early stages of heavy ion collisions, dynamics between color field and plasma particles (gluons) may be important.

Anisotropic momentum distribution $\rightarrow$ plasma instabilities

**Collision time:**

$$t_p \sim \frac{(Q_s \tau)^{2/3}}{Q_s}$$


**Growth rate due to plasma instabilities:**

$$t_{growth} \sim \frac{(Q_s \tau)^{1/2}}{Q_s}$$
Plasmas - the Fourth State of Matter

- Plasmas consist of moving charged particles (electrons and ions).
- Plasmas have unique physical properties, distinct from solids, liquids and gases.

QCD plasma: Fifth state of matter?
Plasma instabilities

Exponential growth of wave solutions: $A \sim e^{\omega t}$

- **Macroscopic instabilities:**
  coordinate space nonuniformity

- **Microscopic instabilities:**
  momentum space anisotropy

Vlasov dynamics is important

S. Mrowczynski: application to high energy heavy ion collisions.
Plasmas in high energy limit: (relativistic weakly coupled plasma)

\[ T \gg g^2/r \approx g^2 T \quad r \sim T^{-1} \]

Average distance between particles. Kinetic energy of the particle >> potential energy

Hard scale: Plasma particles (gluon)

Soft scale: Gluon field

Weak coupling limit

Large scale separation between the hard and soft momentum scales in the weak coupling limit \( g \ll 1 \).
Non-equilibrium dynamics within kinetic theory

Traditional plasma physics L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + g (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p f = C[f]
\]

\[
\dot{\mathbf{E}} = \nabla \times \mathbf{B} - \mathbf{J} \\
\dot{\mathbf{B}} = -\nabla \times \mathbf{E} \\
\mathbf{J} = g \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} f
\]

\(f(x, p)\) One-particle distribution function

- Collision only: Boltzmann equation.
- Mean field only: Vlasov equation.
Linearized Maxwell-Vlasov Theory

\[ \partial_t f + v \cdot \nabla_x f + e(E + v \times B) \cdot \nabla_p f = 0 \]
\[ \partial_v F^{\mu\nu} = e \int_p v^\mu f \]

Assuming that \( E \) and \( B \) are small: \( f(p) = f_0(|p|) + \delta f(p) \)

Linearized Vlasov:
\[ \partial_t \delta f + v \cdot \nabla_x \delta f + e(E + v \times B) \cdot \nabla_p f_0 = 0 \]
\[ \partial_v F^{\mu\nu} = e \int_p v^\mu \delta f \]

Gauge field in the momentum space:
\[ \left[ q^2 g^{\mu\nu} - q^\mu q^\nu - \Pi^{\mu\nu}(q) \right] A_\nu(q) = 0 \]
\[ \Pi^{\mu\nu}(\omega, q) = e^2 \int_p \frac{\partial f}{\partial p^k} v^\mu \left[ g^{k\nu} - \frac{v^\nu q^k}{v \cdot q + i \epsilon} \right] \]
Anisotropic $f(p)$: there can be unstable solutions when $\omega$ has a positive imaginary part.

$$A \sim \exp(\gamma t)$$

Perturbative instability Growth rate as a function of anisotropy

$$[q^2 g^{\mu\nu} - q^\mu q^\nu - \Pi^{\mu\nu}(q)] A_\nu(q) = 0 \quad q = (\omega, \mathbf{q})$$

$$\theta \approx 0.057$$

$$\theta = \frac{p_z}{p} = \nu_z$$

$$\theta = 0.447$$
Filament production of gauge field magnetic instabilities
Non-Abelian Plasma Instabilities

- In Abelian plasma: instabilities stop when the effect of the soft field on the hard particle trajectory becomes non-perturbative, i.e.

\[ gA \approx p_{\text{hard}} \]

- In Non-Abelian plasma, what is the role of non-Abelian interactions between the soft modes?

1D: same as Abelian plasma: Abelianization of gauge fields

3D: \[ D = \partial - igA \sim i(k_{\text{soft}} - gA) \]

\[ gA \sim k_{\text{soft}} \sim m_\infty \]

when \[ gB \sim gk_{\text{soft}} A \sim k_{\text{soft}}^2 \sim m_\infty \]

Nonlinear effects become important.

Scale for the unstable field growth:

\[ m_\infty^2 = 2N_c g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(p)}{|p|} \]
Hard loop 3D simulations


\[ gA \approx k_{\text{soft}} \]

Moderate anisotropy assumed in the simulation.

Instability grows linearly at late times in 3D. Abelianization not seen.
Lattice size dependence of B-field for strong anisotropic plasma

Moderate anisotropy

Strong anisotropy

Completely different behavior from the one that is observed in moderate anisotropic case

D. Bodeker and K. Rummukainen, JHEP 0707, 022 (2007)
Expanding plasma

A. Rebhan, M. Strickland and M. Attems,
``Instabilities of an anisotropically expanding non-Abelian plasma: 1D+3V discretized hard-loop simulations,''

\[ t_{growth} \sim \frac{1}{m_D} \sim \sqrt{\tau} \]

\[ m_D^2 \sim \frac{1}{\tau} \]
Beyond linear approximation (HL)

solving full Vlasov equation for gluons, i.e. dynamics of fields and particles.

\[ p^\mu \left( \frac{\partial}{\partial x^\mu} - g f^{abc} A^b_\mu Q^c - g Q_a F^a_{\mu \nu} \frac{\partial}{\partial p^\nu} \right) f(x, p, Q) = 0 \]

\[ D_\mu F^{\mu \nu} = J^\nu \]

- non-linear effects: saturation of instabilities.
- study isotropization of the system (particles and field) by interactions between particles and field.
- Extreme anisotropies (back reaction important)
- weak and strong initial field.
Particle in cell simulation for colored particles (CPIC)

Test particle method:

\[ f(x,p,Q) = \frac{1}{N_{\text{test}}} \sum_i \delta(x-x_i(t)) \delta(p-p_i(t)) \delta(Q-Q_i(t)) \]

Wong-Yang-Mills equation:

\[ \frac{d x_i}{d t} = v_i, \quad \frac{d p_i}{d t} = g Q_i^a (E_i^a + v_i \times B_i^a), \quad \frac{d Q_i}{d t} = ig v_i^\mu [ A_\mu, Q_i ] \]

\[ D_\mu F^{\mu\nu} = J^\nu = g \sum Q v^\mu \delta(x-x_i(t)) \]

WYM system itself is very complicated. Before doing some realistic simulations in heavy ion collisions, we should check its properties.
Yang-Mills Hamiltonian on the lattice

Numerical implementation of real time simulation in classical YM is well established. Kogut-Susskind Hamiltonian:

\[
H_{YM} = \frac{1}{2} \sum_l E_l^2 + \frac{1}{2} \sum_\Box \left( N_C - \Re Tr U_\Box \right)
\]

Link

\[
U_{j,i} = \exp[-iga A_j(i)]
\]

Plaquette

\[
U_\Box \equiv U_{l,j} U_{m,j+l} U_{l,j+m} U_{m,j}^\dagger
\]

Equations of motion for dynamical variable

\[
\frac{d\nu}{d\tau} = \{H_L, \nu\}
\]
Nearest-Grid-Point (NGP) method for non-Abelian gauge theories

Density: Count the number of particles within a cell
A current is generated only when a particle crosses a cell

\[ J(i) = Q \frac{\delta(t - t_{cross})}{N_{test}} \]

Color charge must be parallel transported:

\[ Q(i+1) = U_x(i)^\dagger Q(i) U_x(i) \]

to satisfy the lattice covariant continuity equation,

\[ \dot{\rho}(i) = \sum_x U_x(i-x)^\dagger J_x(i-x) U_x(i-x) - J_x(i) \]

in order to conserve Gauss law.

Full 3D simulations (Isotropic case)

Applicable range of the Point particle method is limited. (1D simulation is OK) too noisy and unphysical damping.

\[ m_{\infty}^2 = g^2 N_c \int \frac{d^3 p}{(2\pi)^3} \frac{f(p)}{|p|} \]
Color-Particle-in-Cell (CPIC) method

\[ J_x(i, j) = Q \frac{x_{i+1} - x_i}{\delta t} (1 - W_y), \quad J_x(i, j+1) = Q_y \frac{x_{i+1} - x_i}{\delta t} W_y, \]

\[ J_y(i, j) = Q \frac{y_{i+1} - y_i}{\delta t} (1 - W_x), \quad J_y(i+1, j) = Q_x \frac{y_{i+1} - y_i}{\delta t} W_x. \]

Parallel transport of the charge:
\[ Q_x = U_x(i) \dagger Q U_x(i), \quad Q_y = U_y(i) \dagger Q U_y(i) \]

Linear weighting function defined at the midpoint:
\[ W_x = \frac{x_i + x_{i+1}}{2} - i, \quad W_y = \frac{y_j + y_{j+1}}{2} - j \]

Parallel transport satisfies the lattice covariant continuum equation:
\[ \dot{\rho}(i) = \sum_x U_x(i-x) \dagger J_x(i-x) U_x(i-x) - J_x(i) \]

Sites (i,j), (i+1,j), (i,j+1) are consistent with
\[ \rho(i, j) = Q(1-x)(1-y), \quad \rho(i, j+1) = Q_y(1-x)y \]
\[ \rho(i+1, j) = Q_y x(1-y), \quad \rho(i+1, j+1) = Q_{xy} xy \]

Boundaries experiencing current flow

\[ Tr(Q^2) \] is conserved within \( O(a^2) \)

\( \rho(i+1, j+1) \) depends on the path

We use current conservation to determine the increment of the color charge.
Initial conditions for simulations

- **3D-3V simulation:** 3dimension for both space and momentum space.
- Initial field configuration: random white noise for the field.
- Anisotropic initial particle distribution:

\[
f(p, t=0) \sim \delta(p_z) e^{-\sqrt{p_x^2 + p_y^2} / p_{hard}}
\]
U(1) results for 3D+3V: good!

Exponential growth of magnetic field.
Independent of lattice spacing.
Fourier transform of the transverse magnetic field in U(1): 

No UV problem in U(1). Soft modes unstable, hard modes stable.
SU(2) for 3D+3V

Field strength grows with inverse lattice spacing!
Time evolution of the Fourier transformed fields in SU(2) 3D+3V

Soft fields rapidly avalanche to the UV

Phard!

Soft fields rapidly avalanche to the UV
Linear Vlasov results

\[ f \sim \frac{1}{k^2} \]

D. Bodeker and K. Rummukainen,
JHEP 0707, 022 (2007)
Time evolution of longitudinal mom.

\[ m_\infty t < 10 \]

Independent of the lattice size

\[ m_\infty t > 10 \]

depend of the lattice size
Moderate v.s. strong anisotropy

Proper definition of classical dynamics of soft+hard modes in gauge theories is needed.

Dynamical conversion of Field-particle.
Boltzmann-Vlasov for colored particles

- The kinetic equation for classical colored particle:

\[ p^\mu \left( \frac{\partial}{\partial x^\mu} - g f_{abc} A^b_\mu Q^c - g Q_a F^a_{\mu\nu} \frac{\partial}{\partial p_\nu} \right) f(x, p, Q) = C(x, p, Q) \]

- Coupled to the classical YM equation

\[ D_\mu F^\mu_\nu = J^\nu \]

- \( f(x, p, Q) \): hard modes: particle
- \( F^\mu_\nu \): Soft modes: classical field
- \( C(x, p, Q) \): Collision term (hard scattering)

- Soft interactions are treated by mean color field

\[ \sigma_{g g} \sim \frac{\alpha_s^2}{m_D^2} \quad \Rightarrow \quad m \frac{dp^\mu}{d \tau} = g Q^a F^\mu_\nu \]
Simulating collision term among hard plasma particles

Full ensemble method: Lorentz invariant procedure to simulate collisions in Boltzmann equation in a Monte-Carlo fashion

Collision probability for any pairs in a cell:

\[ P_{22} = v_{\text{rel}} \frac{\sigma_{22}}{N_{\text{test}}} \frac{\Delta t}{\Delta^3 x} \]
Cut off free Boltzmann-Vlasov-YM approach

SU(2) $g=2$, $T=4\text{GeV}$, $m_\infty=1.4/\text{fm}$.

- Soft exchange ($q<\pi/a$) via gauge field (Lorentz force)
- Hard exchange ($q>\pi/a$) via collision term
Effects of Collisions


Binary collisions among hard particles reduce the growth rate of unstable field modes by 40% and 15% depending on the lattice spacing, due to different available field modes and random collision rate.
Summary and Outlook

- Particle-field (Vlasov-YM) simulations in QCD.
- Non-Abelican results are totally different from Abelian results. UV avalanche.
- Collective effects due to color field play important role for the pre-equilibrium dynamics.

- Include expansion of the system.
- Detailed study of interplay between Collision term. and color mean-field.
- Proper definition of particle + field dynamics beyond HL in gauge theories.
(Partial) Isotropization

exponential growth of long. Pressure due to deflection of particles

SU(2)

transv.

longit.

pressures

g^{2}T_{\text{part}}^{\text{xx}}
g^{2}T_{\text{part}}^{T}
g^{2}(T_{\text{part}}^{xx} + T_{\text{field}}^{xx})
g^{2}(T_{\text{part}}^{T} + T_{\text{field}}^{T})

initial

final

\frac{p_{x}}{p_{\text{hard}}} \text{ or } \frac{p_{y}}{p_{\text{hard}}}

exponential growth of long. Pressure due to deflection of particles