Ultra-cold Atomic Gases with a Composite-Fermion-Channel Feshbach Resonance

Takushi NISHIMURA, 西村拓史 (Tokyo Metropolitan University)
collaborated with

薮博之 (立命大理工), 鈴木徹 (首都大理工), P. Schuck (IPNO, LPMMC)

23-25, August, 2006 at 京都大学基礎物理学研究所
Introduction: Feshbach resonance (1)

a open & a closed channel \((|\frac{9}{2}, -\frac{9}{2}\rangle + |\frac{9}{2}, -\frac{7}{2}\rangle \& |\frac{9}{2}, -\frac{9}{2}\rangle + |\frac{9}{2}, -\frac{5}{2}\rangle)\)

Zeeman shift: \(\Delta E \sim \Delta m(B - B_c)\)

\(a \sim a_{nor.} \left(1 + \frac{1}{E - E_{res.} + i\hbar \Gamma_{res.}/2}\right) \sim a_{nor.} \left(1 + \frac{\Delta B}{B - B_{res.}}\right)\)

JILA(\(^{40}\)K), PRL90, 230404(2003)
Introduction: Feshbach resonance (2)

- Composite-**Boson**-Channel Feshbach Resonance
  - **F-F** system:
    - JILA ($^{40}$K); ENS, Innsbruck, Rice, MIT, Duke ($^{6}$Li)...

- Composite-**Fermion**-Channel Feshbach Resonance
  - **B-F** system:
    - MIT ($^{23}$Na-$^{6}$Li); JILA, LENS ($^{87}$Rb-$^{40}$K); ENS ($^{7}$Li-$^{6}$Li)...

F-F system (1)

Example: a simple two-channel model

\[ H = H_a - \mu_M |\phi_M|^2 + g \sum_k f_k (\phi_M^* a_{-k} c_k + h.c.) \]

\[ H_a \equiv \sum_k (e_k^{(a)} - \mu_a) a_k^\dagger a_k + \sum_k (e_k^{(c)} - \mu_c) c_k^\dagger c_k \]

\[ \mu_M \equiv \mu_a + \mu_c - dE < 0 \]
F-F system (2)

steady condition: \( \frac{\delta \langle H \rangle}{\delta \phi_M} = 0 \)

\[ \therefore \]

\[ \phi_M = -\frac{1}{\mu_M U_{eff}} \Delta \neq 0, \quad \Delta \equiv -U_{eff} \sum_k f_k \langle a_{-k} c_{+k} \rangle \]

\[ \frac{1}{U_{eff}} \Delta^2 = -\frac{g}{U_{eff}} \phi_M \Delta \quad \Rightarrow \quad U_{eff} = \frac{g^2}{\mu_M} < 0 \]

few molecule limit \(\Rightarrow\) BCS (single-channel model)
B-F system

few molecule limit = nearly free molecules

\[
\downarrow
\]

B-F pairing

unstable for pair generation

A. Storozhenko, P. Schuck, et. al. PRA 71, 063617 (2005)
B-F pairing model (1)

\[ H = \sum_k \left( h_k^{(c)} c_k^\dagger c_k + h_{-k}^{(b)} b_{-k}^\dagger b_{-k} - \Delta F_{-k}^\dagger b_{-k} c_{+k} + \text{h.c.} \right) \]

\[ \Delta \equiv -U_{eff} \sum_k \langle F_{-k}^\dagger b_{-k} c_{+k} \rangle \]

1. F is a virtual fermion operator

2. no atomic BEC

3. \( \Delta \) is real
**B-F pairing model (2)**

Variational ansatz:

\[
|\psi\rangle = N \left[ \prod_k (1 + g_k c_{+k}^\dagger b_{-k}^\dagger F') \right] |\varphi\rangle , \quad |\varphi\rangle = \left[ \prod_k N_{i_k} (b_{-k}^\dagger)^{i_k} \right] F^\dagger |0\rangle
\]

1. It gives the same solution diagonalized by matrix representation method.

2. Analogous to the BCS model.
B-F pairing model (3)

\[ \Delta = 0; \quad H = \sum_k \left( h_k^{(c)} c_k^\dagger c_k + h_k^{(b)} b_{-k}^\dagger b_{-k} \right) \]
**B-F pairing model (4)**

\[ |\Delta| \neq 0 \equiv 1; \quad H = \sum_k \left( h_k^{(c)} c_k^\dagger c_k + h_k^{(b)} b_{-k}^\dagger b_{-k} - F_{-k}^\dagger b_{-k} c_{+k} + \text{h.c.} \right) \]
Bose statistic enhancement

\[ + \]

Bose statistic enhancement make ”many-body pair cluster”

\[ \downarrow \]

1. the discrete structure

2. atomic BEC may not be: \( \mu_b \to 0; n_{crit} \to \infty \)
Gap equation

\[ 1 = -4\pi a_s \sum_k \left[ \frac{(i_k + 1)}{2\sqrt{\left(\frac{h_k^c + h_k^b}{2}\right)^2 + (i_k + 1)\Delta^2}} - \frac{1}{k^2} \right] \]

\[ \mu_F = 1.6, \mu_B = -0.6 \]

\[ \mu_F = 1.1, \mu_B = -0.1 \]

\[ \mu_F = -0.4, \mu_B = -0.1 \]

\[ \mu_F = 1.2, \mu_B = -0.2 \]
If an atomic BEC is

\[ U_{FB} \sim a_{FB} \int dr \ n_{BEC} n_F \] makes collapse?

FIG. 1: a) K atom number left in the trap after 50 ms hold time at a fixed magnetic field \( B \) in nondegenerate mixture \( (T=1.1 \mu K) \); b) theoretical expectation for the fermion-boson scattering length \( a_{FB} \) at the Feshbach resonance; c) width of the Rb distribution after ballistic expansion from the trap. Here the degenerate mixture is prepared at \( B=543.4 \) G (circles) or \( B=551.4 \) G (triangles) and adiabatically brought to the final field \( B \). The heating in the region from \( B=546.7 \) G to \( B=549.4 \) G is due to collapse.

LENS group, cond-mat/0606757
Summary

- pair correlations in Boson-Fermion mixed gases around a CFCFR were discussed.

perspective

- how to include atomic BEC & B-B interaction
- pair motion & Fermi molecule effect