Latent Heat in the Chiral Phase Transition

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Introduction

Phase diagram.

In this study, we compute the latent heat in the chiral phase transition based on QCD-like gauge theory using a mean field approximation.
The partition function ($Z$).

Let us consider the system of QCD:

$$
Z = N \exp \int_0^\beta d\tau \int d^3x \, L_{QCD},
$$

$$
L_{QCD} = \bar{\Psi}(i\gamma^\mu \partial_\mu + \mu \gamma^0 - m)\Psi
- g\bar{\Psi}\gamma^\mu T^a\Psi A^a_\mu + \frac{1}{2} A^{\mu a} D^{-1}_{F \mu \nu} A^{\nu a} + \text{nonlinear terms},
$$

where $\beta = T^{-1}$, $\mu$ stands for the chemical potential, $D^{-1}_{F \mu \nu} = g_{\mu \nu} \partial^\lambda \partial_\lambda - (1 - \alpha^{-1}) \partial_\mu \partial_\nu$. We neglect the nonlinear terms and integrate out the gluon field.

Using Fierz transformation, we can pick up the most attractive terms.

Then, $L_{QCD}$ reduces to $L'_{QCD}$ in Landau gauge ($\alpha = 0$):

$$
L'_{QCD} = \bar{\Psi}(x)(i\gamma^\mu \partial_\mu + \mu \gamma^0 - m)\Psi(x)
$$

$$
+ \int d^4y \frac{g^2}{18} (\bar{\Psi}(x)\Psi(y)) D_F(x - y)(\bar{\Psi}(y)\Psi(x))
$$

where $D_F(x - y) \equiv \int \frac{d^3p}{(2\pi)^3} \frac{1}{-p^2} e^{-ip(x-y)}$, $\int \frac{d^3p}{(2\pi)^3} \frac{1}{-p^2} e^{-ip(x-y)}$, $\omega_n = 2n\pi T$. 

The partition function $Z$ is given from $L'_{QCD}$ as

$$Z = N \int D\bar{\Psi} D\Psi \exp \left( d^4x L'_{QCD} \right).$$

(1)

We now introduce a bilocal auxiliary field $\varphi(x-y)$. We use the following identity:

$$1 = C \int D\varphi^* D\varphi \exp \left[ - \frac{d^4 x d^4 y}{2 g^2} \{ \varphi(x-y) - \bar{\Psi}(x)\Psi(y) \} - \frac{\Box}{18} D_F(x-y) \varphi^*(x-y) - \frac{\Box}{18} \varphi(x-y) D_F(x-y) \} \right].$$

(2)

We substitute the above expression into Eq.(1) and integrate out the fermion field. Then, we get the partition function in the auxiliary field:

$$Z(T, \mu) = N \int D\varphi^* D\varphi \exp[-S_{eff}],$$

(3)

where

$$S_{eff} = S_0 + S_1,$$

$$S_0 \equiv \int d^4 x d^4 y \left[ -\text{tr} \log \left\{ (i \gamma^\mu \partial_\mu - \mu \gamma^0 + m) \delta(x-y) \right. \right.$$}

$$\left. \left. - \frac{g^2}{18} D_F(x-y) \varphi(y-x) - \frac{g^2}{18} \varphi^*(x-y) D_F(x-y) \right\} \right]$$

$$S_1 \equiv \int d^4 x d^4 y \frac{g^2}{18} \left| \varphi(x-y) \right|^2 D_F(x-y),$$

"tr" refers to Dirac, colour SU(3) and flavour SU(3) matrices.
Hereafter we treat the case of chiral limit (m=0).

Let us switch to the momentum space.
We use the following notations:

\[ x = (-i\tau, \mathbf{x}), \quad p = (i\omega_n, \mathbf{p}), \quad \bar{p} \equiv (i\omega_n + \mu, \mathbf{p}), \]

for fermion fields \( \Psi \) and \( \varphi \),

\[ \omega_n = (2n + 1)\pi T, \]

for gluon propagator \( D_F \),

\[ \omega_n = 2n\pi T, \]

with \( n \) an integer.

Using Fourier transformation, we can express the auxiliary field as,

\[ \varphi(x - y) = \mathcal{T} \sum_{n} \frac{d^{3} p}{(2\pi)^{3}} \varphi(p) e^{-ip(x-y)}, \quad (4) \]

where \( \mathcal{T}_n \) denotes Matsubara frequency sum over \( \omega_n \).
We define the effective mass function $\Delta(x - y)$ and $\Delta(p)$ as below,

$$
\Delta(x - y) \equiv \frac{g^2}{9} D_F(x - y) \varphi(y - x)
$$

\[
= T^2 \chi \chi' \mathcal{Z} \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \varphi(p) \frac{g^2}{9} \frac{1}{q^2} e^{-i(p-q)(y-x)}
\]

\[
= T \chi^n \mathcal{Z}^m \int_n \frac{d^3p}{(2\pi)^3} \Delta(p) e^{-ip(y-x)},
\]

where

$$
\Delta(p) \equiv T \chi' \mathcal{Z} \int \frac{d^3q}{(2\pi)^3} \varphi(p+q) \frac{g^2}{9} \frac{1}{q^2}.
$$

(5)

Here, we suppose that $\varphi(p)$ is a real number.

The effective action $S_{eff}$ is transformed as follows,

$$
S_{eff} = S_0 + S_1,
$$

$$
S_0 = -2\xi V \chi \int \frac{d^3p}{(2\pi)^3} \log(\Delta^2(p) - p^2),
$$

$$
S_1 = \frac{1}{2} V \chi \int \frac{d^3p}{(2\pi)^3} \varphi(p) \Delta(p),
$$

where $\xi = N_c \square N_f$, $N_c = 3$ and $N_f = 3$. 6
Chiral phase transition.

In order to determine the mean field, we demand the following stationary condition:

$$\frac{\delta S_{\text{eff}}}{\delta \varphi(p)} |_{\varphi = \varphi^{(0)}} = 0,$$

where $\varphi^{(0)}$ denotes the mean auxiliary field.

Then, we find that the stationary condition can be satisfied insofar as

$$\varphi^{(0)}(p) = 4\xi \frac{\Delta(p)}{\Delta^2(p) - p^2},$$

$$\Delta(p) = 4T \sum_n \int \frac{d^3q}{(2\pi)^3} \frac{g^2}{-(p - q)^2 \Delta^2(q) - q^2} \frac{\Delta(q)}{\Delta^2(q) - q^2}.$$  

Eq. (10) is nothing but the Schwinger-Dyson equation in ladder approximation.

Using eq. (9), we can get the mean field effective action $S_{\text{eff}}^{(0)}$:

$$S_{\text{eff}}^{(0)} = -2\xi V \sum_n \int \frac{d^3p}{(2\pi)^3} \log(\Delta^2(p) - p^2)$$

$$+ \frac{1}{2} V \sum_n \int \frac{d^3p}{(2\pi)^3} \varphi^{(0)}(p) \Delta(p)$$

$$= 2\xi V \sum_n \int \frac{d^3p}{(2\pi)^3} \left[ - \log(\Delta^2(p) - p^2) + \frac{\Delta^2(p)}{\Delta^2(p) - p^2} \right].$$  

(9)
Schwinger-Dyson equation (S-D eq.):

\[ \Delta(p) = 4T \prod_n \frac{d^3q}{(2\pi)^3} \frac{g^2}{(p-q)^2} \frac{\Delta(q)}{\Delta^2(q) - q^2}. \]

S-D eq. has two solutions:

\( \square \) \( \Delta(p) = 0 \); symmetric phase,

\( \square \) \( \Delta(p) \neq 0 \); broken phase \( T < T_c \).

Two kinds of chiral phase transitions:

\( \square \) first-order phase transition \( \longrightarrow \) latent heat

\( \square \) second-order phase transition

The thermodynamical potential \( \Omega \):

\[ \Omega(\mu, T) \equiv -\frac{T}{V} \log Z = \frac{T}{V} S^{(0)}_{eff} \]

\[ = 2\xi T \prod_n \frac{d^3p}{(2\pi)^3} \left[ -\log(\Delta^2(p) - \bar{p}^2) + \frac{\Delta^2(p)}{\Delta^2(p) - \bar{p}^2} \right]. \]

At the critical points,

\[ \Omega_{\mu,T_c}(\Delta(p) \neq 0) = \Omega_{\mu,T_c}(\Delta(p) = 0). \]

We note the following relation: \( d\Omega = -SdT - \rho d\mu \),
where \( S \) and \( \rho \) denote the entropy and the particle density, respectively.

Therefore, using \( \Omega \), we can calculate \( S \) and \( \rho \) as

\[ S = -\frac{\partial \Omega}{\partial T}, \quad \rho = -\frac{\partial \Omega}{\partial \mu}. \]
The entropy gap ($\Delta S$):

$$Q = T_c \Delta S$$

![Graph showing the entropy gap ($\Delta S$) in 1st-order phase transition.](image)

We can calculate the latent heat ($Q_l$) from the entropy gap:

$$Q_l = T_c \Delta S.$$  (10)
\section*{Numerical calculations}

\subsection*{Gap equation}

Let us now adopt the modified running coupling ($\overline{g}$):

\[ g^2 \longrightarrow \overline{g}^2(p^2) = \frac{2\pi^2a}{\log[(-p^2 + p_R^2)/\Lambda^2_{QCD}]} \cdot (11) \]

We use the following asymptotic form of the mass function obtained through the study about $\mu = 0$ case:

\[ \Delta(p) = \frac{\sigma}{-p^2 + p_R^2}(\log[(-p^2 + p_R^2)/\Lambda^2_{QCD}])^{\frac{g}{2} - 1}, \quad (12) \]

where $a = \frac{8}{9}$, $\sigma$ is the order parameter and the parameter $p_R$ is introduced to regulate the infra-red divergence.

The S-D eq. (gap equation) is modified as

\[ \Delta(q) = 4T \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{\overline{g}^2(q^2 + p^2)}{(q - p)^2} \frac{\Delta(p)}{\Delta^2(p) - \overline{p}^2}. \quad (13) \]

Using $\sigma$ as a variational parameter, we intend to solve the gap eq. at

\[ -q^2 = \omega_0^2 = \pi^2 T^2. \quad (14) \]

We set the value of $\Lambda_{QCD}$ at 738 MeV and fix $\log(p_R^2/\Lambda^2_{QCD})$ to 0.1.
The gap equation we ought to solve is as

\[
\frac{1}{\pi^2 T^2 + p_R^2} \left[ \log \left\{ \left( \frac{\pi^2 T^2 + p^2}{\Lambda_{QCD}^2} \right) \right\} \right]^{-\frac{5}{9}}
\]

\[
\chi = 4T \sum_n Z \left| p \right|^2 \frac{\alpha}{\log \left( \frac{\left( \pi^2 T^2 + \omega_n^2 + p^2 + p_R^2 \right)}{\Lambda_{QCD}^2} \right)}
\]

\[
\phi \left( \frac{1}{(\omega_n - \pi T)^2 + p^2 \Delta^2(p) - p^2} \right)
\]

(15)
We have a 2nd-order phase transition at $T_c = 200$ MeV along the $\mu = 0$ line and a 1st-order phase transition at $\mu_c = 400$ MeV along the $T = 0$ line.

The position of the tricritical point (P) is at $(T_P = 100\text{MeV}, \mu_P = 300\text{MeV})$. 
\( \square \) Entropy

We define \( \Delta \Omega \) as follows,

\[
\Delta \Omega \equiv \Omega_{T,\mu}(\sigma \neq 0) - \Omega_{T,\mu}(\sigma = 0).
\]  

We can determine the entropy gap \( \Delta S_{Tc,\mu} \) as below,

\[
\Delta S_{Tc,\mu} = -\frac{d\Delta \Omega}{dT}_{T=Tc}.
\]

where \( T_c \) denotes the critical temperature.
Figure 7: The temperature dependence of $\Delta \Omega$ at $\mu = 350$ MeV.

The slope of the tangent of the curve at the critical point ($\Delta \Omega = 0$) is regarded as $-\Delta S_{T_c,\mu}$. 

The latent heat ($Q_l$): 

$$Q_l = T_c \Delta S_{T_c,\mu}.$$  \hspace{1cm} (18)

Figure 8 shows the dependence of $Q_l$ on $\mu$. The latent heat is produced in 1st-order phase transition ($\mu$ values are over 0.3 GeV). The maximum value is about 140 Mev at the tricritical point ($T_p = 100$MeV, $\mu_p = 300$MeV). The latent heat decreases as $\mu$ increases and becomes zero at ($T_c = 0$MeV, $\mu_c = 400$MeV) point.
Summary and discussions

We computed the latent heat in the chiral phase transition using the mean field method in the framework of the QCD-like gauge field theory.

We found that the latent heat is produced in 1st-order chiral phase transition. The value becomes more than 250 MeV near the tricritical point \((T_p = 100, \mu_p = 300)\) MeV. The latent heat decreases as \(\mu\) increases and become zero at \((T_c = 0, \mu_c = 400)\) MeV point.

It is pointed out that the values of \(T\) and \(\mu\) accomplished in high energy heavy-ion collision experiment may be close to the tricritical point. Therefore, it may be possible to observe some signals due to generated latent heat because of the fact that the latent heat has its peak near the tricritical points.