ハドロン流体の $\eta/s$ の計算

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QGPは完全流体か？

・RHICのデータに対し流体モデルはよくあっているようにみえる。

・完全流体？？
Viscous $\sim l/L$ corrections to hydro

- Ideal hydrodynamics assumes that all the dynamics is local, it ignores the finiteness of the mean free path $l$ (or other nonzero correlation lengths).

- Including the first $O(l/L)$ dissipative terms, e.g. into boost-inv. (Bjorken) and axially symmetric hydro eqn:

$$\frac{1}{\epsilon + p} \frac{d\epsilon}{d\tau} = \frac{1}{s} \frac{ds}{d\tau} = \frac{1}{\tau} \left( 1 - \frac{\Gamma_s}{\tau} \right)$$

one finds in the r.h.s. exactly the combination which also appears in the sound attenuation length $\Gamma_s = \frac{4}{3} \eta/(\epsilon + p) = \frac{4}{3}(\eta/Ts)$

- according to Derek Teaney, hep-ph/0301099 $\Gamma_s \sim .1 \sim .2 fm$
  or $\eta/s \sim .1$, which is much lower than for all known liquids.
The theory of viscosity

- Developed for long time in the weak coupling perturbative framework large, \( \eta/T^3 \sim const/g^4 \log(1/g) \gg 1, const \sim 100 \) at small \( g << 1 \)
- If so, \( l \sim \text{few fm} \) and no hydro at RHIC! (Recall the Gyulassy-Molnar plot here)

- However, in the strong coupling (\( \mathcal{N} = 4 \) supersymmetric Yang-Mills or CFT) Polycastro, Son, Starinets, Phys. Rev. Lett. 87 (2001) 081601 (not in QCD!) their value for the sound attenuation length \( T\Gamma_s = \frac{4\eta}{3s} = \frac{1}{3\pi} \). If used for RHIC QGP \( \Gamma_s \sim 0.1 \text{ fm} \) If so, excellent hydro, but no parton cascades...

Son and Starinets (03) found several gravity backgrounds in which similar calculation is possible and found the same \( \frac{\eta}{s} = \frac{1}{4\pi} \) which is conjectured to be the universal lower bound for the most perfect liquid. The best known liquid, \( He^4 \) at high pressure, has \( \frac{\eta}{s \sim 1} \), water at normal conditions about 40.
Teanev’s Result

\[ \frac{d(\tau \sigma)}{d\tau} = \frac{4 \eta}{3 \tau T} . \]

\[ \Gamma_s \equiv \frac{\eta}{s T} . \]

\[ \frac{\Gamma_s}{\tau} \ll 1 , \]

FIG. 3. Elliptic flow \( v_2 \) as a function of \( p_T \) for different values of \( \Gamma_s/\tau_\circ \). The data points are four-particle cumulant data from the STAR Collaboration [3]. Only statistical errors are shown. The difference between the ideal and viscous curves is linearly proportional to \( \Gamma_s/\tau_\circ \).
FIG. 4. (a) Ideal blast wave fit to the experimental HBT radii $R_D$, $R_S$, and $R_L$ shown in (b) as a function of transverse momentum $K_T$. The solid symbols are from the STAR Collaboration [11] and the open symbols are from the PHENIX Collaboration [12]. For clarity, the experimental points have been slightly shifted horizontally.

FIG. 5. (a) Viscous correction $\delta R^2$ for $R_D$, $R_S$, and $R_L$ relative to ideal blast wave HBT radii $(R^2)^{(0)}$. (b) The HBT radii $R_D$, $R_S$, and $R_L$ including the viscous correction. The viscous correction is linearly proportional to $\Gamma_T/\tau_o$.  

Teaney’s Result
How about hadron gas

- very difficult to calculate
  only a several trials
- Hosoya – Kajantie (1985)
- S. Gavin (1985)
- resent model calculation ..... 

Our result by using URASiMA
URASiMA 1.

Secondary Particles

$\langle E \rangle = \frac{\alpha}{1+\alpha}$

$\frac{1}{1+\alpha}$

$N - N$

周期的な箱に入れる

ハドロン衝突

\[ \vec{x}_0(t_0^*) \]
\[ \vec{p}_j = \frac{\vec{p}_1^*}{F_1^*} \]
\[ \vec{p}_2^* = \frac{\vec{p}_2^*}{F_2^*} \]

where \[ \vec{p}_1^* = -\vec{p}_2^* \]

定常状態を実現

相対論的
分布関数のスロープ
共通の値
“温度”
\[ \varepsilon - P \text{ の関係} \]

Energy density – pressure, \( V = 10^3 \text{ fm}^3 \)

- \( \bullet \), \( n_B = 0.157 \text{ [fm}^{-3} \)
- \( \square \), \( n_B = 0.231 \text{ [fm}^{-3} \)
- \( \lozenge \), \( n_B = 0.315 \text{ [fm}^{-3} \)
- \( \Delta \), \( n_B = 0.157 \text{ [fm}^{-3} \)
- \( \triangleleft \), \( n_B = 0.231 \text{ [fm}^{-3} \)
- \( \triangledown \), \( n_B = 0.315 \text{ [fm}^{-3} \)

\( p' = p - \frac{\partial p}{\partial \varepsilon} \varepsilon \sim 0 \)

\( \eta \nu \sim 0 \)
\[ \kappa = \frac{1}{T} \int d^3x \int_{-\infty}^{t} dt' e^{-\varepsilon(t-t')} (P_x, P_x) \] (1)

\[ \eta_s = \frac{2}{T} \int d^3x \int_{-\infty}^{t} dt' e^{-\varepsilon(t-t')} (\pi_x, y, \pi_x, y) \] (2)
URASiMA 3.

Shear Viscosity

$V = 10^3 \text{ fm}^3$

- $n_B = 0.157 \text{ fm}^{-3}$
- $n_B = 0.231 \text{ fm}^{-3}$
- $n_B = 0.315 \text{ fm}^{-3}$

Graphs showing shear viscosity ($\eta$) as a function of temperature ($T$) and $\ln(T)$.
熱伝導率

\[ V = 10^3 \text{ fm}^3 \]

\[ \kappa \text{ [GeV}^2\text{]} \]

\[ T \text{ [MeV]} \]

\[ \ln \kappa \]

\[ \ln T \]

\[ n_B = 0.157 \text{ fm}^{-3} \]

\[ n_B = 0.231 \text{ fm}^{-3} \]

\[ n_B = 0.315 \text{ fm}^{-3} \]

slope 3

slope 5
Hosoya-Kajantie

- Pure-Gluon Glue-Ball gas

Fig. 2. Behaviour of $\eta$ as a function of $T$ for pure gluon matter. The behaviour in the critical region $T_c = \Lambda$ is unknown: $\eta$ or some of its derivatives may exhibit discontinuities.
Fig. 1. The first and second viscosities $\eta$ and $\zeta$, and the thermal conductivity $\kappa$ are shown, relative to the collision time, for a pion gas ($g = 3$, $m_{\pi} = 140$ MeV) as a function of temperature. The thermal conductivity is relevant only to a gas of conserved pions and is discussed in sect. 3. Note that the second viscosity is multiplied by a factor of $10^3$. 
Fig. 2. The first and second viscosities and thermal conductivity $\eta$, $\zeta$ and $\kappa$ relative to the collision time for a massive quark-anti-quark fluid ($\beta = 6$, $m = 250$ MeV) are given. Results are shown as a function of temperature for $\mu_\text{B} = 100$ MeV (fig. 2a) and $\mu_\text{B} = 150$ MeV (fig. 2b). The first viscosity and thermal conductivity shown are about 10% less than the $m = 0$ values and the second viscosity, which vanishes when $m = 0$, is now non-zero (note that the second viscosity is multiplied by $10^4$). We see, by comparison of figs. 2a and 2b, that $\kappa/\tau$ varies strongly as a function of $\mu_\text{B}$ whereas $\eta/\tau$ and $\zeta/\tau$ are roughly independent of $\mu_\text{B}$ in this temperature range.

Fig. 3. The relaxation times for viscosity and thermal conduction $\tau_\eta$ and $\tau_\zeta$ are shown as a function of temperature for the pion fluid expected in the central rapidity region. The transport coefficients $\eta$ and $\kappa$, multiplied by the temperature, are also given (left-hand scale).
VISCOSITY OF MESON MATTER

FIG. 2. Shear viscosity of the pion gas with a constant scattering amplitude (from Weinberg’s theorem). Since the interaction does not grow with the pion momentum, the viscosity is unacceptably large even for somewhat low temperatures. But this is used to check the low temperature limit.

FIG. 3. Shear viscosity of the pion gas from the simple analytical phase shifts (52) from Welke et al. [15].

both kaons and etas to the gas is finally plotted in Fig. 8. Of course, in a relativistic heavy ion collision we expect the
The density operator of a cell \( \rho_l = \rho(x^l, p^l) \) is defined as follows:

\[
\rho_l = \begin{cases} 
1 : \text{Particle exists in the } l\text{-th cell.} \\
0 : \text{The } l\text{-th cell is empty.}
\end{cases}
\]  

(1)

The definition of the entropy is given by

\[
S = - \text{Tr} \left\{ \langle \rho_l \rangle \ln \langle \rho_l \rangle \right\}, \quad \text{all cells in phase space}
\]

\[
= - \sum_l \langle \rho_l \rangle \ln \langle \rho_l \rangle,
\]

(2)

where \( \langle \rho_l \rangle \) is the ensemble average of the density operator of a cell,

\[
\langle \rho_l \rangle = \frac{1}{\text{number of ensemble states}} \sum_{\text{ensemble}} \rho_l,
\]

(3)
ハドロンガスのエントロピー

Temperature – $S/N_B$, $V = 10^3$ fm$^3$

- $n_B = 0.157$ fm$^{-3}$
- $n_B = 0.231$ fm$^{-3}$
- $n_B = 0.315$ fm$^{-3}$

![Graph](image_url)
$\eta / s$

Shear viscosity / Entropy

Temperature (MeV)

$\eta / s$ as a function of temperature. The graph shows a trend where $\eta / s$ increases with temperature.
まとめ

• ハドロンガスに関しては、

\[ \frac{\eta}{s} \sim O(1) \]

• RHIC で \( \frac{\eta}{s} \ll 1 \) は必要か？
  – Reynard数で見るべきでは？
  – Navier-Stokes を解く
粘性のある場合のScaling解

Reynolds number\textsuperscript{6) } defined by

\[ R = \tau (E + P_s) / \left( \xi_v + \frac{2}{3} \xi_s \right) \]

生成エントロピー量
ラピデティー

\[ \frac{d\Sigma}{d\eta} = \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau} \frac{1}{T} \left( \xi_v + \frac{2}{3} \xi_s \right), \]

G. Baym (1984)

Heat-up